Summer Vacation Assignment - Mathematics

Worksheet 1: Real Numbers

Section A: Multiple Choice Questions

Q. 1. What is the unique prime factorisation of the composite number 140?
A) $2^2 \times 5 \times 7$ B) $2 \times 5^2 \times 7$ C) $2^2 \times 5^2 \times 7$ D) $2 \times 5 \times 7^2$

Q. 2. Identify the irrational number from the following options:
A) $\sqrt{4}$ B) $\sqrt{9}$ C) $\sqrt{5}$ D) $\sqrt{16}$

Q. 3. Calculate the Highest Common Factor (HCF) of the numbers 96 and 404.
A) 2 B) 4 C) 6 D) 8

Q. 4. Determine the Least Common Multiple (LCM) of the numbers 26 and 91.
A) 182 B) 13 C) 91 D) 26

Q. 5. Given that HCF (306, 657) = 9, what is the corresponding LCM (306, 657)?
A) 22338 B) 23328 C) 22383 D) 32238

Q. 6. In the prime factorisation of the number 144, what is the exponent of the prime base 2?
A) 2 B) 4 C) 3 D) 4

Q. 7. Find the LCM of the smallest existing prime number and the smallest existing composite number.
A) 2 B) 4 C) 6 D) 8

Q. 8. If two variables p and q are defined as $p = a \times b^2$ and $q = a^3 \times b$ (where a, b are primes), find their LCM.
A) $a \times b$ B) $a^2 \times b^2$ C) $a^3 \times b^2$ D) $a^3 \times b^3$

Q. 9. Using the same variables $p = a \times b^2$ and $q = a^3 \times b$, what is their HCF?
A) $a^3 \times b^2$ B) $a^3 \times b$ C) $a \times b^2$ D) $a \times b$

Q. 10. Which statement correctly describes the product of a non-zero rational number and an irrational number?
A) Always irrational B) Always rational C) Rational or irrational D) One

Q. 11. Identify the smallest natural number that is completely divisible by all integers from 1 to 5 inclusive.
A) 10 B) 20 C) 60 D) 120

Q. 12. If the HCF of 2520 and 6600 is 40 and their LCM is expressed as 252 × k, find the value of k.
A) 1650 B) 1600 C) 165 D) 16500

Q. 13. What is the largest possible integer that divides 70 and 125, leaving remainders of 5 and 8 respectively?
A) 13 B) 65 C) 875 D) 1750

Q. 14. If the LCM of an unknown number $'a'$ and 18 is 36, and their HCF is 2, what is the value of $'a'$?
A) 2 B) 3 C) 4 D) 1

Q. 15. The LCM of 10, 15, and 20 is:
A) 30 B) 60 C) 90 D) 120

Q. 16. Which of the following is a prime number?
A) 39 B) 51 C) 97 D) 111

Q. 17. For any natural number $'n'$, what is the fixed unit digit of the expression 6^n?
A) 2 B) 4 C) 6 D) 8

Q. 18. What will always be the Highest Common Factor (HCF) of any two consecutive even integers?
A) 1 B) 2 C) 4 D) 0

Q. 19. If it is known that the product of two co-prime numbers is 117, what must their LCM be?
A) 1 B) 117 C) 11 D) 17

Q. 20. Select the correct and complete prime factorization of the number 3825.
A) $3^2 \times 5^2 \times 17$ B) $3 \times 5^2 \times 17$ C) $3^2 \times 5 \times 17$ D) $3^2 \times 5^2 \times 7$

Section B: Very Short Answer Type Questions

Q. 21. Decompose the number 5005 completely into a product of its prime factors.

Q. 22. State whether the expression $(2 \times 3 \times 5 \times 7) + 7$ represents a prime or a composite number.

Q. 23. Calculate the HCF of the triplet (12, 15, 21) using prime factorisation.

Q. 24. Determine the exact LCM of the triplet (12, 15, 21).

Q. 25. Formulate the specific relationship connecting the HCF and LCM of the two numbers 306 and 657.

Q. 26. Evaluate whether the numerical expression 4^n can ever end with the digit 0 for any natural number $'n'$.

Q. 27. Write down the Fundamental Theorem of Arithmetic.

Q. 28. Write the prime factorization of the number 120.

Q. 29. Calculate the LCM of the numbers 8, 9, and 25.

Q. 30. Determine the HCF of the numbers 8, 9, and 25.

Q. 31. Is it logically possible for two unknown numbers to have an HCF of 18 and an LCM of 380? Answer with a brief reason.

