Summer Vacation Assignment - Mathematics

Worksheet 2: Polynomials

Section A: Multiple Choice Questions

Q. 1. Which of the following mathematical expressions correctly represents a valid polynomial?

A) $x^2 + \frac{2}{x}$ B) $\sqrt{x} + 5$ C) $x^3 + 3x^2 - 1$ D) $x^{-1} + 4$

Q. 2. Identify the degree of the given multivariable-like polynomial $p(x) = 7x^4 - 2x^2 + x - 9$.

A) 1 B) 2 C) 3 D) 4

Q. 3. Determine the precise zero of the linear polynomial expressed as $p(x) = 3x - 12$.

A) 4 B) -4 C) $\frac{1}{4}$ D) $-\frac{1}{4}$

Q. 4. Calculate the evaluated numerical value of the quadratic polynomial $p(x) = x^2 - 5x + 6$ exactly at $x = 2$.

A) 0 B) 1 C) 2 D) -1

Q. 5. If a polynomial is defined as $p(x) = 2x + 5$, what is the simplified result of $p(x) + p(-x)$?

A) 10 B) $4x$ C) 0 D) 5

Q. 6. What is the degree of a non-zero constant polynomial?

A) 1 B) 0 C) Not defined D) Infinity

Q. 7. For a generic linear polynomial given by $p(x) = mx + n$ (where m ≠ 0), what is its zero?

A) $\frac{m}{n}$ B) $-\frac{m}{n}$ C) $\frac{n}{m}$ D) $-\frac{n}{m}$

Q. 8. Determine the evaluated value of the polynomial $p(x) = -3x^2 + 4x - 2$ when $x = 0$.

A) 0 B) -2 C) 2 D) 4

Q. 9. What is the evaluated value of the cubic polynomial $p(x) = x^3 - x^2 + x - 1$ when $x = -1$?

A) -4 B) 4 C) 0 D) -2

Q. 10. If it is established that $x = 3$ is a zero of the polynomial $p(x) = kx^2 - 5x + 3$, find the constant k.

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) 1 D) -1

Q. 11. Which of the following listed algebraic expressions does NOT qualify as a linear polynomial?

A) $5x + 7$ B) $\sqrt{2}x - 3$ C) $x^2 + 2$ D) $x - 8$

Q. 12. According to the remainder concept, what is the remainder when $p(x) = x^3 - mx^2 + 2x - m$ is evaluated at $x = m$?

A) m B) $2m$ C) 0 D) $-2m$

Q. 13. If a specific polynomial is defined as $f(y) = 3y^2 - 2y + 5$, what is the exact value of $f(0)$?

A) 0 B) 3 C) -2 D) 5

Q. 14. What is the degree of the 'zero polynomial'?

A) 0 B) 1 C) Infinite D) Not defined

Q. 15. Compute the numerical value of the polynomial expression $4x^2 - 3x + 7$ precisely at $x = 2$.
A) 17 B) 15 C) 9 D) 11

Q. 16. A standard linear polynomial in one variable will always possess exactly how many real zeros?
A) 0 B) 1 C) 2 D) Infinite

Q. 17. If given the polynomial $p(x) = x^2 - 3\sqrt{2}\,x + 4$, what is the evaluated result of $p(3\sqrt{2})$?
A) 0 B) 4 C) $2\sqrt{2}$ D) $8\sqrt{2}$

Q. 18. Identify which of the following provided polynomials features a zero located exactly at $x = -4$.
A) $x - 4$ B) $x + 4$ C) $2x - 4$ D) $x^2 + 4$

Q. 19. If the cubic polynomial is given as $p(x) = 2x^3 + x^2 - 3x + 5$, compute the value of $p(-2)$.
A) -1 B) -5 C) 1 D) -15

Q. 20. What is the zero of the simple proportional polynomial $p(x) = ax$, given the condition that $a \neq 0$?
A) a B) 1 C) 0 D) $-a$

