# Class IX – Mathematics Worksheet: Coordinate Geometry

### Variety 1: Identifying Quadrants

**Concept:** Determining the location of a point based on the positive or negative signs of its coordinates.

**Solved Example:** In which quadrant does the point $(-2, -3)$ lie?

**Solution:** Both the x-coordinate (abscissa) and y-coordinate (ordinate) are negative. Points with the sign pattern $(-, -)$ lie in Quadrant III.

**Practice Questions:** 1. In which quadrant does the point $(4, -7)$ lie?

2. State the quadrant for the point $(-5,2)$.

3. A point has a positive abscissa and a negative ordinate. Where does it lie?

4. In which quadrant do both coordinates have a positive sign?

5. Identify the quadrant for $(-10, -10)$.

### Variety 2: Locating Points on the Axes

**Concept:** Recognizing that points with a zero coordinate fall on the axes rather than in a quadrant.

**Solved Example:** The point $(0, -5)$ lies on:

**Solution:** Since the x-coordinate is 0, the point has no horizontal displacement. It moves down 5 units on the y-axis. Therefore, it lies on the Negative y-axis.

**Practice Questions:** 1. Where does the point $(7,0)$ lie?

2. A point has an ordinate of 0 and a negative abscissa. Where is it located?

3. Where does the origin $(0,0)$ lie?

4. On which axis does $(0,12)$ lie?

5. If a point lies on the negative x-axis, what is its y-coordinate?

### Variety 3: Understanding Abscissa and Ordinate

**Concept:** Mastering the terminology: 'abscissa' refers to the x-coordinate, and 'ordinate' refers to the y-coordinate.

**Solved Example:** The ordinate of a point is its distance from:

**Solution:** The ordinate (y-coordinate) measures how far a point is vertically displaced from the x-axis.

**Practice Questions:** 1. What is the abscissa of the point $(3, -8)$?

2. Write the ordinate of the point $(-4,0)$.

3. What is the abscissa of all points lying on the y-axis?

4. If the ordinate is 5 and the abscissa is $-2$, write the coordinates.

5. Write the abscissa and ordinate of the point $(\sqrt{2}, -5)$.

### Variety 4: Perpendicular Distance from the x-axis

**Concept:** The perpendicular distance of a point from the x-axis is the absolute value of its y-coordinate.

**Solved Example:** If the coordinates of a point are $(3, -4)$, its distance from the x-axis is:

**Solution:** The distance from the x-axis is determined by the y-coordinate. Taking the absolute value of $-4$, the distance is 4 units.

**Practice Questions:** 1. Find the distance of $(7,5)$ from the x-axis.

2. What is the perpendicular distance of $(-2, -9)$ from the x-axis?

3. Find the distance of $(0,6)$ from the x-axis.

4. How far is the point $(-8,3)$ from the horizontal axis?

5. If a point is 10 units away from the x-axis and lies in Quadrant IV, what is its ordinate?

#### Variety 5: Perpendicular Distance from the y-axis

**Concept:** The perpendicular distance of a point from the y-axis is the absolute value of its x-coordinate.

**Solved Example:** The perpendicular distance of the point $P(3,4)$ from the y-axis is:

**Solution:** The distance from the y-axis is determined by the x-coordinate. The absolute value of 3 is 3 units.

**Practice Questions:** 1. Find the distance of $(-5,8)$ from the y-axis.

2. What is the perpendicular distance of $(12, -2)$ from the vertical axis?

3. How far is $(0, -7)$ from the y-axis?

4. Determine the distance of $(-4, -4)$ from the y-axis.

5. A point is 6 units from the y-axis and lies in Quadrant II. What is its abscissa?

#### Variety 6: Distance from the Origin

**Concept:** Using the Pythagorean theorem to find the distance of a point $(x, y)$ from $(0,0)$ using the formula $d = \sqrt{x^2 + y^2}$.

**Solved Example:** The distance of the point $(4,3)$ from the origin is:

**Solution:** Apply the formula: $d = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ units.

**Practice Questions:** 1. Find the distance of $(6,8)$ from the origin.

2. Calculate the distance of $(-5,12)$ from the origin.

3. How far is the point $(8, -15)$ from $(0,0)$?

4. Find the distance of $(-3, -4)$ from the origin.

5. What is the distance of $(1,1)$ from the origin?

#### Variety 7: Distance Between Any Two Points

**Concept:** Applying the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**Solved Example:** Find the distance between $A(0,6)$ and $B(0, -2)$.

