| Class : X | Subject : Mathematics |
|---|---|
| Topic : Linear equation in two variables | Prepared by: Ashish Kumar Jain |

Introduction of linear equation:

If two linear equations have the two same variables, they are called a pair of linear equations in two variables. Following is the most general form of linear equations:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Here, a1, a2, b1, b2, c1 and c2 are real numbers such that;

A pair of linear equations can be represented and solved by the following methods:

a. Graphical method

b. Algebraic method

**Exercise 3.1**

Q 1 Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Let the present age of Aftab be $x$.

And, present age of his daughter = $y$

Seven years ago,

Age of Aftab = $x$ - 7

Age of his daughter = $y$ - 7

According to the question,

$(x-7)=7(y-7)$

$x-7 = 7y-49$

$x-7y=-42 \qquad (1)$

Three years hence,

Age of Aftab = $x$ + 3

Age of his daughter = $y$ + 3

According to the question,

$(x+3)=3(y+3)$

$x+3 = 3y+9$

$x-3y=6 \qquad (2)$

Therefore, the algebraic representation is

$x-7y=-42$

$x-3y=6$

$x-7y=-42$

$x=-42+7y$

The solution table is

| | | | |
|---|---|---|---|
| $x$ | −7 | 0 | 7 |
| $y$ | 5 | 6 | 7 |

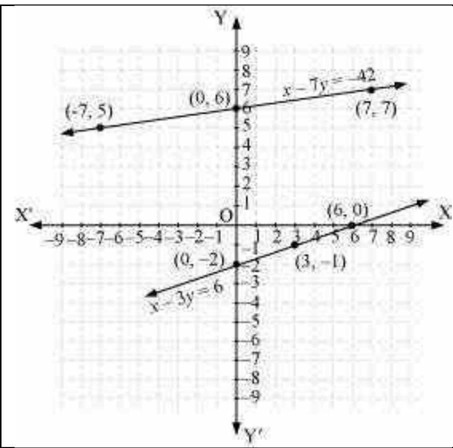
$x-3y=6$

$x=6+3y$

The solution table is

| | | | |
|---|---|---|---|
| $x$ | 6 | 3 | 0 |
| $y$ | 0 | -1 | -2 |

The graphical representation is as follows.




Q 2 The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Let the cost of a bat be Rs *x*.
And, cost of a ball = Rs *y*
According to the question, the algebraic representation is
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$3x+6y=3900$