Q. 32. If the HCF of two numbers is 27, their LCM is 162, and one number is 54, calculate the second number.

Q. 33. Determine the generic HCF of any two consecutive natural numbers.

Q. 34. Determine the generic HCF of any two consecutive odd numbers.

Q. 35. Given the prime factored forms $a = 2^3 \times 3$ and $b = 2 \times 3 \times 5$, find their LCM.

Q. 36. If $'x'$ and $'y'$ are distinct prime numbers, what is their LCM?

Q. 37. Write a single rational number that falls strictly between $\sqrt{2}$ and $\sqrt{3}$.

Q. 38. Find the value of the missing composite number 'Z' if its prime factorization is exactly $2 \times 3^2 \times 5$.

Q. 39. Simplify the radical expression $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ and state if the result is rational or irrational.

Q. 40. What is the lowest common multiple of 15 and 20?

Section C: Short Answer Type Questions

Q. 41. Calculate the HCF and LCM of 26 and 91, and subsequently verify that HCF × LCM equals the product of these two numbers.

Q. 42. Explain with mathematical reasoning why the arithmetic sequence $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5)$ yields a composite number.

Q. 43. Sonia takes 18 minutes to run around a circular track while Ravi takes 12 minutes. If they start simultaneously from the same point, after how many minutes will they cross the starting point together?

Q. 44. Find the greatest 6-digit integer that is perfectly divisible by 24, 15, and 36.

Q. 45. Determine the smallest possible number which, when divided by 28 and 32, leaves identical remainders of 8 and 12 respectively.

Q. 46. Two digital alarms ring at regular intervals of 50 seconds and 48 seconds. If they beep together exactly at noon, predict the precise time of their next simultaneous beep.

Q. 47. A carpenter has three wooden planks measuring 42 m, 49 m, and 63 m. He must cut them into equal smaller planks. What is the greatest possible length he can choose?

Q. 48. Prove mathematically that the binomial expression $3 + 2\sqrt{5}$ is an irrational number.

Q. 49. Find the smallest natural integer by which the number 1200 must be multiplied to ensure its square root becomes a rational number.

Q. 50. The sum of the LCM and HCF of two specific positive numbers is 1260. If their LCM is 900 greater than their HCF, find the product of these two numbers.

Q. 51. Determine the complete prime factorization of the number 1008, and calculate the sum of all the exponents of its prime factors.

Q. 52. Check whether 15^n can end with the digit 0 for any natural number $\boldsymbol{n}$.

Q. 53. Using the prime factorization method independently, compute the Highest Common Factor (HCF) of 408 and 1032.

Q. 54. A sweet shop owner wishes to stack 420 Kaju barfis and 130 Badam barfis such that each stack has the same number of sweets and uses minimal tray area. How many sweets go in one stack?

Q. 55. Find the LCM of 96 and 104 using the prime factorization method.

Q. 56. Find the largest integer that divides both 2053 and 967 while leaving remainders of 5 and 7 respectively.

Q. 57. Check systematically whether the exponential expression 6^n can end with the digit 0 for any natural number n.

Q. 58. Find the LCM of the smallest two-digit composite number and the absolute smallest composite number.

Q. 59. Check if the numbers 15 and 28 are co-prime numbers by finding their HCF.

Q. 60. The LCM of two numbers is exactly 14 times their HCF. If the sum of the LCM and HCF is 600, and one of the numbers is 280, calculate the unknown number.

