

Parallel Plate capacitor -

Let $A \rightarrow$ area of each plate

$d \rightarrow$ distance between the two

$\pm\sigma \rightarrow$ uniform surface charge density on the two plates

$\pm Q = \pm \sigma A =$ total charge on the plates

In the outer regions above the upper plate and below the lower plate, the electric fields due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

In the inner region between two capacitor plates, the electric fields due to the two charged plates add up. The net field is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Potential difference between the plates

= Electric field \times distance between the plates

$$V = Ed = \frac{\sigma d}{\epsilon_0}$$

Capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d / \epsilon_0} \text{ or, } C = \boxed{\frac{\epsilon_0 A}{d}}$$

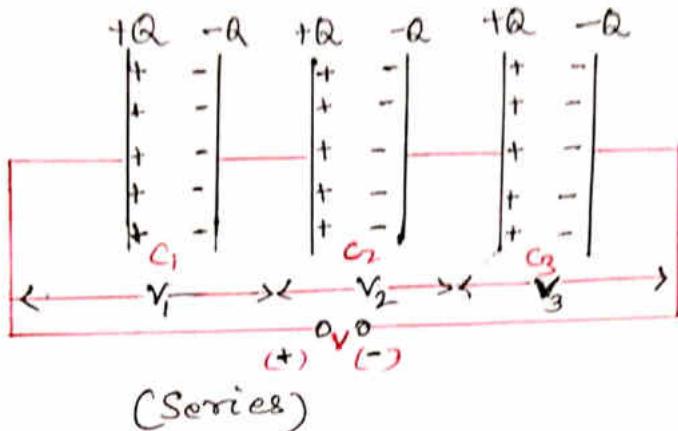
Factors on which the capacitance of a parallel plate capacitor depends -

\rightarrow Area of the plates ($C \propto A$)

\rightarrow Distance between the plates ($C \propto \frac{1}{d}$)

\rightarrow Permittivity of the medium between the plates ($C \propto \epsilon_0$)

Combination of capacitors in series and in parallel -



$$C_p = C_1 + C_2 + C_3$$

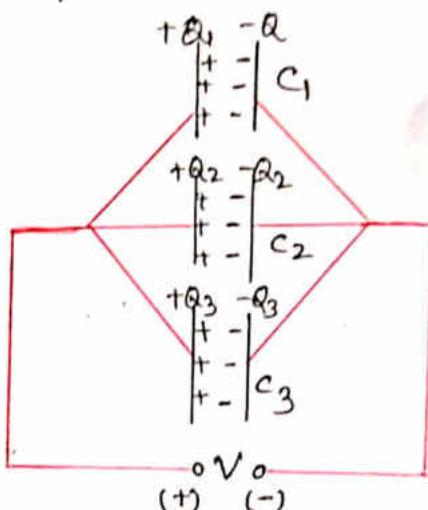
→ The P.D. across each capacitor is same.

→ The charge on each capacitor is proportional to its capacitance.

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

→ The charge on each capacitor is same

→ The P.D. across any capacitor is inversely proportional to its capacitance.



(Parallel)

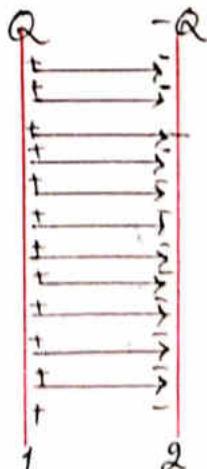
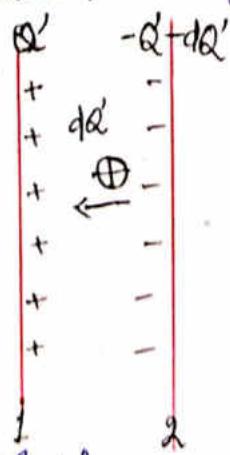
Energy stored in a capacitor -

The work done in charging the capacitor is stored as its electrical potential energy.
(This energy is supplied by the battery at the expense of its stored chemical energy.)

Let us consider a capacitor of capacitance C . Originally, its two plates are uncharged.

Let positive charge is transferred from plate 2 to plate 1 bit by bit.

So, external work is to be done as plate 1 is at higher potential always.



Prepared at home

Contin... 2

Let at any instant plates 1 and 2 have charges Q' and $-Q'$ respectively. Then the potential difference between the plates

$$V' = \frac{Q'}{C}$$

Let a small additional charge dQ' be transferred from plate 2 to plate 1. The work done will be

$$dW = V' \cdot dQ' = \frac{Q'}{C} \cdot dQ'$$

$$\Rightarrow W = \int dW = \int_0^Q \frac{Q'}{C} dQ' = \left[\frac{Q'^2}{2C} \right]_0^Q = \frac{1}{2} \frac{Q^2}{C}$$

The workdone is stored as electrical potential energy U of the capacitor.

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

Energy Density: - Let in parallel plate capacitor, plate area 'A' and plate separation 'd'.