$x=\frac{3900-6y}{3}$

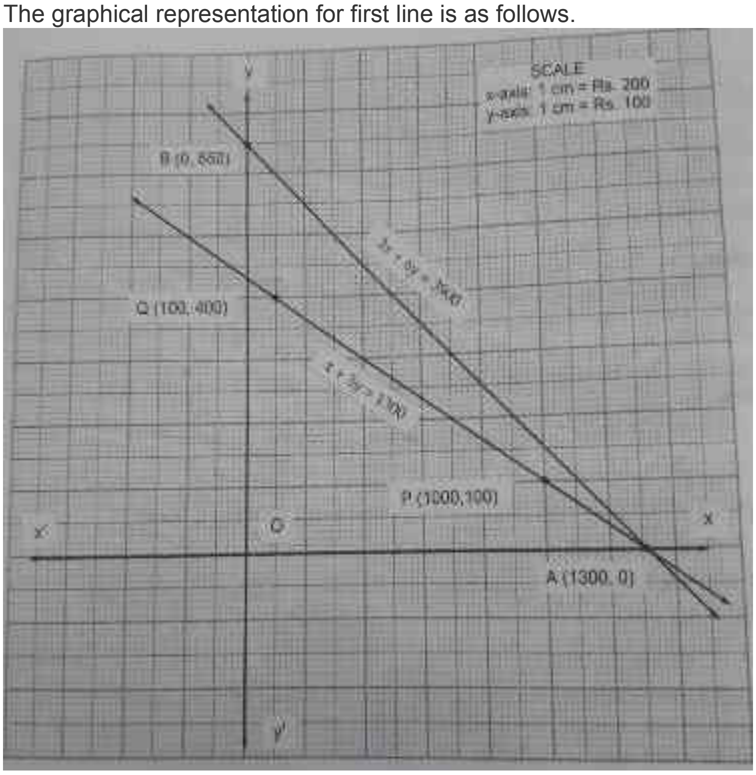
The solution table is

| | | | |
|---|---|---|---|
| X | 300 | 100 | -100 |
| y | 500 | 600 | 700 |

For
x+ 3y = 1300,
x = 1300 - 2y
The solution table is

| | | | |
|---|---|---|---|
| X | 400 | 700 | 1000 |
| y | 300 | 200 | 100 |

The graphical representation for first line is as follows.



| | |
|---|---|
| Q 3 | **The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.** |

Let the cost of 1 kg of apples be Rs *x*.
And, cost of 1 kg of grapes = Rs *y*
According to the question, the algebraic representation is

$$2x + y = 160$$
$$4x + 2y = 300$$

For $2x + y = 160$,
$y = 160 - 2x$

The solution table is

| | | | |
|---|---|---|---|
| x | 50 | 60 | 70 |
| y | 60 | 40 | 20 |

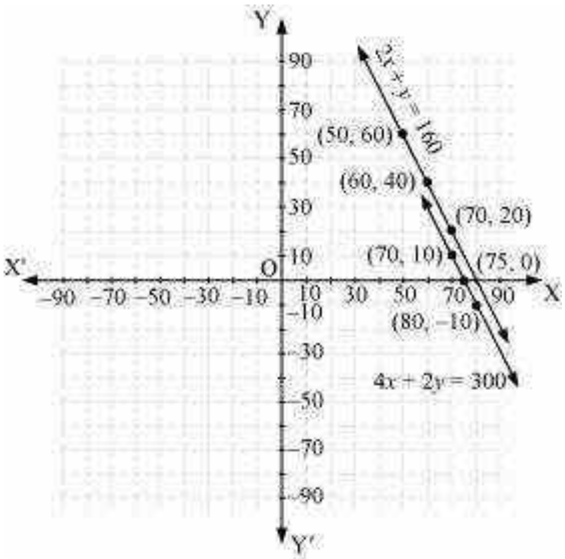
For 4*x* + 2*y* = 300,

$$y = \frac{300 - 4x}{2}$$

The solution table is

| | | | |
|---|---|---|---|
| x | 70 | 80 | 75 |
| y | 10 | - 10 | 0 |

The graphical representation is as follows.

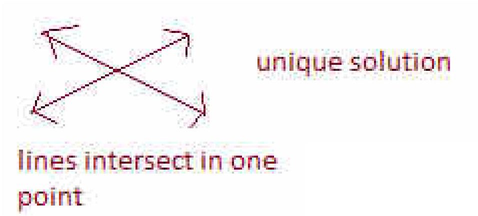



## Exercise 3.2

In this exercise, you will learn the following points.
1) An equation of the form ax + by + c = 0 where a, b & c € R, a≠ 0, b≠0 & x, y are variables, is called a linear equation in two variables.
2) The graph of a linear equation is a straight line.
3) General solution & conditions for solvability.
   A system of linear equation

$a_1x + b_1y + c_1 = 0$ &
$a_2x + b_2y + c_2 = 0$ has

i) A unique solution
When $a_1/a_2 \neq b_1/b_2$



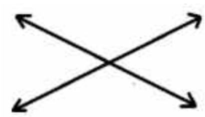

ii) Infinitely many solution , if
$a_1/a_2 = b_1/b_2 = c_1/c_2$


Lines are coinciding

iii) No solution , if
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$


The following table will give you the concept of unique solution, infinitely many solutions and no solution.

