CLASS NOTES

Class: XI	Topic: Set Theory Part 1
Subject: Mathematics	

Content Developed/Prepared Absolutely From Home

Set Theory

Introduction

The concept of set serves as a fundamental part of the present day mathematics. Today this concept is being usedin almost every branch of mathematics. Sets are used to define the concepts of relations and functions. The study of geometry, sequences, probability, etc. requires the knowledge of sets. The theory of sets was developed by German mathematician Georg Cantor (1845-1918). He first encountered sets while working on "problems on trigonometric series". In this Chapter, we discuss some basic definitions and operations involving sets.

Sets and their Representations

In everyday life, we often speak of collections of objects of a particular kind, such as, a pack of cards, a crowd of people, a cricket team, etc. In mathematics also, we come across collections, for example, of natural numbers, points, prime numbers, etc. More specially, we examine the following collections:
(i) Odd natural numbers less than 10, i.e., 1, 3, 5, 7, 9
(ii) The rivers of India
(iii) The vowels in the English alphabet, namely, *a, e, i, o, u*
(iv) Various kinds of triangles
(v) Prime factors of 210, namely, 2,3,5 and 7
(vi) The solution of the equation: $x^2 - 5x + 6 = 0$, viz, 2 and 3.
We note that each of the above example is a well-defined collection of objects in the sense that we can definitely decide whether a given particular object belongs to a given collection or not. For example, we can say that the river Nile does not belong to the collection of rivers of India. On the other hand, the river Ganga does belong to this colleciton.
We give below a few more examples of sets used particularly in mathematics, viz.
N : the set of all natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of real numbers
Z+ : the set of positive integers
Q+ : the set of positive rational numbers, and
R+ : the set of positive real numbers.
The symbols for the special sets given above will be referred to throughout this text.
Again the collection of five most renowned mathematicians of the world is not well-defined, because the criterion for determining a mathematician as most renowned may vary from person to person. Thus, it is not a well-defined collection.
We shall say that ***a set is a well-defined collection of objects.***
The following points may be noted :
(i) Objects, elements and members of a set are synonymous terms.
(ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.
(iii) The elements of a set are represented by small letters *a, b, c, x, y, z,* etc.
If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$ (epsilon) is used to denote the phrase '*belongs to*'. Thus, we write $a \in$ A. If 'b' is not an element of a set A, we write $b \notin$ A and read "b does not belong to A". Thus, in the set V of vowels in the English alphabet, $a \in$ V but $b \notin$ V. In the set P of prime factors of 30, $3 \in$ P but $15 \notin$ P.
There are two methods of representing a set :
(i) Roster or tabular form
(ii) Set-builder form.
(i) In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

Some more examples of representing a set in roster form are given below :(a) The set of all natural numbers which divide 42 is {1, 2, 3, 6, 7, 14, 21, 42}. In roster form, the order in which the elements are listed is immaterial. Thus, the above set can also be represented as {1, 3, 7, 21, 2, 6, 14, 42}.
(b) The set of all vowels in the English alphabet is {*a, e, i, o, u*}.
(c) The set of odd natural numbers is represented by {1, 3, 5, . . .}. The dots tell us that the list of odd numbers continue indefinitely.