Capacitance

$$C = \frac{\epsilon_0 A}{d}$$

If ' σ ' is the surface charge density on the capacitor plates, then electric field between the plates is $E = \frac{\sigma}{\epsilon_0}$ or, $\sigma = \epsilon_0 E$

Energy stored in the capacitor is

$$U = \frac{Q^2}{2C} = \frac{(\epsilon_0 EA)^2}{2 \cdot \epsilon_0 A} = \frac{1}{2} \epsilon_0 E^2 Ad.$$

But Ad = volume of the capacitor between the plates.

So, energy stored per unit volume or energy density is given by

$$u = \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

Energy stored would be additive in Series and parallel both :-

For series combination, $Q = \text{constant}$.

Total energy,

$$U = \frac{Q^2}{2} \cdot \frac{1}{C} = \frac{Q^2}{2} \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \right]$$
$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \dots$$

$$\Rightarrow U = U_1 + U_2 + U_3 + \dots$$

for parallel combination, $V = \text{constant}$.

Total energy,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} [C_1 + C_2 + C_3 + \dots] V^2$$
$$= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \frac{1}{2} C_3 V^2 + \dots$$

$$\Rightarrow U = U_1 + U_2 + U_3 + \dots$$

Redistribution of charges :-

'A' has charge

$$Q_1 = C_1 V_1$$

'B' has charge

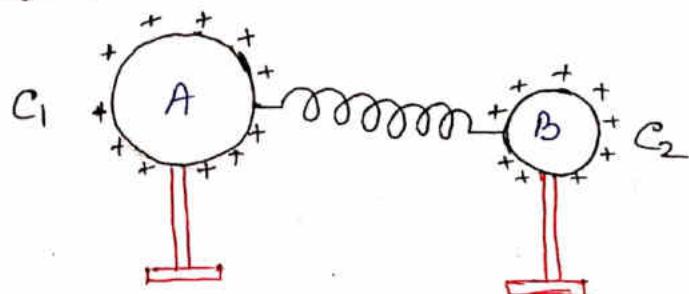
$$Q_2 = C_2 V_2$$

$$\text{Total charge} = Q_1 + Q_2$$

Total capacitance of two spheres which are separated from each other = $C_1 + C_2$

Common potential = $\frac{\text{Total charge}}{\text{Total capacitance}}$

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$



Loss of energy in redistribution of charges

Let C_1 and C_2 be the capacitances and V_1 and V_2 be the potentials of the two conductors before connection. Total potential energy before connection is

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

After connection, let V be their common potential.

Then, $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

Potential energy after connection is

$$\begin{aligned} U_f &= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2} (C_1 + C_2) V^2 \\ &= \frac{1}{2} (C_1 + C_2) \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right]^2 = \frac{1}{2} \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)^2 \end{aligned}$$

Loss in energy

$$\begin{aligned} U &= U_i - U_f = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2} \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)^2 \\ &= \frac{1}{2 (C_1 + C_2)} [C_1^2 V_1^2 + C_2^2 V_2^2 + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2] \\ &= \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} (V_1^2 + V_2^2 - 2V_1 V_2) \\ &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2 \end{aligned}$$

This is positive in all cases. So, when two charged conductors are connected, charges flow from higher to lower potential till both are equally charged.

There is always loss of potential energy in the form of heat due to the flow of charges in the wire.

Dielectrics — is a substance which does not allow the flow of charges through it; but can conduct when external electric field is applied.

Polar and Non-polar dielectrics :-

A molecule in which the centre of mass of positive charges does not coincide with the centre of mass of negative charges is called polar molecule. The dielectric having polar molecules is polar dielectric.

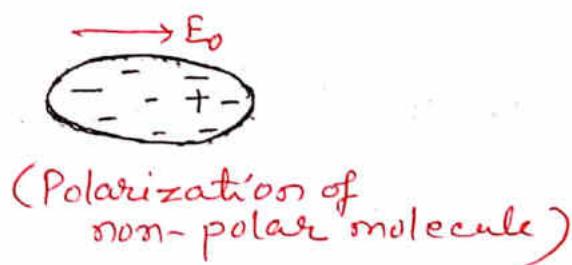
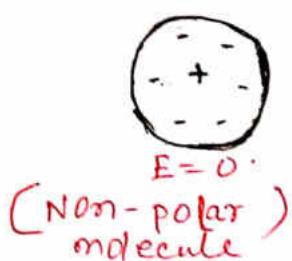
Ex - H_2O , HCl , NH_3 , CO etc.

They have permanent dipole moment of order 10^{-30} Cm.

A molecule in which the centre of mass of positive charge coincides with the centre of mass of negative charges is called non-polar molecules.

They have normally zero dipole moment.

Ex - H_2 , N_2 , O_2 , CO_2 , CH_4 etc.