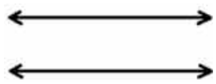
(i) a unique solution
& lines intersect at one point
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$



(ii) Infinitely many solutions
& lines are coinciding
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



(iii) No solution
& lines are parallel
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



Q 1 Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

(i) Let the number of girls be $x$ and the number of boys be $y$.

According to the question, the algebraic representation

$x + y = 10$

$x = 10 - y$

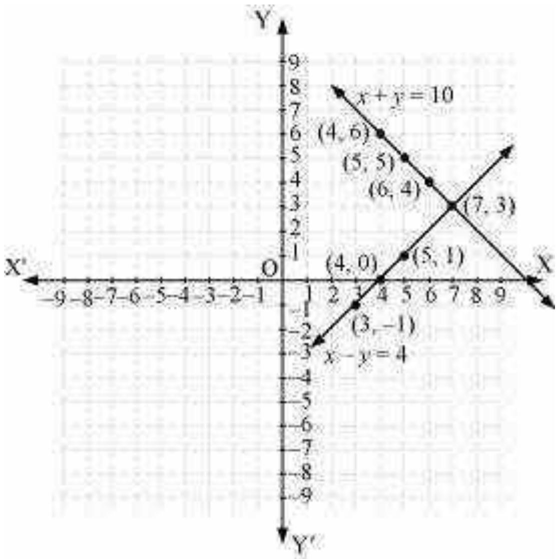
| | | | |
|---|---|---|---|
| x | 5 | 4 | 6 |
| y | 5 | 6 | 4 |

$x - y = 4$

$x = 4 + y$

| | | | |
|---|---|---|---|
| x | 5 | 4 | 3 |
| y | 1 | 0 | - 1 |

Hence, the graphic representation is as follows




From the figure, it can be observed that these lines intersect each other at point (7, 3). Therefore, the number of girls and boys in the class are 7 and 3 respectively.

ii) (i) Let the cost of 1 pencil be Rs $x$ and the cost of 1 pen be Rs $y$.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

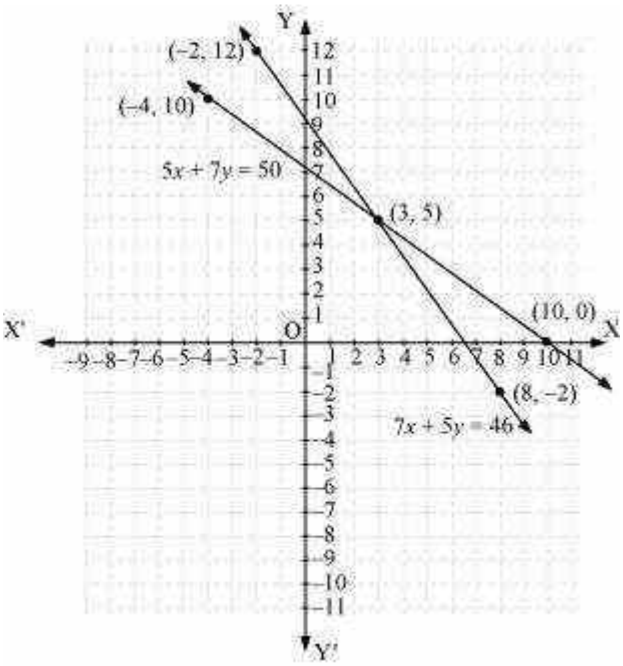
| | | | |
|---|---|---|---|
| $x$ | 3 | 1 | -4 |
| $y$ | 5 | 0 | 1 |

$7x+5y=46$

$$x = \frac{46 - 5y}{7}$$

| | | | |
|---|---|---|---|
| x | 8 | 3 | - 2 |

| | | | |
|---|---|---|---|
| y | - 2 | 5 | 12 |




From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Q 2

On comparing the ratios $a_1/a_2$, $b_1/b_2$ and $c_1/c_2$, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