Section B: Very Short Answer Type Questions

Q. 21. Calculate the zero of the linear polynomial defined as $p(x) = 2x - 8$.

Q. 22. Calculate the zero of the linear polynomial defined as $q(x) = 5x + 15$.

Q. 23. Evaluate the exact numerical value of the polynomial $p(x) = x^2 + 4x + 4$ by substituting $x = 0$.

Q. 24. Evaluate the exact numerical value of the polynomial $f(y) = 3y^2 - 7y + 2$ at $y = 1$.

Q. 25. Identify and state the degree of the algebraic polynomial expression $9x^6 - 4x^3 + 2x - 1$.

Q. 26. Determine mathematically whether $x = 3$ is a valid zero of the polynomial $p(x) = 2x - 6$.

Q. 27. Find the value of the unknown parameter $'k'$ if $x = 2$ is a proven zero of $p(x) = kx - 10$.

Q. 28. Analyze the algebraic expression $\frac{1}{x^2} + 3x$ and state whether it classifies as a polynomial.

Q. 29. Determine the mathematical remainder when the polynomial $p(x) = x^3 - 3x^2 + 2x + 5$ is evaluated directly at $x = 1$.

Q. 30. Determine the evaluated value of the constant polynomial $p(x) = -25$ specifically when $x = 1000$.

Q. 31. Construct a generic linear polynomial in the variable 't' that has its zero located exactly at t = 9.

Q. 32. Calculate the value of $p(-2)$ for the given quadratic polynomial $p(x) = x^2 - 4$.

Q. 33. Determine the precise zero of the uniquely formatted polynomial $p(z) = 7 - 2z$.

Q. 34. Find the numerical value of the polynomial expression $4x^2 - 2x - 5$ specifically at the point $x = -1$.

Q. 35. What is the remainder calculated when the linear polynomial $p(x) = 4x + 7$ is evaluated at $x = -\frac{7}{4}$?

Q. 36. Can the quadratic polynomial $x^2 + 9$ have any real zero? Explain briefly.

Q. 37. If a polynomial is defined as $p(x) = x^2 - 2x$, calculate the sum of the evaluated values $p(0)$ and $p(2)$.

Q. 38. Provide a clear mathematical example of a quadratic polynomial in one variable 'y'.

Q. 39. State the degree of any non-zero constant number like 42.

Q. 40. Evaluate the generic polynomial $f(t) = 5t^2 - 2t + 8$ by substituting the variable t with $'b'$.

Section C: Short Answer Type Questions

Q. 41. Compute the zero of the linear polynomial $p(x) = 3x - 7$ and explicitly mathematically verify your answer.

Q. 42. Given the quadratic polynomial $p(x) = x^2 - 6x + 8$. Find the values of $p(2)$ and $p(4)$. What specific conclusion can you draw from these results?

Q. 43. If $x = -2$ is confirmed as a zero of the polynomial $p(x) = ax^2 - 2(a-1)x - 4$, calculate the exact numerical value of a.

Q. 44. Evaluate the cubic polynomial $f(x) = 3x^3 - 5x^2 + 2x + 8$ by substituting $x = 2$ and $x = -1$ consecutively.

Q. 45. Find the explicit remainder when the polynomial $p(x) = 2x^4 + x^3 - 3x^2 + 5x + 1$ is evaluated directly at $x = 1$.

Q. 46. Systematically check whether $x = 3$ and $x = 0$ are valid zeros of the quadratic polynomial $p(x) = x^2 - 3x$.

Q. 47. Find the specific value of k, if $x = 1$ is known to be a root (evaluates to zero) of the polynomial $p(x) = 3x^2 + kx + 2\sqrt{2}$.

Q. 48. If a polynomial is defined as $p(t) = t^3 - 4t^2 + 5t - 20$, find the evaluated value of $p(4)$. Based on this, is $t = 4$ a zero?

Q. 49. Find the zero of the linear polynomial $4x + 9$, and then evaluate this same polynomial at the distinct point $x = -3$.

Q. 50. Calculate the distinct values of the polynomial $2x - 3x^2 + 5$ at three separate points: $x = 0$, $x = -1$ and $x = 2$.

Q. 51. Verify mathematically using substitution whether the numbers 4 and -1 are zeros of the polynomial $q(x) = x^2 - 3x - 4$.

Q. 52. If the polynomial is formulated as $p(y) = 3y^3 - 2y^2 + y - 4$, calculate the exact value of the expression $p(2) - p(-1)$.

Q. 53. A linear polynomial is given by the standard form $p(x) = mx + c$. If it is known that $p(0) = 5$ and $p(1) = 8$, determine the values of m and c.