Section D: Long Answer Type Questions

Q. 61. Find the prime factorization of 336 and 54. (a) Use it to find their HCF. (b) Use it to find their LCM. (c) Verify that HCF × LCM = 336 × 54.

Q. 62. Find the prime factorization of 404 and 96. (a) Use it to find their HCF. (b) Use it to find their LCM. (c) Verify that HCF × LCM = 404 × 96.

Q. 63. A fruit seller has 120 apples and 150 oranges. He wants to pack them in identical boxes with the same number of fruits. (a) Find the HCF of 120 and 150. (b) What is the maximum number of fruits in each box? (c) How many boxes are needed in total?

Q. 64. Traffic lights at three crossings change after 40, 60, and 80 seconds. (a) Find their LCM. (b) Convert the LCM from seconds into minutes. (c) If they change at 8:00 AM together, at what time will they change together next?

Q. 65. A tailor has three rolls of cloth measuring 24 m, 36 m, and 48 m. He wants to cut equal maximum length pieces from them. (a) Find the HCF of 24, 36, and 48. (b) What is the length of each piece? (c) How many pieces will he get in total?

Q. 66. A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. (a) Convert the dimensions to cm. (b) Find the HCF to get the largest side of the square tile. (c) Find the total number of tiles needed.

Q. 67. The HCF and LCM of two numbers are 13 and 1989 respectively. (a) Write the mathematical formula connecting them. (b) If one number is 117, find the second number. (c) Multiply the two numbers and verify your answer.

Q. 68. There is a circular path around a field. Sonia takes 18 minutes for one round and Ravi takes 12 minutes. They start at the same time and place. (a) Find the prime factorization of 18 and 12. (b) Find their LCM. (c) After how many minutes will they meet again?

Q. 69. Prove that $\sqrt{2}$ is an irrational number. (a) Start by assuming it is a rational number $\frac{p}{q}$. (b) Square both sides and show that p is even. (c) Write the final statement showing the contradiction.

Q. 70. Prove that $7 - 2\sqrt{3}$ is an irrational number. (a) Assume it is equal to a rational number $\frac{p}{q}$. (b) Isolate $\sqrt{3}$ on one side of the equation. (c) Explain why a rational number cannot be equal to an irrational number.

Q. 71. Explain why $5 \times 7 \times 11 \times 13 + 7$ is a composite number. (a) Take the number 7 common from the expression. (b) Simplify the numbers inside the bracket. (c) State why having more than two factors makes it composite.

Q. 72. Find the greatest number that divides 445, 572, and 699 leaving remainders 4, 5, and 6 respectively. (a) Subtract the remainders from the numbers. (b) Find the prime factors of the new numbers. (c) Find their HCF.

Q. 73. Three measuring rods are 64 cm, 80 cm, and 96 cm in length. (a) Find their prime factors. (b) Find their LCM. (c) What is the least length of cloth that can be measured exactly using any rod?

Q. 74. Find the smallest number which leaves a remainder of 5 when divided by 14 and 21. (a) Find the prime factors of 14 and 21. (b) Find their LCM. (c) Add 5 to your answer to get the final number.

Q. 75. Find the smallest 4-digit number which is exactly divisible by 18, 24, and 32. (a) Find the LCM of 18, 24, and 32. (b) Write the multiples of this LCM. (c) Identify the smallest multiple that has 4 digits.

Q. 76. Find the HCF and LCM of 504 and 980 using prime factorization. (a) Write the prime factors of 504. (b) Write the prime factors of 980. (c) Calculate both the HCF and the LCM.

Q. 77. Check if the number 24^n can end with the digit 0 for any natural number $\boldsymbol{n}$. (a) Write the prime factors of 24. (b) State the rule that a number must have factors 2 and 5 to end with 0. (c) Write your final conclusion.

Q. 78. The prime factorization of a natural number N is $2^3 \times 3^2 \times 5^2 \times 7$. (a) Pair up the factors of 2 and 5. (b) How many consecutive zeros will be at the end of N? (c) Calculate the exact value of N.