**Solution:** $d = \sqrt{(0 - 0)^2 + (-2 - 6)^2} = \sqrt{0 + (-8)^2} = \sqrt{64} = 8$ units.

**Practice Questions:** 1. Find the distance between $(2,3)$ and $(5,7)$.

2. Calculate the distance between $(-1,-1)$ and $(2,3)$.

3. How far apart are the points $(4,0)$ and $(0,-3)$?

4. Find the distance between $(a,b)$ and $(-a,-b)$.

5. Calculate the distance between $(1,2)$ and $(4,6)$.

**Variety 8: Midpoint Formula**

**Concept:** Finding the exact center of a line segment using $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

**Solved Example:** The midpoint of the line segment joining $(2,4)$ and $(6,8)$ is:

**Solution:** Midpoint $= (\frac{2+6}{2}, \frac{4+8}{2}) = (\frac{8}{2}, \frac{12}{2}) = (4,6)$.

**Practice Questions:** 1. Find the midpoint of $(0,0)$ and $(8,10)$.

2. Calculate the midpoint of $(-3,5)$ and $(7,-1)$.

3. Find the midpoint of $(a+b, a-b)$ and $(a-b, a+b)$.

4. Determine the center of a circle if the endpoints of its diameter are $(4,-2)$ and $(-4,2)$.

5. Find the midpoint between $(-5,-5)$ and $(5,5)$.

**Variety 9: Finding an Unknown Coordinate using Midpoint**

**Concept:** Working backward from the midpoint formula to find a missing endpoint or variable.

**Solved Example:** Find $k$ if the midpoint of $(k,4)$ and $(2,6)$ is $(4,5)$.

**Solution:** Using the x-coordinate midpoint formula: $\frac{k+2}{2} = 4$. Multiplying by 2: $k+2=8$, so $k=6$.

**Practice Questions:** 1. If the midpoint of $(x,3)$ and $(5,7)$ is $(2,5)$, find $x$.

2. The midpoint of $A(-2,y)$ and $B(4,8)$ is $(1,4)$. Find $y$.

3. If the origin is the midpoint of $(a,4)$ and $(-3,b)$, find $a$ and $b$.

4. Find the coordinates of point $P$ if $Q(3,2)$ is the midpoint of $PR$ and $R$ is $(7,6)$.

5. If the midpoint of $(2k,4)$ and $(6,8)$ is $(5,6)$, what is the value of $k$?

**Variety 10: Collinearity of Three Points**

**Concept:** Points are collinear if they lie on the same straight line, meaning the sum of two distances equals the third.

**Solved Example:** Determine if the points $(1,5)$, $(2,3)$, and $(-2,-11)$ are collinear.

**Solution:** $AB = \sqrt{5}$. $BC = 2\sqrt{53}$. $AC = \sqrt{265}$. Since $AB + BC \neq AC$, they are not collinear.

**Practice Questions:** 1. Check if $(1,1)$, $(2,2)$, and $(3,3)$ are collinear.

2. Determine if $(-1,-1)$, $(0,0)$, and $(5,5)$ form a straight line.

3. Are the points $(0,2)$, $(2,0)$, and $(1,1)$ collinear?

4. Show that points $(1,2)$, $(-5,6)$, and $(a,-2)$ are collinear if they lie on a straight line.

5. Prove that $(3,1)$, $(6,4)$, and $(8,6)$ are collinear.

**Variety 11: Equidistant Points**

**Concept:** Setting two distance formulas equal to each other to find an unknown coordinate.

**Solved Example:** Find a point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$.

**Solution:** Let point be $(x,0)$. $(x-2)^2+25=(x+2)^2+81$. $-8x=56 \Rightarrow x=-7$. Point is $(-7,0)$.

**Practice Questions:** 1. Find a point on the y-axis equidistant from $(3,4)$ and $(-3,6)$.

2. What point on the x-axis is equidistant from $(5,4)$ and $(-2,3)$?

3. Find $y$ if $(0,y)$ is equidistant from $(1,2)$ and $(3,4)$.

4. Determine the point on the x-axis equidistant from $(7,6)$ and $(-3,4)$.

5. Find a point on the y-axis equidistant from $(-5,-2)$ and $(3,2)$.

**Variety 12: Identifying Triangles**

**Concept:** Calculating the lengths of all three sides to determine if a triangle is equilateral, isosceles, scalene, or right-angled.

**Solved Example:** Check whether the triangle with vertices $(2,2)$, $(-2,-2)$ and $(-2\sqrt{3},2\sqrt{3})$ is equilateral. **Solution:** All three sides calculate to a length of $4\sqrt{2}$. Yes, it is equilateral.

**Practice Questions:** 1. Show that the points $(0,0)$, $(3,0)$, and $(0,4)$ form a right-angled triangle.

2. Determine the type of triangle formed by $(1,1)$, $(4,1)$, and $(1,5)$.

3. Check if $(-3,0)$, $(3,0)$, and $(0,4)$ form an isosceles triangle.

4. Do the vertices $(2,4)$, $(2,6)$, and $(2,8)$ form a triangle?

5. Identify the triangle with vertices $(0,0)$, $(a,0)$, and $(0,a)$.

**Variety 13: Identifying Quadrilaterals**

**Concept:** Using distance and midpoint formulas to prove shapes like squares, rectangles, and rhombuses.

**Solved Example:** Show that the points $(1,7)$, $(4,2)$, $(-1,-1)$ and $(-4,4)$ are vertices of a square.

**Solution:** All sides equal $\sqrt{34}$. Both diagonals equal $\sqrt{68}$. It is a square.

**Practice Questions:** 1. Prove that $(0,0)$, $(4,0)$, $(4,3)$, and $(0,3)$ form a rectangle.

2. Show that $(-2,-1)$, $(1,0)$, $(4,3)$, and $(1,2)$ form a parallelogram.

3. Check if $(2,1)$, $(5,4)$, $(4,7)$, and $(1,4)$ form a rhombus.

4. Do the points $(1,1)$, $(1,5)$, $(5,5)$, and $(5,1)$ form a square?

5. Determine the shape formed by $(0,0)$, $(a,0)$, $(a,b)$, and $(0,b)$.

**Variety 14: Finding the Fourth Vertex of a Parallelogram**

**Concept:** Utilizing the property that the diagonals of a parallelogram bisect each other.

**Solved Example:** Find fourth vertex $D$ if three vertices are $A(1,2)$, $B(4,3)$, and $C(6,6)$.

**Solution:** Midpoint of $AC=(3.5,4)$. Equating to midpoint of $BD(x,y)$: $\frac{4+x}{2}=3.5 \Rightarrow x=3$. $\frac{3+y}{2}=4 \Rightarrow y=5$. Vertex is $(3,5)$.