Q. 54. Calculate the remainder when the cubic polynomial $2x^3 - 4x^2 + x - 7$ is evaluated precisely at the point $x = 3$.

Q. 55. Demonstrate mathematically that the cubic polynomial $p(x) = x^3 - 64$ has a definitive zero exactly at $x = 4$.

Q. 56. If a function is defined as $f(x) = 2x^2 - 3x + 1$, calculate and evaluate the final sum of $f(3)$ and $f(-2)$.

Q. 57. Compute the zeroes of the two proportional linear polynomials $p(x) = 5x - 15$ and $q(x) = 2x - 6$. Compare them and state your observation.

Q. 58. Systematically verify if the quadratic polynomial $p(x) = x^2 - x + 1$ possesses a valid zero at the coordinate $x = 1$.

Q. 59. If a generic linear polynomial is defined as $p(x) = kx + r$ (where $k \neq 0$), evaluate the polynomial at $x = -\frac{r}{k}$ to mathematically prove it is a zero.

Q. 60. Calculate the exact numerical value of the quadratic polynomial $4x^2 - 6x + 3$ by accurately substituting $x = \frac{1}{2}$.

Section D: Long Answer Type Questions

Q. 61. Given the cubic polynomial $p(x) = x^3 - 7x^2 + 14x - 8$. Evaluate the specific values for $p(1)$, $p(2)$, and $p(4)$. Based on your comprehensive calculations, what can you conclusively state about the numbers 1, 2, and 4 regarding this polynomial?

Q. 62. If $x = 3$ and $x = 0$ are both confirmed and verified zeros of the polynomial $f(x) = 2x^3 - 7x^2 + ax + b$, logically find the precise values of the unknown constants $\boldsymbol{a}$ and b.

Q. 63. Let a quadratic polynomial be mathematically defined as $p(x) = ax^2 + bx + c$. If it is provided that $p(0) = 4$, $p(1) = 9$, and $p(-1) = 3$, formulate a system of equations to isolate and find the exact values of $\boldsymbol{a}$, b and **c**.

Q. 64. Find the complete set of zeroes for the quadratic polynomial $f(x) = x^2 - (\sqrt{5} + 2)x + 2\sqrt{5}$ by substituting $x = \sqrt{5}$ and $x = 2$ to verify them in a step-by-step manner.

Q. 65. A fourth-degree polynomial is given by $p(x) = x^4 - 3x^3 + 2x^2 - ax + 2a - 5$. If the evaluated numerical value of this polynomial is exactly 25 at $x = -1$, calculate the value of the parameter $'a'$.

Q. 66. Thoroughly evaluate the higher-order polynomial $p(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$ at the points $x = 1$ and $x = 2$. Are these specific values zeros of the given polynomial? Justify your answer.

Q. 67. If two distinct quadratic polynomials are defined as $p(x) = x^2 - 5x + 4$ and $q(x) = x^2 - 7x + 10$, find the specific coordinate $\boldsymbol{x}$ for which $p(x)$ perfectly equals $q(x)$. Evaluate both polynomials at this found value to confirm their intersection.

Q. 68. For the given cubic polynomial equation $p(x) = 3x^3 - 2x^2 + 5x - 7$, calculate the exact numerical value of the complex algebraic expression $[p(2) - p(-1)]$ divided by the value of $p(0)$.

Q. 69. Determine the full algebraic expression for a quadratic polynomial $p(x)$ such that its initial value $p(0) = 5$ and it is explicitly known to have zeros located at $x = 2$ and $x = -4$.

Q. 70. Find the exact mathematical zero of the linearly parameterized polynomial $f(x) = (m - n)x + (n - p)$. After successfully finding the expression for the zero, evaluate the function at $f(1)$.

Q. 71. Let a specific cubic polynomial be defined as $p(x) = x^3 + 4x^2 - x - 4$. Demonstrate through step-by-step substitution that $x = 1$, $x = -1$, and $x = -4$ are indeed all valid zeros of $p(x)$.

Q. 72. If a generalized quadratic polynomial is defined as $f(x) = x^2 - mx + n$, and it is given as a premise that $f(1) = 0$ and $f(3) = 0$, find the specific numerical values of $\boldsymbol{m}$ and $\boldsymbol{n}$. Subsequently, evaluate the function at $f(4)$.