Q. 79. There are 156, 208, and 260 students in groups A, B, and C. They are to be seated in buses with an equal number of students from the same group. (a) Find the HCF. (b) How many students will be in one bus? (c) How many buses are needed?

Q. 80. Find the largest number which divides 615 and 963 leaving a remainder of 6 in each case. (a) Subtract 6 from both numbers. (b) Write the prime factors of the new numbers. (c) Calculate their HCF.

Section E: Case Study-Based Questions

Q. 81. A school librarian wants to arrange 144 mathematics books and 192 science books on shelves. She wants to put the same number of books on each shelf, and each shelf should only have one subject.

(i) To find the maximum books per shelf, should we use HCF or LCM?

(ii) Write the prime factorization of 144 and 192.

(iii) What is the maximum number of books that can be put on each shelf?

(iv) How many total shelves will be needed for all the books?

Q. 82. The traffic lights at three different road crossings change after 48 seconds, 72 seconds, and 108 seconds.

(i) Should we use HCF or LCM to find when they will change together next?

(ii) Find the LCM of 48, 72, and 108.

(iii) If they changed together at 8:00 AM, at what time will they change together again?

(iv) Convert this waiting time into minutes and seconds.

Q. 83. A teacher wants to form teams for a quiz. There are 24 boys and 36 girls. She wants each team to have the same number of boys and the same number of girls, with no students left out.

(i) Which concept (HCF or LCM) helps to find the maximum number of teams she can make?

(ii) Write the prime factorization of 24 and 36.

(iii) What is the maximum number of equal teams that can be formed?

(iv) How many boys and how many girls will be in each team?

Q. 84. A worker is tiling a rectangular floor that is 1517 cm long and 902 cm wide. He wants to use the largest possible square tiles without cutting any.

(i) Should he find the HCF or LCM of the floor's length and width to get the tile size?

(ii) Write the prime factorization of 1517 and 902.

(iii) What will be the side length of the largest square tile?

(iv) How many tiles will he need in total?

Q. 85. Three gears with 15, 20, and 25 teeth are connected. A mark on each gear is lined up. As they spin, they will eventually line up again.

(i) Write the prime factorization for the tooth counts 15, 20, and 25.

(ii) Find the LCM of 15, 20, and 25.

(iii) How many teeth must pass before all three marks line up again?

(iv) How many full turns will the smallest gear (15 teeth) make when they line up?

Q. 86. A dairy has two tanks of milk with 850 liters and 680 liters. The manager needs a measuring bucket to empty both tanks exactly, without any milk left.

(i) Which mathematical tool is needed to find the largest bucket size?

(ii) Find the HCF of 850 and 680.

(iii) What is the largest capacity of the measuring bucket?

(iv) How many buckets of milk are there in total in both tanks?

Q. 87. A group is packing relief boxes. They have 56 first-aid kits and 72 water bottles. They want to make identical boxes with both items, leaving nothing behind.

(i) Will HCF or LCM give the maximum number of identical boxes?

(ii) Find the HCF of 56 and 72.

(iii) How many first-aid kits and water bottles will be in each box?

(iv) What is the maximum number of boxes they can pack?

Q. 88. A student draws a right-angled triangle where the base is 1 unit and the height is 1 unit. She needs to find the length of the longest side (hypotenuse).

(i) Using Pythagoras theorem ($a^2 + b^2 = c^2$), what is the length of the hypotenuse?

(ii) Is this length a rational or an irrational number?

(iii) Give a simple reason why it is classified this way.

(iv) Can this number be written as a simple fraction like p/q?

Q. 89. A sports coach is buying equipment. He buys packs of cricket balls that have 6 balls each, and packs of tennis balls that have 8 balls each. He wants to buy an equal number of both types of balls.

(i) Should the coach find the HCF or LCM to buy the minimum equal number of balls?

(ii) Find the LCM of 6 and 8.

(iii) What is the minimum total number of each type of ball he must buy?

(iv) How many packs of cricket balls and tennis balls will he need to buy?