**Practice Questions:** 1. Three vertices of a parallelogram are $(0,0)$, $(3,0)$, and $(4,4)$. Find the fourth.

2. Find the fourth vertex if three are $(-1,2)$, $(3,1)$, and $(2,-3)$.

3. A rectangle has vertices at $(0,0)$, $(5,0)$, and $(0,8)$. Find the fourth vertex.

4. Find the missing vertex of a rhombus with known vertices $(1,1)$, $(3,4)$, and $(5,1)$.

5. If the origin is the center of a square, and one vertex is $(4,4)$, find the opposite vertex.

#### Variety 15: Area of a Triangle using Coordinates

**Concept:** Calculating area using $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$.

**Solved Example:** The vertices are $(1,4)$, $(2,-3)$, and $(-1,-2)$. Find its area.

**Solution:** Area $= 0.5|1(-3-(-2))+2(-2-4)+(-1)(4-(-3))| = 0.5|-1-12-7| = 10$ sq units.

**Practice Questions:** 1. Find the area of a triangle with vertices $(0,0)$, $(4,0)$, and $(0,3)$.

2. Calculate the area of a triangle at $(1,1)$, $(5,1)$, and $(3,6)$.

3. Determine the area of the triangle formed by $(-2,4)$, $(3,-1)$, and $(1,5)$.

4. What is the area of a triangle with vertices $(0,5)$, $(0,-5)$, and $(4,0)$?

5. Prove that the area of a triangle with collinear vertices is 0 using points $(1,1)$, $(2,2)$, and $(3,3)$.

#### Variety 16: Reflection across the x-axis

**Concept:** Reflecting across the x-axis flips the sign of the y-coordinate.

**Solved Example:** Reflecting the point $(2,-3)$ in the x-axis yields:

**Solution:** Keep $x = 2$, change $y$ to 3. The reflected point is $(2,3)$.

**Practice Questions:** 1. Find the reflection of $(4,7)$ across the x-axis.

2. What is the image of $(-5,-2)$ after a reflection in the x-axis?

3. Reflect $(0,6)$ across the horizontal axis.

4. If a point $(a,b)$ is reflected across the x-axis, write its new coordinates.

5. Find the reflection of $(-8,1)$ across the x-axis.

#### Variety 17: Reflection across the y-axis and Origin

**Concept:** Reflecting across the y-axis flips the x-coordinate. Reflecting across the origin flips both.

**Solved Example:** What is the image of $P(x,y)$ across the origin?

**Solution:** A reflection across the origin negates both coordinates. The image is $(-x,-y)$.

**Practice Questions:** 1. Reflect the point $(3,5)$ across the y-axis.

2. Find the image of $(-4,7)$ reflected across the origin.

3. What is the reflection of $(-2,-9)$ across the vertical axis?

4. Reflect $(6,0)$ across the y-axis.

5. If $(x,-y)$ is reflected across the origin, what are the new coordinates?

**Variety 18: Points on Basic Linear Equations**

**Concept:** Checking if a coordinate pair satisfies a given algebraic equation.

**Solved Example:** Which point lies on the line $y = x$?

**Solution:** The abscissa and ordinate must be identical, such as (3,3).

**Practice Questions:** 1. Which of these points lies on $x + y = 0$: (2,2) or $(2, -2)$?

2. Verify if (3,4) lies on the line $y = x + 1$.

3. Find a point that lies on the line $x = 5$.

4. Does the origin lie on the line $y = 2x$?

5. Which point satisfies $y = 2x - 1$: (1,1) or (2,4)?

**Variety 19: Ratio of Division (Section Formula concept)**

**Concept:** Finding the ratio in which an axis divides a line segment.

**Solved Example:** Find the ratio in which the y-axis divides the segment joining $(5, -6)$ and $(-1, -4)$.

**Solution:** x-coordinate is 0. $0 = \frac{k(-1)+1(5)}{k+1} \Rightarrow k = 5$. Ratio is 5: 1.

**Practice Questions:** 1. Find the ratio in which the x-axis divides the segment joining (2,3) and $(4, -6)$.

2. In what ratio does the y-axis divide the segment from $(-3,5)$ to (4,7)?

3. Find the ratio in which the x-axis cuts the line between $(-1, -4)$ and (5,2).

4. Determine the ratio in which the vertical axis divides the line from (6,1) to $(-2,3)$.

5. Find the ratio in which the origin divides the segment joining (2,2) and $(-2, -2)$.

**Variety 20: Points on a Circle**

**Concept:** Verifying if a point lies on a circle by checking its distance to the center.

**Solved Example:** Verify if the point (3,4) lies on the circle with center origin and radius 5.

**Solution:** Distance $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Since distance equals radius, it lies on the circle.