Q. 73. Calculate the exact mathematical remainder when the given polynomial expression $4x^4 - 3x^3 - 2x - 5$ is evaluated individually and sequentially at $x = 1$, $x = -1$, and $x = 2$.

Q. 74. If it is firmly given that x = 3/2 is a verified zero of the cubic polynomial f(x) = $4x^3$ - $8x^2$ + kx - 15, calculate the correct and precise value of the constant k.

Q. 75. Verify mathematically, showing all steps, that the numbers $2, -2$, and $\frac{1}{2}$ are all definitive zeros of the cubic polynomial $p(x) = 2x^3 - x^2 - 8x + 4$.

Q. 76. Evaluate the cubic polynomial expression $3y^3 + 2y^2 - 3y - 2$ at the specific values $y = 1$ and $y = -1$. What does this evaluation concretely tell you about the properties of the polynomial?

Q. 77. If two separate and distinct polynomials are given as $p(x) = 3x^2 - 2x + 4$ and $q(x) = 2x^2 + x - 3$, calculate the combined value of the complex expression $3.p(1) - 2.q(-1)$.

Q. 78. A student evaluating the polynomial $p(x) = x^2 - kx + 12$ at $x = 3$ correctly receives a remainder of 0. Utilizing this determined value of k, evaluate the exact value of the polynomial at $p(-3)$.

Q. 79. Demonstrate mathematically that the given polynomial $p(x) = x^4 + 3x^2 + 7$ has absolutely no real zeros by systematically evaluating its terms for positive, negative, and zero real values.

Q. 80. Given the parameter-based cubic polynomial $p(x) = x^3 - b^2x + 2x + 5$. If the evaluated value of $p(b)$ is exactly 11, find the precise numerical value of the unknown parameter $\boldsymbol{b}$.

Section E: Case Study-Based Questions

Q. 81. A corporate financial analyst models a company's projected quarterly revenue. The revenue R (in millions of dollars) generated by selling s thousand premium software licenses is modeled by the linear polynomial $R(x) = 15x - 45$.

(i) Calculate the exact zero of the polynomial $R(x)$.

(ii) What does this calculated zero physically signify in the context of the company's sales and revenue?

(iii) Evaluate the projected revenue if the company successfully sells exactly 5 thousand licenses.

(iv) Calculate the financial deficit (represented as negative revenue) if the company sells only 1 thousand licenses.

Q. 82. A meteorologist tracks the temperature variations during a severe winter storm. The temperature T (in degrees Fahrenheit) at hour '$\boldsymbol{h}$' after midnight is closely modeled by the quadratic polynomial $T(h) = h^2 - 8h + 12$.

(i) Determine the initial starting temperature exactly at midnight (evaluated at $h = 0$).

(ii) Evaluate the recorded temperature exactly at $h = 2$ hours and $h = 6$ hours.

(iii) Based on your previous calculations, identify the zeros of the polynomial $T(h)$.

(iv) Evaluate and find the exact temperature at the midpoint of $h = 4$ hours.

Q. 83. A physics instructor demonstrates gravity by dropping a sensor from a hovering helicopter. The sensor's height $\boldsymbol{H}$ (in meters) above the ground after $\boldsymbol{t}$ seconds is modeled by the quadratic polynomial $H(t) = -5t^2 + 20t + 105$.

(i) What is the total altitude of the helicopter from which the sensor is dropped? (Evaluate $\boldsymbol{H}$ at $t = 0$).

(ii) Find the exact height of the sensor above the ground after exactly 2 seconds of freefall.

(iii) Verify mathematically if $t = 7$ seconds represents a zero of the polynomial $H(t)$.

(iv) Evaluate and record the precise height of the sensor exactly after 4 seconds.

Q. 84. An urban planner studies the population growth of a newly developed suburb. The population increase P (in thousands) over $'y'$ years is modeled by the linear polynomial $P(y) = 3y + 12$.

(i) Determine the mathematical zero of the polynomial $P(y)$.

(ii) Evaluate the expected population increase after a span of exactly 5 years.

(iii) If the polynomial $P(y)$ evaluates to exactly 30, solve for the number of years $'y'$.