(i) $5x-4y+8=0$ (ii) $9x+3y+12=0$ (iii) $6x-3y+10=0$

$7x+6y-9=0$ $\quad 18x+6y+24=0$ $\quad 2x-y+9=0$

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1x+b_1y+c_1=0$

and $a_2x+b_2y+c_2=0$, we obtain

$a_1=5, \quad b_1=-4, \quad c_1=8$

$a_2=7, \quad b_2=6, \quad c_2=-9$

$$\frac{a_1}{a_2}=\frac{5}{7}$$

$$\frac{b_1}{b_2}=\frac{-4}{6}=\frac{-2}{3}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and

$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$

$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and we obtain,

$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$

$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Q 3 **On comparing the ratios, $a_1/a_2$, $b_1/b_2$ and $c_1/c_2$ find out whether the following pair of linear equations are consistent, or inconsistent.**

(i) $3x+2y=5;\quad 2x-3y=7$ (ii) $2x-3y=8;\quad 4x-6y=9$

(iii) $\frac{3}{2}x+\frac{5}{3}y=7;\quad 9x-10y=14$ (iv) $5x-3y=11;\ -10x+6y=-22$

(v) $\frac{4}{3}x+2y=8;\quad 2x+3y=12$

(i) $3x + 2y = 5$
$2x - 3y = 7$

$$\frac{a_1}{a_2}=\frac{3}{2},\quad \frac{b_1}{b_2}=\frac{-2}{3},\quad \frac{c_1}{c_2}=\frac{5}{7}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(ii)$2x - 3y = 8$
$4x - 6y = 9$

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x+\frac{5}{3}y=7$

$9x-10y=14$

$$\frac{a_1}{a_2}=\frac{\frac{3}{2}}{9}=\frac{1}{6},\quad \frac{b_1}{b_2}=\frac{\frac{5}{3}}{-10}=\frac{-1}{6},\quad \frac{c_1}{c_2}=\frac{7}{14}=\frac{1}{2}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

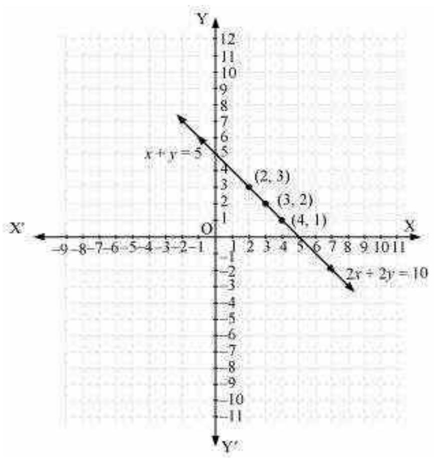
Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(iv)$5x - 3y = 11$
$-10x + 6y = -22$

$$\frac{a_1}{a_2}=\frac{5}{-10}=\frac{-1}{2},\quad \frac{b_1}{b_2}=\frac{-3}{6}=\frac{-1}{2},\quad \frac{c_1}{c_2}=\frac{11}{-22}=\frac{-1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

| | |
|---|---|
| v) | (v) $\frac{4}{3}x + 2y = 8$<br>$2x + 3y = 12$<br>$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$<br>$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$<br>Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent. |
| Q 4 | **Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:**<br>(i) $x + y = 5, \quad 2x + 2y = 10$<br>(ii) $x - y = 8, \quad 3x - 3y = 16$<br>(iii) $2x + y - 6 = 0, \; 4x - 2y - 4 = 0$<br>(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$ |
| | (i)$x + y = 5$<br>$2x + 2y = 10$<br>$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$<br>$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$<br>Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.<br><br><br>From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.<br>(ii)$x - y = 8$<br>$3x - 3y = 16$ |

$$\frac{a_1}{a_2}=\frac{1}{3}, \quad \frac{b_1}{b_2}=\frac{-1}{-3}=\frac{1}{3}, \quad \frac{c_1}{c_2}=\frac{8}{16}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii)$2x + y - 6 = 0$
$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}, \quad \frac{b_1}{b_2}=\frac{-1}{2}, \quad \frac{c_1}{c_2}=\frac{-6}{-4}=\frac{3}{2}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

iv)The given pair is

$2x -2y -2 = 0$

$4x – 4y -5 = 0$

$a_1/a_2=2/4=1/2$

$b_1/b_2=-2/-4=1/2$

$c_1/c_2 =-2/-5=2/5$

Therefore ½=1/2≠2/5

$a_1/a_2= b_1/b_2 \neq c_1/c_2$

Hence , given pair of system have consistent solution.

