

Electric current → The flow of electric charges through a conductor constitutes an electric current.

Electric current = $\frac{\text{Electric charge}}{\text{Time}}$

$$I = \frac{Q}{t} \quad (\text{if the current is steady})$$

$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$ (if the charge ΔQ passes through an area in time t to $t + \Delta t$, then current I at time t is)

SI unit of current →

$$1 \text{ mA} = 10^{-3} \text{ A}$$

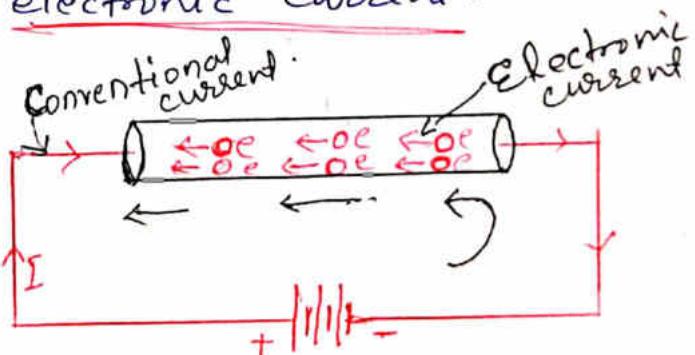
$$1 \text{ nA} = 10^{-9} \text{ A}$$

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

Direction of current → The current constituted by the flow of electrons along the direction of electrons is known as electronic current.

The direction of flow of current opposite to the direction of flow of electrons is known as conventional current.

(From +ve terminal of battery to -ve terminal)

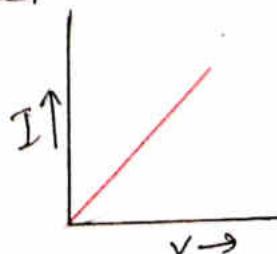


Ohm's Law - The current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided temperature and other physical conditions remain unchanged.

$$V \propto I \Rightarrow V = RI \Rightarrow R = \frac{V}{I}$$

SI unit of resistance is ohm (Ω)

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}$$



Factors affecting the resistance

1. Resistance is directly proportional to length.
 $R \propto l$.

2. Area of cross-section -
 $R \propto \frac{1}{A}$

3. Nature of the material -

The resistance of a conductor depends on the nature of the material.

Combining all, $R \propto \frac{l}{A} \Rightarrow R = \rho \frac{l}{A}$

$\Rightarrow \rho = \frac{RA}{l}$ where ' ρ ' is constant of proportionality, known as resistivity or specific resistance.

If $I = 1$ unit, $A = 1$ square unit, then $R = \rho$.

The resistivity or specific resistance of a material may be defined as the resistance of a conductor of that material, having unit length and unit area of cross-section.

SI unit of $\rho \rightarrow \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm} \times \text{m}$.

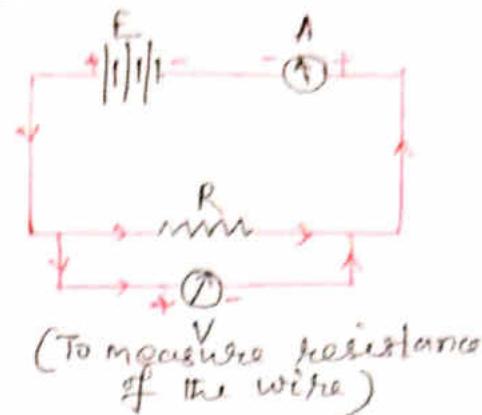
Current Density - The current density at any point inside a conductor is defined as the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.

If $I \rightarrow$ current flowing uniformly.

$A \rightarrow$ area of cross-section of the conductor.

current density $j = \frac{q/t}{A} = \frac{I}{A}$.

It is a vector quantity having the direction of conventional current -



(To measure resistance of the wire)

Conductance - Reciprocal of resistance and denoted by G .

$$G = \frac{1}{R}$$

SI unit of conductance is ohm^{-1} or mho or siemens.