(iv) Calculate the initial baseline population increase factor evaluated at year $y = 0$.

Q. 85. A civil engineer monitors the water flow rate through an aging dam spillway. The flow rate F (in cubic meters per second) at time $'m'$ (in minutes) is given by the quadratic polynomial $F(m) = -m^2 + 10m - 16$.

(i) Calculate the initial water flow rate at the start of the observation ($m = 0$).

(ii) Evaluate the exact water flow rate at $m = 2$ minutes and $m = 8$ minutes.

(iii) Identify the zeros of the polynomial $F(m)$ based on your evaluations.

(iv) Determine the water flow rate at exactly $m = 5$ minutes.

Q. 86. An energy consultant analyzes the daily electricity consumption of a large server farm. The consumption E (in megawatts) at hour $'t'$ of the day is modeled by the linear polynomial $E(t) = 4t + 50$.

(i) Determine the base fixed power consumption when $t = 0$.

(ii) Evaluate the total electricity consumption at the specific hour $t = 10$.

(iii) Calculate the consumption at the specific hour $t = 15$.

(iv) If the consumption hits exactly 90 megawatts, mathematically solve for the hour $'t'$.

Q. 87. An automotive engineer tests a new hybrid engine's fuel efficiency. The efficiency rating R (in miles per gallon) at a sustained speed $'s'$ (in tens of mph) is modeled by $R(s) = -s^2 + 12s + 20$.

(i) What is the baseline efficiency rating when the vehicle is stationary ($s = 0$)?

(ii) Evaluate the efficiency rating R exactly when $s = 4$.

(iii) Evaluate the efficiency rating R exactly when s = 8.

(iv) Verify if s = 14 represents a zero of this efficiency polynomial.

Q. 88. An agricultural scientist estimates the crop yield of a newly engineered wheat strain. The yield Y (in tons) based on $'f'$ units of specialized fertilizer is modeled by $Y(f) = -2f^2 + 16f$.

(i) What is the expected yield if absolutely zero fertilizer is applied ($f = 0$)?

(ii) Evaluate the exact crop yield when 3 units of fertilizer are applied.

(iii) Find the mathematical zeros of the polynomial $Y(f)$.

(iv) Evaluate the exact crop yield when 5 units of fertilizer are applied.

Q. 89. A quantitative analyst models the daily price fluctuation of a volatile tech stock. The stock's price change V (in dollars) at hour $'x'$ of trading is modeled by the quadratic polynomial $V(x) = x^2 - 9x + 14$.

(i) What is the price change exactly at the opening bell ($x = 0$)?

(ii) Find the precise zeros of the polynomial $V(x)$.

(iii) Evaluate the exact stock price change at trading hour $x = 5$.

(iv) Evaluate the exact stock price change at trading hour $x = 1$.

Q. 90. An acoustic engineer measures the sound wave intensity in a concert hall. The intensity I (in decibels) at a distance $'d'$ (in meters) from the speaker is given by the linear polynomial $I(d) = -5d + 120$.

(i) What is the maximum sound intensity directly at the speaker's origin ($d = 0$)?

(ii) Calculate the exact sound intensity at a distance of $d = 10$ meters.

(iii) Calculate the exact sound intensity at a distance of $d = 20$ meters.

(iv) Find the exact distance $'d'$ (the zero of the polynomial) where the sound intensity drops to 0.

Q. 91. A chemist studies the reaction rate of a new catalyst. The rate of reaction C (in moles per second) at temperature $'k'$ (in tens of degrees Celsius) is modeled by $C(k) = -k^2 + 14k - 45$.

(i) Calculate the reaction rate at a baseline temperature of $k = 0$.

(ii) Evaluate the reaction rate precisely at $k = 5$ and $k = 9$.

(iii) Identify the zeros of the polynomial $C(k)$.

(iv) Calculate the exact reaction rate at the optimal temperature $k = 7$.

Q. 92. A structural engineer examines the tension profile of a bridge suspension cable. The tension T (in kilonewtons) at horizontal distance $'x'$ (in meters) is modeled by $T(x) = x^2 - 12x + 36$.

(i) What is the cable tension at the anchor point where $x = 0$?

(ii) Determine the mathematical zero of this polynomial expression.

