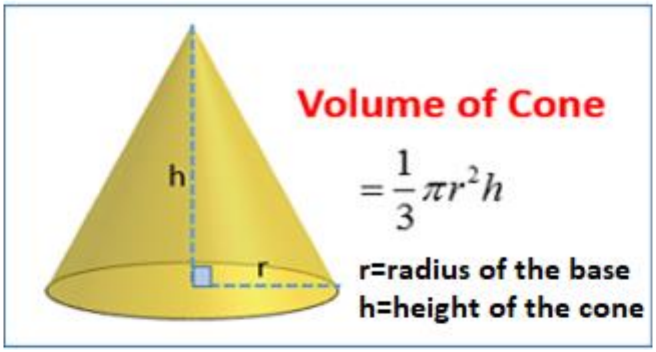
CLASS NOTES

Class: IX	Topic: CH-13, Surface area and volume Ex.13.7
Subject: Mathematics	

KEY POINT : Volume of a right circular cone = One-third of a right circular cylinder



Ex-13.7

Q.1 Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm
(ii) radius 3.5 cm, height 12 cm.

Solution:

(i)

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7 \text{ cm}^3$

$= 22 \times 2 \times 6 \text{ cm}^3 = 264 \text{ cm}^3$

(ii) Here, radius of the cone (r) = 3.5 cm $= \frac{35}{10}$ cm

Height (h) = 12 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{35}{10}\right)^2 \times 12$

$\frac{1}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times 12 \text{ cm}^3$

$= 11 \times 7 \times 2 \text{ cm}^3 = 154 \text{ cm}^3$

Q.2 Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm
(ii) height 12 cm, slant height 13 cm

Solution:

$\Rightarrow h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = 24$ cm

$\therefore$ Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\ cm^3$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24\ cm^3$

$= 22 \times 7 \times 8\ cm^3 = 1232\ cm^3$

$= \frac{1232}{1000}\ l = 1.232\ l$ $\quad [\because 1000\ cm^3 = 1\ l]$

Thus, the required capacity of the conical vessel is 1.232 l.

(ii) Here, height (h) = 12 cm and l = 13 cm

$\therefore r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144}$

$= \sqrt{25} = 5$ cm

Q.3 The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base.

Solution:

Here, height of the cone (h) = 15 cm

Volume of the cone (v) = 1570 cm^3

Let the radius of the base be 'r' cm.

$\therefore \frac{1}{3}\pi r^2 \times h = 1570$

$\Rightarrow \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$\Rightarrow \frac{1}{3} \times \frac{314}{100} \times r^2 \times 15 = 1570$

$\Rightarrow r^2 = \frac{1570 \times 3 \times 100}{314 \times 15}$

$= \frac{5 \times 3 \times 100}{15} = 100$

$\Rightarrow r^2 = 10^2$ cm

$\Rightarrow r = 10$ cm

Thus, the required radius of the base is 10 cm.

Q.4 If the volume of a right circular cone of height 9 cm is 48π cm^3, find the diameter of its base.

Solution:

Volume of the cone = 48 π cm^3

Height of the cone (h) = 9 cm

Let 'r' be its base radius.

$\therefore \frac{1}{3}\pi r^2 h = 48\pi$

$\Rightarrow \frac{1}{3}\pi r^2 \times 9 = 48\pi$

Q.5 A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Solution:

Here, diameter of the conical pit = 3.5 m

Radius (r) = $\frac{3.5}{2}$m = $\frac{35}{20}$m

Depth (h) = 12 m

Volume (capacity) = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{35}{20}\right)^2 \times 12 \text{ m}^3$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times 12 \text{ m}^3$

$= \frac{11 \times 35}{10} \text{m}^3 = \frac{385}{10} \text{m}^3$

= 38.5 m^3

$1m^3 = 1000l$

$1kl = 1000l$

$1m^3 = 1kl$

Thus, the capacity of the conical pit is 38.5 kl.

Q.6 The volume of right circular cone is 9856 cm^3. If the diameter of the be is 28 cm, find

(i) height of the cone (ii) slant height of the cone

(iii) curved surface area of the cone.

Solution:

Volume of the cone (v) = 9856 cm^3

Diameter of the base = 28 cm

$\Rightarrow$ Radius of the base = $\frac{28}{2}$ cm = 14 cm

(i) To find the height

Let the height of the cone be 'h' cm.

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h$

Thus, the required height is 48 cm.

(ii) To find the slant height

Let the slant height be 'l' cm.