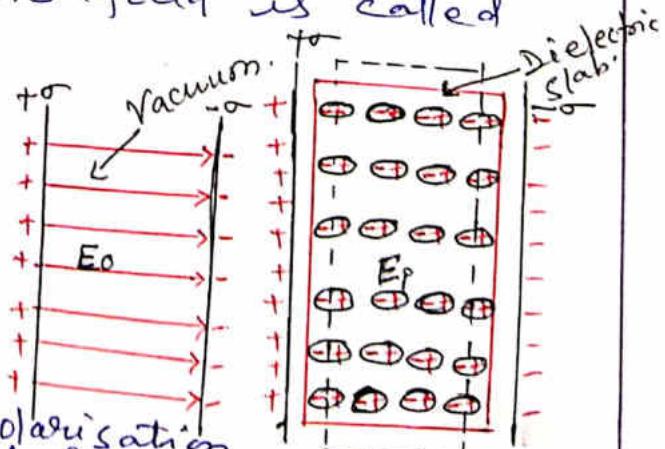


Polarisation of a polar dielectric in an external electric field.

The stretching of dielectric atoms due to displacement of charges in the atoms under the action of the applied electric field is called polarisation.

Polarisation of a Dielectric Slab :-
when vacuum is there between the two parallel plates electric field $E_0 = \frac{\sigma}{\epsilon_0}$.

When a dielectric slab of non-polar atoms is introduced between the plates, due to polarisation electric field E_p is produced in opposite to E_0 .
Prepared at Home.



Conti - ...

The resultant dielectric field in the dielectric is $E = E_0 - E_p$. (reduced electric field)

Dielectric constant - The ratio of the strength of the applied electric field to the strength of the reduced value of electric field is called dielectric constant.

$$K = \frac{E_0}{E}$$

Capacitance of a parallel plate capacitor, when a dielectric slab partially fills the space between plates -

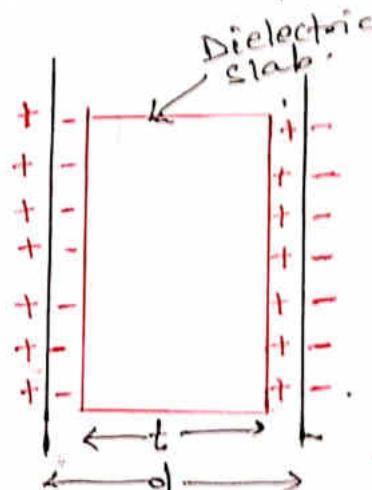
Parallel plates, each with area $\rightarrow A$.

Separation $\rightarrow d$.

When there is vacuum between the plates,

$$C_0 = \frac{\epsilon_0 A}{d}$$

When the capacitor is connected to a battery, electric field of strength E_0 is produced. When dielectric slab of thickness t ($t < d$) is introduced between the plates, polarisation takes place.



E is the electric field at a distance t

E_0 is the electric field at a distance $(d-t)$

$V \rightarrow$ the potential difference between the plates of the capacitor.

$$V = Et + E_0(d-t) \quad \text{But } \frac{E_0}{E} = k$$

$$\text{or, } E = \frac{E_0}{K} \Rightarrow V = \frac{E_0}{K}t + E_0(d-t) = E_0(d-t + \frac{t}{K})$$

$$\text{Now, } E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

$$V = \frac{q}{\epsilon_0 A} (d-t + \frac{t}{K})$$

$$\Rightarrow C = \frac{q}{V} = \frac{q}{\frac{q}{\epsilon_0 A} (d-t + \frac{t}{K})} = \frac{\epsilon_0 A}{d-t + \frac{t}{K}} \Rightarrow C = \frac{\epsilon_0 A}{d-t(1-\frac{1}{K})}$$

Prepared at Home.

Contin...

If the dielectric slab wholly fills the space between the plates, then $t = d$.

$$C = \frac{\epsilon_0 A}{d - d(1 - \frac{1}{k})} \Rightarrow C = \frac{\epsilon_0 k A}{d}$$

$$\Rightarrow C = k C_0$$

When conducting slab is introduced between the plates of a capacitor:-

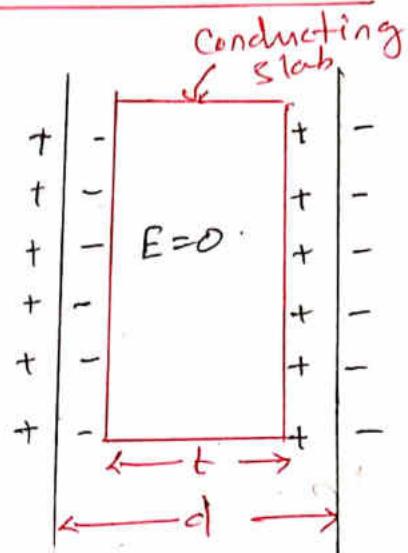
$$C_0 = \frac{\epsilon_0 A}{d}$$

Electric field inside the conducting slab is zero.

$$V = \epsilon_0 (d - t)$$

$$V = \frac{d}{\epsilon_0 A} (d - t)$$

$$C = \frac{q}{V} = \frac{q}{\frac{d}{\epsilon_0 A} (d - t)} = \frac{\epsilon_0 A}{d - t}$$



$$C = \frac{\epsilon_0 A}{d(1 - \frac{t}{d})} \Rightarrow C = \frac{C_0}{(1 - \frac{t}{d})}$$

If $t = d$ (slab fills the space wholly)
then,

$$C = \frac{C_0}{1 - \frac{d}{d}} = \frac{C_0}{0} = \infty$$