**Practice Questions:** 1. Does (5,12) lie on a circle centered at the origin with radius 13?

2. Verify if $(-6,8)$ lies on a circle centered at (0,0) with radius 10.

3. Check if (1,1) lies on a circle with center (0,0) and radius 2.

4. A circle has its center at (2,2) and radius 3. Does (2,5) lie on it?

5. Prove that $(a, b)$ lies on the circle $x^2 + y^2 = r^2$ if $a^2 + b^2 = r^2$.

**Variety 21: Symmetry and Mirror Lines**

**Concept:** Plotting points to visually identify the line of symmetry between them.

**Solved Example:** Plot (2,3) and $(2, -3)$. What line are they symmetric about?

**Solution:** They share the same x-coordinate but opposite y-coordinates. They are symmetric about the x-axis.

**Practice Questions:** 1. What line are $(-4,5)$ and $(4,5)$ symmetric about?

2. Find the line of symmetry for $(1,2)$ and $(-1,-2)$.

3. Points $(a,b)$ and $(a,-b)$ are symmetric about which axis?

4. What line acts as a mirror for $(0,5)$ and $(0,-5)$?

5. Are $(3,3)$ and $(-3,-3)$ symmetric about the x-axis, y-axis, or origin?

#### Variety 22: Generating Coordinates based on Distance Limits

**Concept:** Finding all possible locations based on absolute distances from the axes.

**Solved Example:** A point is 4 units from the x-axis and 3 units from the y-axis. Find all possible coordinates.

**Solution:** $|y| = 4$, $|x| = 3$. Combinations: $(3,4), (-3,4), (3,-4), (-3,-4)$.

**Practice Questions:** 1. Find all points 5 units from the y-axis and 2 units from the x-axis.

2. A point is 7 units from the horizontal axis and 0 units from the vertical. What are its coordinates?

3. List the coordinates of points 1 unit away from both axes.

4. A point lies in Quadrant II, 4 units from the x-axis and 6 units from the y-axis. Find it.

5. If a point is 8 units from the y-axis and lies on the x-axis, what are its possible coordinates?

#### Variety 23: Geometric Proofs using Coordinates

**Concept:** Using general coordinates to prove universal geometric theorems.

**Solved Example:** Prove diagonals of a rectangle with vertices $(0,0), (a,0), (a,b), (0,b)$ are equal.

**Solution:** Diagonals are $\sqrt{(a-0)^2+(b-0)^2} = \sqrt{a^2+b^2}$ and $\sqrt{(0-a)^2+(b-0)^2} = \sqrt{a^2+b^2}$. They are equal.

**Practice Questions:** 1. Prove that the diagonals of a square at $(0,0), (a,0), (a,a)$,and $(0,a)$ are equal.

2. Show that the midpoint of the hypotenuse of a right triangle at $(0,0), (a,0)$,and $(0,b)$ is equidistant from all vertices.

3. Prove the diagonals of the rectangle in the solved example bisect each other.

4. Use coordinates to prove that a triangle with vertices $(0,0), (2a,0)$,and $(a,a\sqrt{3})$ is equilateral.

5. Prove that the line joining the midpoints of two sides of a triangle at $(0,0), (a,0)$,and $(b,c)$ is parallel to the base.

#### Variety 24: Real-World Case Study - Distance Modeling

**Concept:** Applying the distance formula to real-world map grids.

**Solved Example:** A drone flies from $(1,1)$ to $(7,9)$. Find the travel distance.

**Solution:** $d = \sqrt{(7-1)^2+(9-1)^2} = \sqrt{36+64} = 10$ units.

**Practice Questions:** 1. A fountain is at $(0,0)$ and a bench is at $(4,5)$. Find the distance between them.

2. A boat travels from $(-2,-3)$ to $(1,1)$. How far did it travel?

3. Calculate the distance a robot moves from $(5,2)$ to $(17,7)$.

4. A laser is pointed from $(0,5)$ to $(12,0)$. What is the length of the beam?

5. Find the distance between two archaeological artifacts located at $(10,10)$ and $(13,14)$.

**Variety 25: Real-World Case Study - Midpoint Navigation**

**Concept:** Using the midpoint formula to find halfway points in real-world scenarios.

**Solved Example:** Rahul is at $(2,4)$ and Sam is at $(8,4)$. Find coordinates of the flag placed exactly in the middle.

**Solution:** Midpoint $= (\frac{2+8}{2}, \frac{4+4}{2}) = (5,4)$.

**Practice Questions:** 1. A drone flies from $(1,1)$ to $(7,9)$. Find the coordinates of the photo-stop at the exact midpoint.

2. A hospital is at $(2,3)$ and a school at $(10,9)$. Find the coordinates of a fire station at the midpoint.

3. Two players are standing at $(-4,-6)$ and $(6,8)$. Where should the referee stand to be exactly between them?

4. A bridge spans from $(0,10)$ to $(20,10)$. What are the coordinates of the middle pillar?

5. Find the halfway resting point for a hiker traveling straight from basecamp $(0,0)$ to the peak $(14,24)$.

**Variety 26: Real-World Case Study - Shape Identification**

**Concept:** Identifying polygons formed by physical landmarks on a grid.

**Solved Example:** Artifacts are at $A(1,2)$, $B(3,4)$, and $C(5,6)$. Are they in a straight line?

**Solution:** Slope $AB = \frac{4-2}{3-1} = 1$. Slope $BC = \frac{6-4}{5-3} = 1$. Because slopes are equal, they form a straight line.

**Practice Questions:** 1. Digital Art: A logo has vertices $(0,0)$, $(2,3)$, $(4,0)$, and $(2,-3)$. Identify the quadrilateral.

