Class Notes

Class:12	Topic: INTEGRATION - 2
Subject: MATHEMATICS	

Definition – A function $g(x)$ is said to be integral of a function $f(x)$ if $\frac{d}{dx}g(x) = f(x)$

Integration is of two types

1. Indefinite Integration
2. Definite Integration

Indefinite Integration – Let f(x) be a function. Then the family of all its anti-derivative is called the indefinite integration.

Denoted by $\int f(x)dx$

So, $\frac{d}{dx}g(x) = f(x) \Leftrightarrow \int f(x)\, dx = g(x)$

FORMS OF INTEGRATION

1. $\int \frac{ax+b}{(cx+d)^n} dx$

In order to evaluate this type of integration we have to use the following algorithm

Step 1- Write ax +b in terms of cx +d in following way

$$ax + b = \lambda(cx + d) + \mu$$

Step 2 – Find λ *and* μ by equating the coefficient of like power of x and constant

Step 3 – Replace $ax + b$ by $\lambda(cx + d) + \mu$ in the given integration we get

$$\int \frac{ax + b}{(cx + d)^n} dx = \lambda \int \frac{cx + d}{(cx + d)^n} dx + \mu \int \frac{1}{(cx + d)^n} dx$$

Now integrate by using the formula we get the result

Example - $\int \frac{2x+3}{(x-1)^2} dx$

Solution – Let $2x + 3 = \lambda(x - 1) + \mu$

$$2 = \lambda \; and \; 3 = -\lambda + \mu$$

$$3 = -2 + \mu$$

So, $\mu = 5$

Therefore, $\int \frac{2x+3}{(x-1)^2} dx = 2\int \frac{x-1}{(x-1)^2} dx + 5\int \frac{1}{(x-1)^2} dx$

$$= 2\int \frac{1}{x-1} dx + 5\int \frac{1}{(x-1)^2} dx$$

$$= 2log(x-1) + 5\frac{(x-1)^{-2+1}}{-2+1}$$

$$= 2log(x-1) - \frac{5}{x-1} + c$$

FORMS OF INTEGRATION

$$\int \frac{(ax+b)^n}{(cx+d)^r} dx \qquad \text{Where } n > r$$

In order to evaluate this type of integration we have to use the following algorithm

Step 1 – Open the polynomial by binomial

Step 2 -Divide the polynomial and write it in mixed fraction

Step 3 – put the value and integrate

$$Example \int \frac{x^3}{(x+2)^2}\,dx$$

$$I = \int \frac{x^3}{(x+2)^2}\,dx$$

$$= \int \frac{x^3}{x^2+4x+4}\,dx$$

Let $\frac{x^3}{x^2+4x+4} = x - 4 + \frac{12x+16}{x^2+4x+4}$

$$I = \int x - 4 + \frac{12x+16}{x^2+4x+4}\,dx$$

$$= \int x dx - 4\int dx + \int \frac{12x+16}{(x+2)^2}\,dx$$

$$= \int x dx - 4\int dx + 4\int \frac{3x+4}{(x+2)^2}\,dx$$

$$I = \frac{x^2}{2} - 4x + 4I_1 \text{.......(1)}$$

$$I_1 = \int \frac{3x+4}{(x+2)^2}\,dx$$

$$3x + 4 = \lambda(x+2) + \mu$$

$$\lambda = 3 \; and \; 4 = 2\lambda + \mu$$

$$-2 = \mu$$

$$I_1 = 3\int \frac{x+2}{(x+2)^2}dx - 2\int \frac{1}{(x+2)^2}dx$$

$$I_1 = 3log(x+2) + 2\frac{1}{x+2}$$

Putting the value of I_1 in (1) we get

$$I = \frac{x^2}{2} - 4x + 12log(x+2) + \frac{8}{x+2}$$

FORMS OF INTEGRATION

$$\int (ax+b)\sqrt{cx+d}\,dx \;\; and \; \int \frac{ax+b}{\sqrt{cx+d}}dx$$

In order to evaluate this type of integration we have to use the following algorithm

Step 1- Write ax +b in terms of cx +d in following way

$$ax + b = \lambda(cx+d) + \mu$$

Step 2 – Find $\lambda \; and \; \mu$ by equating the coefficient of like power of x and constant

Step 3 – Replace $ax + b$ by $\lambda(cx + d) + \mu$ in the given integration we get

$$\int (ax + b)\sqrt{cx + d}\, dx = \int (\lambda(cx + d) + \mu)\sqrt{cx + d} dx$$

$$= \lambda \int (cx + d)\sqrt{cd + d} dx + \mu \int \sqrt{cx + d}\, dx$$

$$= \lambda \int (cx + d)^{\frac{3}{2}} dx + \mu \int \sqrt{cx + d}\, dx$$

OR

$$\int \frac{ax + b}{\sqrt{cx + d}} dx = \lambda \int \frac{cx + d}{\sqrt{cx + d}} dx + \mu \int \frac{1}{\sqrt{cx + d}} dx$$

Now integrate by using the formula we get the result

$\int (x + 2)\sqrt{3x + 5}$dx

$$Solution \;\; - Let\ I = \int (x+2)\sqrt{3x+5}\,dx$$

$$Then, x + 2 = \lambda(3x+5) + \mu$$

$$1 = 3\lambda \ and\ 2 = 5\lambda + \mu$$

$$\frac{1}{3} = \lambda \ and\ 2 = 5\left(\frac{1}{3}\right) + \mu$$

$$2 = \frac{5}{3} + \mu$$

$$\mu = \frac{1}{3}$$

$$I = \int (x+2)\sqrt{3x+5}\,dx$$

$$= \frac{1}{3}\int (3x+5)\sqrt{3x+5}\,dx + \frac{1}{3}\int \sqrt{3x+5}\,dx$$

$$= \frac{1}{3}\int (3x+5)^{\frac{3}{2}}\,dx + \frac{1}{3}\int (3x+5)^{\frac{1}{2}}\,dx$$

$$= \frac{1}{3}\frac{(3x+5)^{\frac{5}{2}}}{3\times\frac{5}{2}} + \frac{1}{3}\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$

Question solves from NCERT