(iii) Evaluate the exact cable tension at a distance of $x = 4$ meters.

(iv) Evaluate the exact cable tension at a distance of $x = 8$ meters.

Q. 93. A land developer calculates the usable area of a series of terraced plots. The area A (in square meters) based on the terrace width $'w'$ is given by the linear polynomial $A(w) = 25w - 150$.

(i) Calculate the mathematical zero of the polynomial $A(w)$.

(ii) What does this zero indicate regarding the minimum viable width for a plot?

(iii) Evaluate the exact usable area if the terrace width is planned at $w = 10$ meters.

(iv) Solve for the required width $'w'$ if a specific plot must have exactly 350 square meters of area.

Q. 94. A packaging designer optimizes the volume of a new shipping box. The volume V (in cubic inches) based on a dimension factor $'x'$ is evaluated using a cubic polynomial $V(x) = x^3 - 5x^2 + 4x$.

(i) Evaluate the volume of the box when the dimension factor $x = 2$.

(ii) Evaluate the volume of the box when the dimension factor $x = 3$.

(iii) Verify systematically if $x = 4$ is a zero of the polynomial $V(x)$.

(iv) Verify systematically if $x = 1$ is a zero of the polynomial $V(x)$.

Q. 95. A traffic analyst models the braking deceleration speed of a tested vehicle. The speed S (in meters per second) at time $'t'$ (in seconds) during braking is given by $S(t) = -4t + 28$.

(i) What is the initial cruising speed of the vehicle right when braking begins ($t = 0$)?

(ii) Calculate the exact speed of the vehicle after t=3 seconds of braking.

(iii) Determine the mathematical zero of the polynomial $S(t)$.

(iv) What physical event does this determined zero represent in the real world?

Q. 96. An epidemiologist models the early transmission rate of a seasonal virus in a closed community. The infection rate I (in cases per day) at day $'d'$ is given by the quadratic $I(d) = -d^2 + 16d - 48$.

(i) Evaluate the infection rate precisely at day $d = 4$ and day $d = 12$.

(ii) Identify the zeros of the polynomial $I(d)$ based on your results.

(iii) Evaluate the peak expected infection rate exactly at day $d = 8$.

(iv) What is the theoretical infection rate at day $d = 0$ according to the model?

Q. 97. A publishing house calculates the operational printing cost for a new textbook run. The cost C (in dollars) for $'p'$ thousand pages is modeled by the linear polynomial $C(p) = 12p + 400$.

(i) Identify the fixed baseline setup cost when absolutely zero pages are printed ($p = 0$).

(ii) Calculate the total operational cost for printing exactly 50 thousand pages.

(iii) Calculate the total operational cost for printing exactly 100 thousand pages.

(iv) If the total budgeted cost is exactly 1600 dollars, solve for the number of pages $'p'$ that can be printed.

Q. 98. A bakery manager analyzes the daily sales trajectory of their signature cake. The sales volume S (in units) at hour $'h'$ after opening is modeled by the polynomial $S(h) = -2h^2 + 20h$.

(i) What is the sales volume exactly at the opening hour ($h = 0$)?

(ii) Evaluate the exact sales volume at $h = 3$ hours after opening.

(iii) Find the mathematical zeros of the sales polynomial $S(h)$.

(iv) Evaluate the exact sales volume at S hours after opening.

Q. 99. A software developer tracks the average high score in a new mobile game over its first week. The score curve M (in thousands) at day $'d'$ is given by the quadratic polynomial $M(d) = d^2 - 10d + 24$.

(i) Determine the initial base score recorded exactly at launch ($d = 0$).

(ii) Find the exact zeros of the polynomial $M(d)$.

(iii) Evaluate the average high score exactly at day $d = 3$.

(iv) Evaluate the average high score exactly at day $d = 7$.

Q. 100. An accountant calculates the depreciating value of a suite of office computers. The value D (in dollars) after $'y'$ years of use is modeled by the linear polynomial $D(y) = -300y + 1500$.

(i) Determine the initial total purchase value of the computers at $y = 0$.

(ii) Calculate the exact retained value of the computers after $y = 2$ years.

(iii) Find the zero of the polynomial $D(y)$ to determine when the computers become financially worthless.

(iv) Calculate the exact retained value of the computers after $y = 4$ years.