∵ $(\text{Slant height})^2 = (\text{Radius})^2 + (\text{Height})^2$

∴ $l^2 = 14^2 + 48^2 = 196 + 2304 = 2500 = (50)^2$

$\Rightarrow l = 50$

Thus, the required height = 50 cm.

(iii) To find the curved surface area

∵The curved surface area of a cone is given by πrl

∴ Curved surface area = $\frac{22}{7} \times 14 \times 50 \text{ cm}^2 = 22 \times 2 \times 50 \text{ cm}^2 = 2200 \text{ cm}^2$

Thus, the curved surface area of the cone is 2200 cm^2.

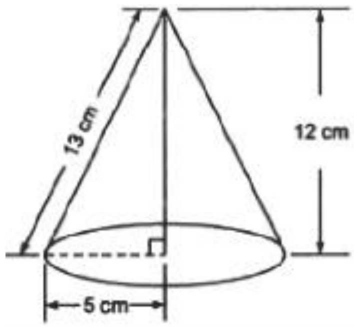
Q.7 A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Solution:

Sides of the right triangle are 5 cm, 12 cm and 13 cm.

∵The right angled triangle is revolved about the 12 cm side.

∴ Its height is 12 cm and base is 5 cm.



Thus, we have Radius of the base of the cone so formed (r) = 5 cm

Height (h) = 12 cm

Slant height = 13 cm

$\therefore$ Volume of the cone so obtained = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12 \text{ cm}^3$$

= π × 100 cm^3

= 100π cm^3

Thus, the required volume of the cone is 100π cm^3.

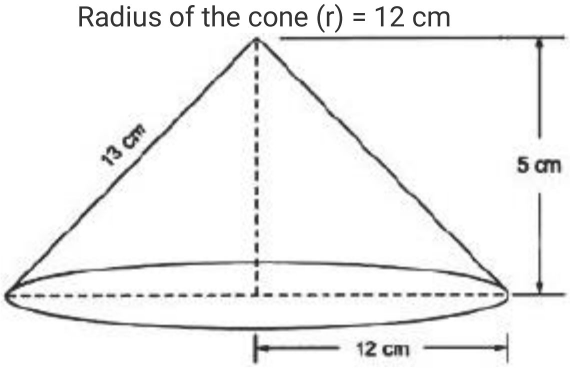
Q.8 If the triangle ABC in the question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8

Solution:

Since the right triangle is revolved about the side 5 cm.

∴ Height of the cone so obtained (h) = 5 cm

Radius of the cone (r) = 12 cm



$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times (12)^2 \times 5 \text{ cm}^3$$

$$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5 \text{ cm}^3$$

$$= \pi \times 240 \text{ cm}^3 = 240\pi \text{ cm}^3$$

Now, $\frac{[\text{Volume of the cone having } r = 5 \text{ cm}]}{[\text{Volume of the cone having } r = 12 \text{ cm and } h = 5 \text{ cm}]} = \frac{100\pi \text{cm}^3}{240\pi \text{cm}^3}$

$$= \frac{5}{12} = 5 : 12$$

Thus, the required ratio is 5 : 12.

Q.9 A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Solution:

Here the heap of wheat is in the form of a cone such that

Base diameter = 10.5 m

$\Rightarrow$ Base radius (r) $= \frac{10.5}{2} m = \frac{105}{20} m$

Height (h) = 3 m

Volume of the heap $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{105}{20}\right)^2 \times 3\ m^3$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{105}{20} \times \frac{105}{20} \times 3\ m^3 = \frac{11 \times 15 \times 105}{200} m^3 = \frac{86625}{200} m^3 = 86.625\ m^3$

Thus, the required volume = 86.625 m^3

Area of the canvas

$\because$The area of the canvas to cover the heap must be equal to the curved surface area of the conical heap.

$\therefore$ Area of the canvas $= \pi r l$ where $l = \sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{10.5}{2}\right)^2 + 3^2} = \sqrt{\frac{110.25}{4} + 9} = \sqrt{\frac{146.25}{4}}$

$= \sqrt{36.5625} = 6.046$ m (approx.)

$= 6.05$ m (approx.)

Now, $\pi r l = \frac{22}{7} \times \frac{10.5}{2} \times 6.05\ m^2 = 11 \times 1.5 \times 6.05\ m^2$

$= 99.825\ m^2$

Thus, the required area of the canvas is 99.825 m^2.

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ASSIGNMENT : Practice related sums from R.D.Sharma

NOTE: Students are advised to write these solutions in their Mathematics notebook.

“ CONTENT ABSOLUTELY PREPARED AT HOME “