Q 5 **Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

Let the width of the garden be $x$ and length be $y$. According to the question, $y - x = 4$……….. (1)
$y + x = 36$………. (2)

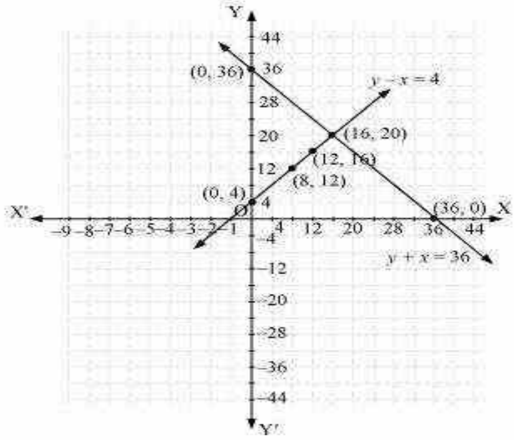
$y - x = 4$
$y = x + 4$

| | | | |
|---|---|---|---|
| x | 0 | 8 | 12 |
| y | 4 | 12 | 16 |

$y + x = 36$

| | | | |
|---|---|---|---|
| x | 0 | 36 | 16 |
| y | 36 | 0 | 20 |

Hence, the graphic representation is as follows.




From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Q 6

**Given the linear equation $2x + 3y - 8 = 0$, write another linear equations in two variables such that the geometrical representation of the pair so formed is:**

**(i) intersecting lines (ii) parallel lines (iii) coincident lines**

(i)Intersecting lines:

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line is

(ii) Parallel lines:

For this condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

$4x + 6y - 8 = 0$

$$\text{as} \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-8} = 1$$

$$\text{And clearly,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$2x + 4y - 6 = 0 \quad \text{as} \quad \frac{a_1}{a_2} = \frac{2}{2} = 1, \quad \frac{b_1}{b_2} = \frac{3}{4} \text{ and } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}.$$

(iii)Coincident lines:

For coincident lines,

$$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$$
$$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$$

Hence, the second line can be

$6x + 9y - 24 = 0$

as $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q7 **Draw the graphs of the equations *x* - *y* + 1 = 0 and 3*x* + 2*y* - 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the *x*-axis, and shade the triangular region.**

$x - y + 1 = 0$

$x = y - 1$

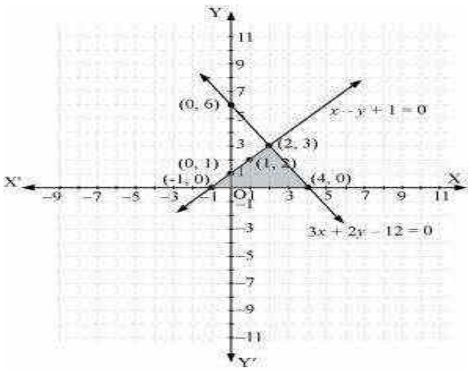
| | | | |
|---|---|---|---|
| x | 0 | 1 | 2 |
| y | 1 | 2 | 3 |

$3x + 2y - 12 = 0$

$$x = \frac{12 - 2y}{3}$$

| | | | |
|---|---|---|---|
| x | 4 | 2 | 0 |
| y | 0 | 3 | 6 |

Hence, the graphic representation is as follows.




From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and *x*-axis at ( - 1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), ( - 1, 0), and (4, 0).

Area of ▲ = ½[base X altitude]

= ½ [5 X 3] = ½[15] = 7.5 unit$^2$

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