2. Sports Day: Rahul is at $(2,4)$, Sam at $(8,4)$, and Tina at $(5,8)$. Identify the type of triangle they form.

3. Architect's Grid: A fountain is at $(0,0)$, a bench at $(4,5)$, and a swing at $(-4,5)$. Identify the shape formed.

4. A farm is plotted with corners at $(0,0)$, $(0,100)$, $(50,100)$, and $(50,0)$. What shape is the farm?

5. Four boundary markers are placed at $(2,2)$, $(6,2)$, $(6,6)$, and $(2,6)$. Verify if the plot is a square.

# Class IX - Mathematics Worksheet: Introduction to Polynomials & Linear Models

**Variety 1: Differentiating Algebraic Expressions and Polynomials**

**Concept:** A polynomial is an algebraic expression where all variables have non-negative integer exponents.

**Solved Example:** Differentiate between an algebraic expression and a polynomial using an example.

**Solution:** $x^{\frac{1}{2}} + 2$ is an algebraic expression but not a polynomial because the exponent is a fraction. $x^2 + 2$ is both an algebraic expression and a polynomial.

**Practice Questions:** 1. Is $\sqrt{y} + 5$ a polynomial or just an algebraic expression?

2. Why is $\frac{3}{x^2}$ considered an algebraic expression but not a polynomial?

3. Classify $5x^3 - 2x$ as an expression, a polynomial, or both.

4. Give an example of an expression with two variables that is not a polynomial.

5. Explain why $2^x$ is not a polynomial.

**Variety 2: Identifying Valid Polynomial Expressions**

**Concept:** Determining which expressions strictly meet the definition of a polynomial.

**Solved Example:** Is the expression $4x^{-2} + 3$ a polynomial? State yes or no.

**Solution:** No, because it contains a negative integer exponent $(-2)$.

**Practice Questions:** 1. Which of the following is a valid polynomial expression: $3x^2 + \frac{1}{x}$ or $7x^3 - 2x + 9$?