Conductivity - The reciprocal of the resistivity of a material is called its conductivity and denoted by σ .

$$\sigma = \frac{1}{\rho}$$

SI unit of conductivity is $\text{ohm}^{-1} \text{m}^{-1}$ or mho m⁻¹.

Drift velocity and Relaxation time - The free electrons in a metal move with velocities of the order of 10^5 m/s . These velocities are in random directions.

If $\vec{u}_1, \vec{u}_2, \vec{u}_3, \dots, \vec{u}_N$ are the random velocities of N free electrons, then average velocity of electrons will be $\vec{U} = \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} \approx 0$.

In presence of an external field \vec{E} , each electron experiences a force $-e\vec{E}$ in the opposite direction of \vec{E} .

The acceleration

$$\vec{a} = \frac{-e\vec{E}}{m}$$

where 'm' is the mass of an electron.

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$$

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2$$

$$\vdots$$

$$\vec{v}_N = \vec{u}_N + \vec{a}\tau_N$$

The average velocity \vec{V}_d of all the N electrons will be

$$\vec{V}_d = \frac{\vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \dots + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

$$= \frac{\vec{U}_1 + \vec{U}_2 + \dots + \vec{U}_N}{N} + \vec{a} \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N}$$

$$= 0 + \vec{a}\tau$$

where $\tau = \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N}$ is the average time between two successive collisions; which is known as relaxation time.

For most of the conductors, it is of the order $\sim 10^{-14}$ sec.

$$\text{Now, } \vec{V}_d = \vec{a}\tau = -\frac{eE\tau}{m}$$

\vec{V}_d is called drift velocity of electrons. It is defined as the average velocity gained by the free electrons of a conductor in the direction opposite to the applied electric field.

Relation between electric current and drift velocity

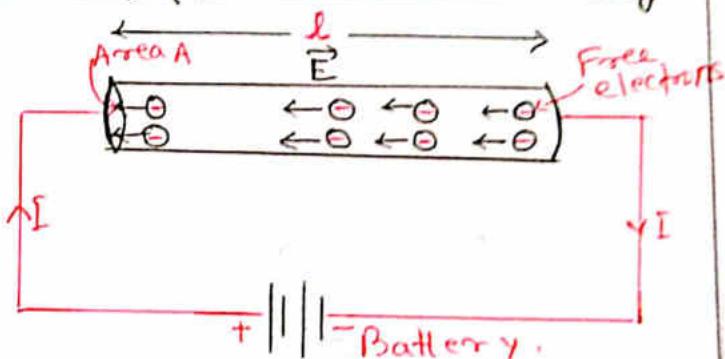
Derivation of Ohm's law-

Length of conductor $\rightarrow l$.

Applied electric field $\rightarrow \vec{E}$

Area of cross-section of the conductor $\rightarrow A$

Potential difference $\rightarrow V$.



$$E = \frac{V}{l}$$

No. of electrons per unit volume i.e. electron density
charge on electron $\rightarrow e$

Total no. of electrons $= nAl$

Total charge in the conductor $= q = enAl$

$$\text{time } t = \frac{\text{distance}}{\text{velocity}} = \frac{l}{V_d}$$

$$\text{Current } I = \frac{q}{t} = \frac{enAl}{l/V_d} \Rightarrow I = neAV_d$$

This is the relation between I and V_d .

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Conti ...

Deduction of Ohm's law → when a potential difference V is applied across a conductor of length l , the drift velocity

$$V_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

The current through the conductor will be

$$I = n e A V_d = n e A \cdot \frac{eV\tau}{ml}$$

or, $\boxed{\frac{V}{l} = \frac{ml}{ne^2 \tau A}}$

At a fixed temperature, the quantities n, l, σ, e, τ and A , all have constant values for a given conductor.

$$\text{So, } \frac{V}{l} = R \Rightarrow \boxed{R = \frac{ml}{ne^2 \tau A}}$$

Resistivity in terms of electron density and relaxation time -

The resistance R is given by

$$R = \rho \frac{l}{A}$$

But, $R = \frac{ml}{ne^2 \tau A}$

where τ is relaxation time.
Comparing, we get

$$\boxed{\rho = \frac{m}{ne^2 \tau}}$$

ρ does not depend upon dimensions of a conductor.
but depends upon → no. of free electrons per unit volume.