It may be noted that (i) while writing the set in roster form an element is not generally repeated, i.e., all the elements are taken as distinct. For example, the set of letters forming the word 'SCHOOL' is { S, C, H, O, L} or {H, O, L, C, S}. Here, the order of listing elements has no relevance. (ii) In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {*a, e, i, o, u*}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = {$x : x$ is a vowel in English alphabet}
It may be observed that we describe the element of the set by using a symbol x (any other symbol like the letters y, z, etc. could be used) which is followed by a colon " : ". After the sign of colon, we write the characteristic property possessed by the elements of the set and then enclose the whole description within braces. The above description of the set V is read as "the set of all x such that x is a vowel of the English alphabet". In this description the braces stand for "the set of all", the colon stands for "such that". For example, the set A = {$x : x$ is a natural number and $3 < x < 10$} is read as "the set of all x such that x is a natural number and x lies between 3 and 10." Hence, the numbers 4, 5, 6, 7, 8 and 9 are the elements of the set A.
If we denote the sets described in (*a*), (*b*) and (*c*) above in roster form by A, B,C, respectively, then A, B, C can also be represented in set-builder form as follows:
A= {$x : x$ is a natural number which divides 42}
B= {$y : y$ is a vowel in the English alphabet}
C= {$z : z$ is an odd natural number}
Example Write the solution set of the equation $x^2 + x – 2 = 0$ in roster form.
Solution The given equation can be written as $(x – 1)(x + 2) = 0$, i. e., $x = 1, – 2$
Therefore, the solution set of the given equation can be written in roster form as {1, – 2}.
Example Write the set {$x : x$ is a positive integer and $x2 < 40$} in the roster form.
Solution The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is
{1, 2, 3, 4, 5, 6}.
Example Write the set A = {1, 4, 9, 16, 25, . . . } in set-builder form.
Solution We may write the set A as A = {$x : x$ is the square of a natural number} Alternatively, we can write A = {$x : x = n^2$, where $n \in \mathbf{N}$}
Example Rewrite each of the sets from the roster form into the set-builder form :
(i) {P, R, I, N, C, A, L} Ans {$x : x$ is a letter of the word PRINCIPAL}
(ii) { 0 } Ans {$x : x$ is an integer and $x + 1= 1$}
(iii) {1, 2, 3, 6, 9, 18} Ans { $x : x$ is a positive integer and is a divisor of 18}
(iv) {3, –3} Ans { $x : x$ is an integer and $x^2 – 9 = 0$}

Dear students here provided some practice questions. You are advised to solve them in your fair class note book (if you have one with you, else you can use the left pages in any previous year notebook also for the same). We will upload their solutions also through our next class notes, yet you are advised to try these questions by yourselves first. And if you don't get any question you can discuss the same with any of the mathematics teachers of our school through Phone call/ Video call also.

EXERCISE 1.1

1. Which of the following are sets ? Justify your answer.
(i) The collection of all the months of a year beginning with the letter J.
(ii) The collection of ten most talented writers of India.
(iii) A team of eleven best-cricket batsmen of the world.
(iv) The collection of all boys in your class.
(v) The collection of all natural numbers less than 100.
(vi) A collection of novels written by the writer Munshi Prem Chand.
(vii) The collection of all even integers.
(viii) The collection of questions in this Chapter.
(ix) A collection of most dangerous animals of the world.

2. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:
(i) 5. . .A (ii) 8 . . . A (iii) 0. . .A
(iv) 4. . . A (v) 2. . .A (vi) 10. . .A

3. Write the following sets in roster form:
(i) A = $\{x : x$ is an integer and $-3 \le x < 7\}$
(ii) B = $\{x : x$ is a natural number less than 6$\}$
(iii) C = $\{x : x$ is a two-digit natural number such that the sum of its digits is 8$\}$
(iv) D = $\{x : x$ is a prime number which is divisor of 60$\}$
(v) E = The set of all letters in the word TRIGONOMETRY
(vi) F = The set of all letters in the word BETTER

4. Write the following sets in the set-builder form :
(i) (3, 6, 9, 12} (ii) {2,4,8,16,32} (iii) {5, 25, 125, 625}
(iv) {2, 4, 6, . . .} (v) {1,4,9, . . .,100}

5. List all the elements of the following sets :
(i) A = $\{x : x$ is an odd natural number$\}$
(ii) C = $\{x : x$ is an integer, $x^2 \le 4\}$
(iii) D = $\{x : x$ is a letter in the word "LOYAL"$\}$
(iv) E = $\{x : x$ is a month of a year not having 31 days$\}$
(v) F = $\{x : x$ is a consonant in the English alphabet which precedes k $\}$.

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(I) {1, 2, 3, 6}	(A) {X: X IS A PRIME NUMBER AND A DIVISOR OF 6}
(II) {2, 3}	(B) {X: X IS AN ODD NATURAL NUMBER LESS THAN 10}
(III) {M, A,T, H, E, I,C, S}	(C) {X: X IS NATURAL NUMBER AND DIVISOR OF 6}
(IV) {1, 3, 5, 7, 9}	(D) {X: X IS A LETTER OF THE WORD MATHEMATICS}