2. Is $x^{\frac{1}{3}} + 4$ a polynomial?

3. Is $\sqrt{3}x^2$ a valid polynomial? Why or why not?

4. Determine if $5x^4 - 2x^3 + 0x^2 + 7$ is a valid polynomial.

5. Is $\frac{5}{x^2} + x$ a polynomial?

**Variety 3: Determining the Degree of a Polynomial**

**Concept:** The degree is the highest power of the variable present.

**Solved Example:** Identify the degree of the polynomial: $12x^4 - 5x^6 + 2x$.

**Solution:** The highest power of $x$ is 6, so the degree is 6.

**Practice Questions:** 1. What is the degree of the polynomial $8x^5 - 3x^2 + 12$?

2. Determine the degree of $7x - x^3$.

3. Find the degree of the zero polynomial.

4. What is the degree of $y^3 - 2y^2 + y - 1$?

5. State the degree of $8 - 4x^5 + x^2 - 3x$.

**Variety 4: Identifying Coefficients**

**Concept:** A coefficient is the numerical multiplier of a specific variable term.

**Solved Example:** Write the coefficient of $x^3$ in the polynomial: $8 - 4x^3 + x^2$.

**Solution:** The coefficient of the $x^3$ term is $-4$.

**Practice Questions:** 1. The coefficient of the $x^2$ term in $-6x^3 + x^2 - 4x + 2$ is what?

2. State the coefficient of $x$ in $5 - 3x + x^2$.

3. State the coefficient of $x$ in $\frac{x}{4} + 7$.

4. Identify the coefficient of $x^4$ in $9x^4 - x^3 + 2x - 5$.

5. What is the leading coefficient of $-2x^4 + 5x^3 - x + 9$?

**Variety 5: Identifying the Constant Term**

**Concept:** The constant term does not contain any variables.

**Solved Example:** Write the constant term of the polynomial $-7x^3 + 4x - 9$.

**Solution:** The constant term is $-9$.

**Practice Questions:** 1. Identify the constant term in $3 - 5x^2 + x$.

2. What is the constant term in $8 - 4x^3 + x^2$?

3. What is the constant term in $9x^4 - x^3 + 2x - 5$?

4. Find the constant term in $y^3 - 2y^2 + y - 1$.

5. Find the constant term in $-2x^4 + 5x^3 - x + 9$.

**Variety 6: Classifying Polynomials by Number of Terms**

**Concept:** Classifications include monomials (1), binomials (2), or trinomials (3).

**Solved Example:** Classify the polynomial $3x^2 - 7$ as a monomial, binomial, or trinomial.

**Solution:** It has two terms, so it is a binomial.

**Practice Questions:** 1. If a polynomial has only two terms, what is it called?

2. How many terms does the polynomial $x^5 - 1$ have?

3. Classify $7x - x^3$ based on its number of terms.

4. Write a monomial of degree 7.

5. Identify the terms in $5a - 3b + 7$.

**Variety 7: Classifying Polynomials by Degree**

**Concept:** Classifications include constant (0), linear (1), quadratic (2), and cubic (3).

**Solved Example:** Which of these is a polynomial of degree 0: $0, x, 14, \frac{1}{x}$?

**Solution:** 14 is a constant polynomial of degree 0.

**Practice Questions:** 1. Classify $3x^2 - 2x + 5$ by its degree.

2. Classify $y = -9x + 4$ by its degree.

3. Classify $8 - 4x^3 + x^2$ by its degree.

4. Classify the geometric perimeter polynomial $4x + 6$ by its degree.

5. Formulate a binomial of degree 3 that has a leading coefficient of $-4$.

**Variety 8: Writing Polynomials in Standard Form**

**Concept:** Arranging terms from the highest exponent to the lowest.

**Solved Example:** What is the standard form of the polynomial $3 - 5x^2 + x$?

**Solution:** $-5x^2 + x + 3$.

**Practice Questions:** 1. Write the polynomial $4x - 2x^3 + 5$ in standard form.

2. Rewrite $8 - 4x^5 + x^2 - 3x$ in standard form.

3. Write $p(x) = -2x^4 + 5x^3 - x + 9$ in standard form.

4. Arrange $9x - 2x^5 + 4x^3 - 7 + x^2$ in standard form and list its coefficients in descending order.

5. Rewrite $x^2 + 4x - 2 - x^3$ in standard form.

**Variety 9: Evaluating a Polynomial at a Specific Value**

**Concept:** Substituting a numerical value for the variable to compute a result.

**Solved Example:** Evaluate the polynomial $p(x) = 3x - 4$ at $x = 0$.

**Solution:** $p(0) = 3(0) - 4 = -4$.

**Practice Questions:** 1. In the polynomial $p(x) = 4x - 7$, what is $p(2)$?

2. Find the value of $y$ when $x = 2$ for the equation $y = -3x + 10$.

3. Evaluate $p(x) = -4x + 12$ at $x = -1$.

4. Evaluate $p(y) = y^3 - 2y^2 + y - 1$ at $y = 2$.

5. If $p(t) = 2t^2 - t + 3$, calculate $p(\frac{1}{2})$.

**Variety 10: Comparing Values of Two Polynomials**

**Concept:** Evaluating multiple polynomials at a given point to determine relative magnitude.

**Solved Example:** Evaluate $p(x) = 3x^3 - 4x^2 + 2x - 5$ and $q(x) = 2x^3 + x^2 - x + 1$ at $x = 2$. Which yields a higher value?

**Solution:** $p(2) = 3(8) - 4(4) + 2(2) - 5 = 24 - 16 + 4 - 5 = 7$.

$q(2) = 2(8) + 4 - 2 + 1 = 16 + 4 - 2 + 1 = 19$. $q(x)$ yields a higher value.

**Practice Questions:** 1. Evaluate $A = 4x^3 - 2x^2 + 7x - 1$ and $B = -4x^3 + 5x^2 - 3x + 6$ at $x = 1$. Which is greater?