→ the relaxation time τ .

Mobility of charge carriers :- The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field.

$$\boxed{M_e = \frac{V_d}{E}}$$

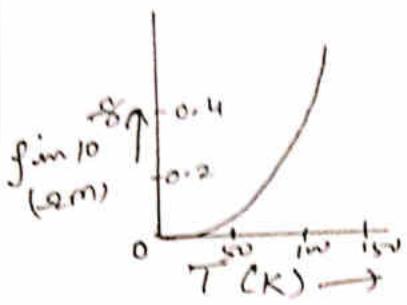
Temperature Dependence of Resistivity

Resistivity is given by $\rho = \frac{m}{ne^2}$

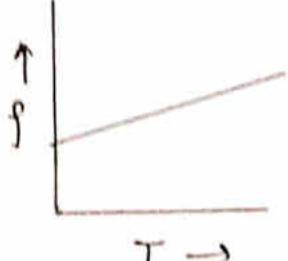
For Metals \rightarrow

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

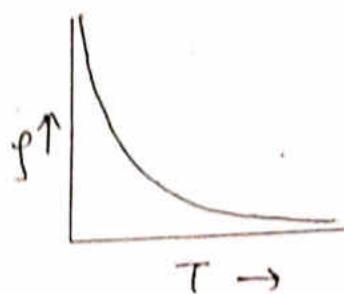
$$\alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)}, \text{ where } \alpha \text{ is temperature coefficient of resistivity.}$$



(Copper)



(Nichrome)



(Semiconductor)

For most of the metals, resistivity increases linearly with the increase in temperature.

Alloys have high resistivity. The resistivity of nichrome has weak temperature dependence.

$$R = \rho \frac{l}{A} \Rightarrow R \propto \rho.$$

$$R_t = R_0(1 + \alpha t)$$

Semiconductors and insulators — In case of semiconductors and insulators, the relaxation time τ does not change with temperature.

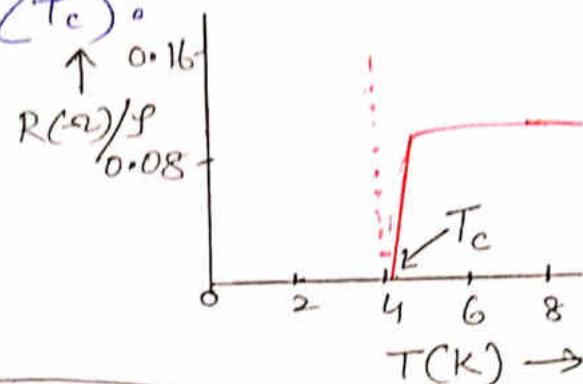
The no. density of electrons at temperature T is given by $n(T) = n_0 e^{-E_g/k_B T}$ where k_B is the Boltzmann constant and E_g is the energy gap.

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Confidence - - -

Superconductivity - The phenomenon of complete loss of resistivity by certain metals and alloys when they are cooled below certain temperature is called superconductivity. The temperature at this point is called critical temperature (T_c).

Resistivity of mercury is zero at 4.2 K and becomes superconductor.

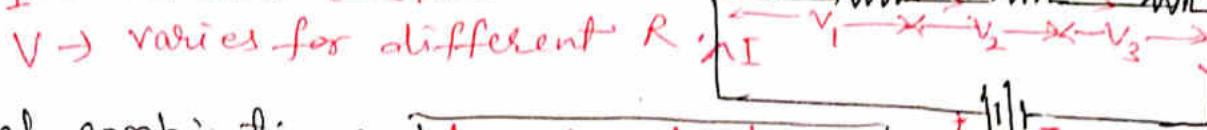


Combination of resistances

Series combination →

$$R_s = R_1 + R_2 + R_3 + \dots$$

$I \rightarrow$ remains constant.