Q. 90. For a school camp, 60 students from Class 8, 84 from Class 9, and 108 from Class 10 are attending. They need to be put into tents. Each tent must have the same number of students, and all students in a tent must be from the same class.

(i) Which concept minimizes the number of tents needed?

(ii) Find the HCF of 60, 84, and 108.

(iii) How many students will sleep in each tent?

(iv) How many tents are needed in total?

Q. 91. Three moons orbit a planet. Moon A takes 10 days, Moon B takes 12 days, and Moon C takes 15 days for one orbit. Today, they are lined up in a straight line.

(i) Find the LCM of 10, 12, and 15.

(ii) After how many days will the three moons line up again?

(iii) How many orbits will Moon A complete by that time?

(iv) Write the prime factors of 15.

Q. 92. A sweet shop has 420 chocolates and 130 candies. The shopkeeper wants to stack them on a tray so that each stack has the same number of sweets, taking up the least space.

(i) Should we use HCF or LCM to find the number of sweets in each stack?

(ii) Find the HCF of 420 and 130.

(iii) How many sweets will be in each stack?

(iv) How many stacks will there be in total?

Q. 93. A gardener wants to plant 24 apple trees, 36 mango trees, and 48 guava trees in rows. Each row must have the same number of trees, and only one type of tree per row.

(i) Find the HCF of 24, 36, and 48.

(ii) What is the maximum number of trees that can be planted in a single row?

(iii) How many rows of mango trees will there be?

(iv) What is the total number of rows in the garden?

Q. 94. Two machines in a factory make a loud beep. Machine A beeps every 60 seconds. Machine B beeps every 62 seconds.

(i) Write the prime factorization of 60 and 62.

(ii) Find the LCM of 60 and 62.

(iii) If both machines beep together at 12:00 PM, how many seconds will pass before they beep together again?

(iv) Convert this time into minutes and seconds.

Q. 95. A lighthouse has three colored lights. The red light flashes every 15 seconds, yellow every 20 seconds, and white every 25 seconds.

(i) Find the LCM of 15, 20, and 25.

(ii) If all three flash together at midnight, when will they flash together next?

(iii) How many times will they flash together in 1 hour?

(iv) Write the prime factorization of 25.

Q. 96. A carpenter has two pieces of wood measuring 45 meters and 60 meters. He wants to cut them into the longest possible equal pieces without wasting any wood.

(i) Which concept is used to find the best cut length?

(ii) Find the HCF of 45 and 60.

(iii) How many pieces will he get from the 45-meter wood?

(iv) How many equal pieces will he have in total?

Q. 97. In a marathon, there is a water station every 12 km and a medical tent every 16 km. The race starts at 0 km.

(i) Find the LCM of 12 and 16.

(ii) At which distance will a runner first see both a water station and a medical tent together?

(iii) If the race is 86 km long, how many times will both stations be together?

(iv) What is the distance between these combined stations?

Q. 98. Two pendulums are swinging. Pendulum X completes a swing in 18 seconds. Pendulum Y takes 24 seconds.

(i) Write the prime factorization of 18 and 24.

(ii) Which concept tells us when they will swing together again?

(iii) How many seconds before both pendulums swing together?

(iv) How many swings will Pendulum X make in that time?

Q. 99. Three old clocks tick at different speeds. Clock A ticks every 63 seconds, Clock B every 70 seconds, and Clock C every 77 seconds.

(i) Write the prime factorization of 63, 70, and 77.

(ii) Find the LCM of these times.

(iii) If they tick together now, how many seconds until they tick together again?

(iv) How many times will Clock A tick before they sync up again?

Q. 100. A student measures a rectangle. Its length is $1 + \sqrt{5}$ meters.

(i) Knowing that $\sqrt{5}$ is irrational, is $1 + \sqrt{5}$ rational or irrational?

(ii) Give a simple reason why.

(iii) If the width is $1 - \sqrt{5}$, find the area (length × width).

(iv) Is the final area a rational or irrational number?