2. Compare $p(x) = x^2 - 4$ and $q(x) = 2x - 4$ at $x = 0$.

3. Compare $y = 5x + 10$ and $y = 2^x$ at $x = 2$.

4. Compare $p(t) = 2t^2 - t + 3$ and $q(t) = -4t + 12$ at $t = 1$.

5. Evaluate $C(x) = 120x + 5000$ and $R(x) = 250x + 500$ at $x = 10$. Which is higher?

**Variety 11: Finding Roots of a Polynomial**

**Concept:** A root is a value that makes the expression evaluate to zero.

**Solved Example:** Check if $x = 2$ is a root (evaluates to 0) of the polynomial $p(x) = x^2 - 4$.

**Solution:** $p(2) = (2)^2 - 4 = 4 - 4 = 0$. Yes, it is a root.

**Practice Questions:** 1. Is $x = 3$ a root of $p(x) = -4x + 12$?

2. Is $x = \frac{7}{4}$ a root of $p(x) = 4x - 7$?

3. Is $t = 25$ a root of $h(t) = 100 - 4t$?

4. Is $d = 20.83$ a root of $V(d) = 25000 - 1200d$?

5. Find the root of $y = 2x - 4$.

**Variety 12: Properties of Constant and Zero Polynomials**

**Concept:** Understanding polynomials with no variables (degree 0) or the zero polynomial (undefined degree).

**Solved Example:** Why is the degree of any non-zero constant polynomial 0?

**Solution:** A constant $c$ can be written as $c \cdot x^0$. The highest power of $x$ is 0.

**Practice Questions:** 1. What is the slope of a horizontal line representing a constant polynomial $y = 8$?

2. What is the degree of the zero polynomial?

3. Does $y = 8$ represent a straight line or a curve?

4. If a polynomial evaluates to 14 for all $x$, what is its degree?

5. What is the y-intercept of $y = 0$?

**Variety 13: Identifying the Slope in Linear Equations**

**Concept:** In $y = ax + b$, '$a$' is the slope.

**Solved Example:** The slope of the linear relationship $y = 15 + 3x$ is what?

**Solution:** The slope is 3.

**Practice Questions:** 1. What is the slope in the linear equation $y = -9x + 4$?

2. Find the slope of $y = -4x + 7$.

3. Identify the slope in $y = 50 - 0.08x$.

4. What is the slope of $y = 12$?

5. Graph the linear equation $y = 2x - 4$ by identifying its slope.

**Variety 14: Identifying the Y-Intercept in Linear Equations**

**Concept:** In $y = ax + b$, '$b$' is the y-intercept.

**Solved Example:** In the linear equation $y = -4x + 7$, what is the y-intercept?

**Solution:** The y-intercept is 7.

**Practice Questions:** 1. Identify the y-intercept in the equation $y = 6x - 11$.

2. Evaluate the y-intercept of the polynomial $p(x) = -2x + 15$.

3. Identify the y-intercept of $y = 15 + 3x$.

4. State the y-intercept of $y = 2x - 4$.

5. Find the y-intercept of $C(x) = 120x + 5000$.

**Variety 15: Differentiating Linear Growth and Decay**

**Concept:** A positive slope indicates growth; a negative slope indicates decay.

**Solved Example:** If $y = ax + b$ represents linear growth, what must be true about '$a$'?

**Solution:** $a > 0$.

**Practice Questions:** 1. Which of the following models linear decay: $y = 5x + 10$ or $y = -3x + 20$?

2. State whether $y = 14x$ represents linear growth or linear decay.

3. Differentiate between linear growth and linear decay based on the value of the slope.

4. A water tank drains at 5 litres per minute. This is an example of what?

5. Is $V(t) = -2000t + 30000$ linear growth or decay?

**Variety 16: Modelling Linear Growth (Word Problems)**

**Concept:** Formulating models where quantities increase constantly over time.

**Solved Example:** A plant starts at 5 cm and grows 2 cm per week. The linear model for its height is what?

**Solution:** $y = 2x + 5$.

**Practice Questions:** 1. Write a linear polynomial that models a growth rate of 5 units per day starting from 0.

2. A fitness club charges a ₹ 30 membership fee plus ₹ 15 per month. Formulate the linear polynomial for total cost.

3. Write a linear model for a savings account starting with ₹ 500 and adding ₹ 75 every month.

4. A tree is 1.5 meters tall and grows 0.2 meters every month. Model this linear growth.

5. Kiran opens a bank account with ₹ 8,000 and deposits ₹ 1,500 every month. Write a linear polynomial representing the balance.

**Variety 17: Modelling Linear Decay (Word Problems)**

**Concept:** Formulating models where quantities decrease constantly over time.

**Solved Example:** A car's fuel tank has 50 litres. It consumes 0.08 litres per km. The linear model for the remaining fuel is what?

**Solution:** $y = 50 - 0.08x$.

**Practice Questions:** 1. Give an example of a linear decay equation in the form $y = ax + b$.

2. If a population decreases by 100 people a year from 5000, write the linear model.

3. A balloon is descending at 4 meters per second from a height of 150 meters. Model its height.

4. A candle is 25 cm tall and burns at a rate of 2 cm per hour. Write a linear polynomial to model its height.

5. A laptop battery is fully charged at 100% and discharges at 12% per hour. Write a linear decay polynomial.

**Variety 18: Real-World Interpretation of Slope**

**Concept:** The slope physically represents the constant rate of change per unit of the independent variable.

**Solved Example:** A city cab service charges a fixed base fare of ₹ 50, plus ₹ 15 for every kilometer traveled modeled by $y = 15d + 50$. Explain what the coefficient represents physically.

**Solution:** The coefficient (15) represents the rate of change: the cost per kilometer traveled.

**Practice Questions:** 1. Explain the meaning of the slope in the context of a real-world linear model.

2. In $C(x) = 120x + 5000$, what is the cost per unit?

3. In $h(t) = 100 - 4t$, what does -4 represent?

4. In $C(m) = 0.05m + 20$, what does 0.05 represent?

5. How do changes in 'a' (large vs small) affect the visual graph of a line?

**Variety 19: Real-World Interpretation of Y-Intercept**

**Concept:** The y-intercept represents the initial value before any rate of change is applied.

**Solved Example:** Discuss how the y-intercept acts as the 'initial value' in practical linear models.

**Solution:** At $x = 0$, the $ax$ term becomes 0, leaving only $b$. It represents the starting quantity or fixed cost.

**Practice Questions:** 1. What does the variable 'b' represent in the linear model $y = ax + b$?

2. In $C(x) = 120x + 5000$, what is the fixed setup cost?

3. In $h(t) = 100 - 4t$, what does 100 represent?

4. What does ₹500 represent geometrically in a savings model starting with ₹500?

5. Graphically, what would the y-intercept of the balloon model $A(m) = -150m + 2000$ represent?

**Variety 20: Identifying Independent and Dependent Variables**

**Concept:** Distinguishing the input variable (independent) from the output variable (dependent).

**Solved Example:** Which variable represents the independent variable in the model $y = 4x + 10$?

**Solution:** The independent variable is $x$.

**Practice Questions:** 1. Identify the independent and dependent variables in $Cost = 5 \times Items + 20$.

2. In $V(d) = 25000 - 300d$, determine the dependent and independent variables.

3. In $H(n) = 40n + 500$, identify the independent variable.

4. In $C(m) = 0.05m + 20$, which is the dependent variable?

5. In $y = 15d + 50$, which is the independent variable?

**Variety 21: Finding Future Values (Evaluating Linear Models)**

**Concept:** Predicting future outcomes based on constant rates of change.

**Solved Example:** If the revenue from selling 'n' items is $R(n) = 25n$, find the revenue for 40 items.

**Solution:** $R(40) = 25(40) = 1000$.

**Practice Questions:** 1. A taxi charges $y = 2.50x + 5$. Calculate the cost for a 10-mile trip.

2. Find the car's value after 7 years if $V(t) = -2000t + 30000$.

3. Calculate the remaining battery percentage after 4.5 hours if $P(h) = -12h + 100$.

4. Find the total cost of printing 1,000 copies if $C(c) = 80c + 10000$.

5. Calculate the balloon's altitude after 8 minutes of descent if $A(m) = -150m + 2000$.

**Variety 22: Solving for the Independent Variable (Time to Target)**

**Concept:** Finding when a specific target value is reached by solving the linear equation for $x$.

**Solved Example:** A car's value is $V(t) = -2000t + 30000$. In what year will the car's value be ₹ 10,000?

**Solution:** $10000 = -2000t + 30000 \Rightarrow 20000 = 2000t \Rightarrow t = 10$ years.

**Practice Questions:** 1. How many months will it take for a tree to reach 5.5 meters if $h(m) = 0.2m + 1.5$?

2. If a user has a budget of ₹ 50, how many full minutes can they talk if $C(m) = 0.05m + 20$?

3. Determine how many days until the reservoir is completely empty if $V(d) = 50000 - 1200d$.

4. When will the falling object hit the ground if $h(t) = 100 - 4t$?

5. If the battery model is $P(h) = -12h + 100$, when will it reach 40%?

**Variety 23: Modifying Expressions to Form Valid Polynomials**

**Concept:** Altering invalid algebraic expressions (like fractional exponents) to meet the rules of polynomials.

**Solved Example:** Analyze $5x^{\frac{5}{2}} - 3x + 8$. Modify only the exponents to turn it into a valid quadratic trinomial.