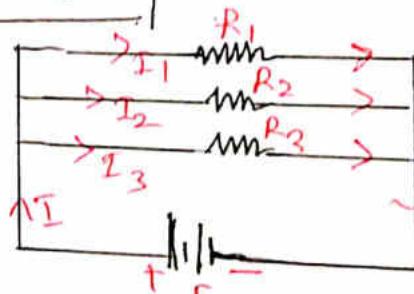
Parallel combination →

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$V \rightarrow$ remains constant.

I varies for different R .

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$



Internal Resistance of a cell -

The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal resistance of the cell.

It depends on →

→ nature of electrolyte

→ directly proportional to the concentration of the electrolyte.

→ directly proportional to the distance between the two electrodes.

→ varies inversely as the common area of the electrodes immersed.

→ increases with decrease in temperature of the electrolyte.

[The internal resistance of a freshly prepared cell is low, its value increases as more and more current is drawn from it.]

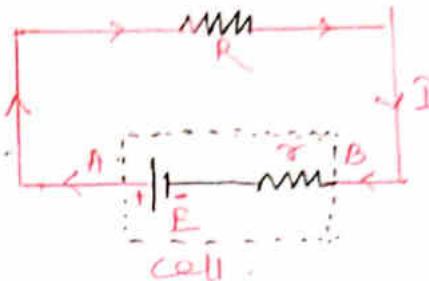
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Conti... .

Relation between τ , E and V → The potential drop across the terminals of a cell when a current is being drawn, is called terminal potential difference (V).

A cell has e.m.f. E and internal resistance τ connected to an external resistance R .

E = Work done to bring a unit charge along the closed circuit by the cell.



= Work done in carrying a unit charge from A to B against external resistance R

+ Work done in carrying a unit charge from B to A against internal resistance τ .

$$\text{or, } E = V + V' \quad \text{But, } V = IR \text{ and } V' = I\tau$$

$$\Rightarrow E = I(R + \tau) \Rightarrow I = \frac{E}{R + \tau}$$

$$\text{Now, } V = IR = \frac{ER}{R + \tau}$$

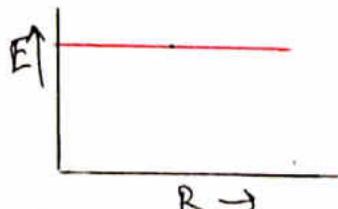
$$V' = E - V = E - I\tau \Rightarrow \tau = \frac{E - V}{I} = \left(\frac{E - V}{V}\right)R$$

Special cases

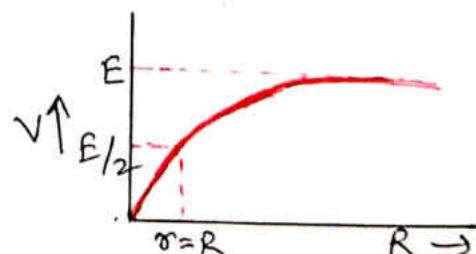
→ when cell is in open circuit $\Rightarrow I = 0$.

$$V_{\text{open}} = E$$

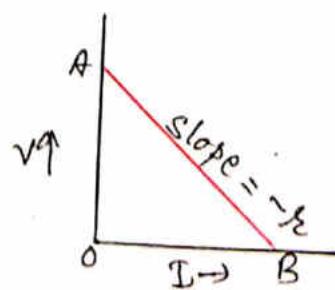
→ a real cell has always some internal resistance.
So, $V < E$.



($E \propto R$ graph)
of a cell



($V \propto R$ graph)
of a cell



($V \propto I$ graph of cell)

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