**Solution:** Change the fractional exponent $\frac{5}{2}$ to an integer 2: $5x^2 - 3x + 8$.

**Practice Questions:** 1. Modify $4x^{-2} + 3$ to make it a linear binomial.

2. Modify $x^{\frac{1}{2}} - 5$ to make it a linear polynomial.

3. Modify $\frac{5}{x^2} + x$ to make it a quadratic binomial.

4. Modify $\sqrt{y} + 5$ to make it a linear polynomial.

5. Modify $3x^2 + \frac{1}{x}$ to make it a cubic binomial.

**Variety 24: Adding and Subtracting Polynomials**

**Concept:** Combining matching variable terms by adding/subtracting coefficients.

**Solved Example:** Simplify the expression $(3x^2 - 2x + 5) - (x^2 + 4x - 2)$.

**Solution:** $3x^2 - x^2 - 2x - 4x + 5 + 2 = 2x^2 - 6x + 7$.

**Practice Questions:** 1. Find the sum $A + B$ for $A = 4x^3 - 2x^2 + 7x - 1$ and $B = -4x^3 + 5x^2 - 3x + 6$.

2. Simplify $(5x^2 + 3) + (2x^2 - x)$.

3. Simplify $(7x^3 - 2x + 9) - (4x^3 - 5)$.

4. Add $3 - 5x^2 + x$ and $-5x^2 + x + 3$.

5. Subtract $x^2 - 4$ from $2x^2 - 4$.

**Variety 25: Analyzing Degree Changes After Polynomial Operations**

**Concept:** Assessing the new degree when adding polynomials, specifically when leading terms cancel out.

**Solved Example:** Given $A = 4x^3 - 2x^2 + 7x - 1$ and $B = -4x^3 + 5x^2 - 3x + 6$. Analyze the resulting sum: what is its degree, and why did the degree change?

**Solution:** The sum is $3x^2 + 4x + 5$. The degree changed from 3 to 2 because the cubic terms ($4x^3$ and $-4x^3$) cancelled each other out.

**Practice Questions:** 1. State the degree of the result of $(3x^2 - 2x + 5) - (x^2 + 4x - 2)$.

2. Find the degree of $(2x^4 + x) + (-2x^4 + x^2)$.

3. Find the degree of $(x^3 - x) - (x^3 - 2x)$.

4. Prove that subtracting a quadratic from a cubic results in a cubic.

5. Does adding two linear polynomials always result in a linear polynomial?

**Variety 26: Formulating Geometric Polynomials (Perimeter/Area)**

**Concept:** Writing polynomials to represent dimensions and perimeters of geometric shapes.

**Solved Example:** Write a polynomial representing the perimeter of a rectangle with length '$x + 3$' and width '$x$'.

**Solution:** $Perimeter = 2(L + W) = 2((x + 3) + x) = 2(2x + 3) = 4x + 6$.

**Practice Questions:** 1. Write the polynomial for the perimeter of a rectangle with length $2x + 5$ and width $x - 3$.

2. Write a polynomial for the perimeter of a square with side length $3x - 1$.

3. Write a polynomial for the perimeter of a triangle with sides $x$, $x + 1$, and $x + 2$.

4. Write a polynomial for the area of a rectangle with length $x$ and width $x + 2$.

5. Model the perimeter of an equilateral triangle with side length $2x$.

**Variety 27: Evaluating Geometric Polynomials**

**Concept:** Substituting a measurement to calculate a concrete geometric value.

**Solved Example:** Given a rectangle has length $2x + 5$ and width $x - 3$. If $x = 6$ cm, calculate the numerical perimeter.

**Solution:** The perimeter polynomial is $6x + 4$. Evaluating at $x = 6$: $6(6) + 4 = 36 + 4 = 40$ cm.

**Practice Questions:** 1. Calculate the perimeter $4x + 6$ if $x = 4$.

2. Calculate the square perimeter $12x - 4$ if $x = 2$.

3. Calculate the triangle perimeter $3x + 3$ if $x = 5$.

4. Calculate the perimeter $10x + 4$ if $x = 3$.

5. Evaluate the perimeter of an equilateral triangle modeled by $6x$ if $x = 10$.

**Variety 28: Mathematical Proofs involving Linear Polynomials**

**Concept:** Building deductive logic to prove properties related to polynomials.

**Solved Example:** Prove mathematically that the sum of two linear polynomials will always result in a linear polynomial or a constant polynomial. Provide examples for both scenarios.

**Solution:** Let $P = ax + b$ and $Q = cx + d$. Sum = $(a + c)x + (b + d)$. If $a + c \neq 0$, the sum is linear (Example: $2x + 3x = 5x$). If $a + c = 0$, the sum is a constant (Example: $2x + (-2x) = 0$).

**Practice Questions:** 1. Prove that the sum of two constant polynomials is a constant polynomial.

2. Prove that subtracting a linear polynomial from itself yields the zero polynomial.

3. Show mathematically why $2(ax + b)$ remains a linear polynomial.

4. Prove that multiplying a linear polynomial by $x$ changes its degree to 2.

5. Prove that the difference between two binomials of degree 1 can be a monomial of degree 0.