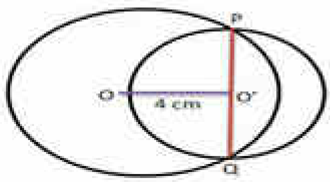
Class Notes

Class: IX	Topic: CIRCLES EX.10.4
Subject: MATHEMATICS	

Question- 1

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Solution:



We know that if two circles intersect each other at two points, then the line joining their centres is the perpendicular bisector of their common chord.

∴ Length of the common chord = PQ = 2O'P

= 2 × 3 cm = 6 cm.

Question- 2

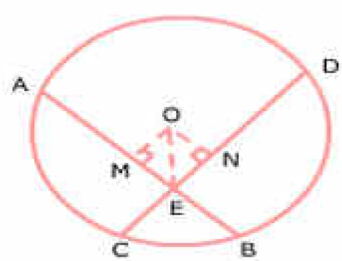
If two equal chords of a circle intersect within the circle, prove that the segments of one chord is equal to corresponding segments of the other chord.

Solution:

Given: A circle with centre O. Its two equal chords AB and CD intersect at E.

To prove: AE = DE and CE = BE

Construction: Draw OM ⊥ AB and ON ⊥ CD. Join OE.



Proof: In ΔOME and ΔONE,

OM = ON (Equal chords of a circle are equidistant from the centre)

OE = OE (Common)

$\therefore \Delta OME \cong \Delta ONE$ (R.H.S.)

$\Rightarrow ME = NE$ (CPCT)

$\Rightarrow AM + ME = DN + NE$ ($\because AM = DN = \frac{1}{2}AB = \frac{1}{2}CD$)

$\Rightarrow AE = DE$

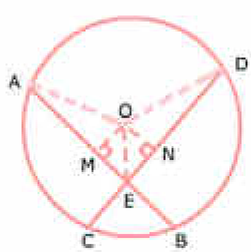
$\Rightarrow AB - AE = CD - DE$ (Given AB = CD)

$\Rightarrow BE = CE.$

Question- 3

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the center makes equal angles with the chords.

Solution:



Given: Two equal chords AB and CD of a circle with centre O intersect within the circle. Their point of intersection is E.

To Prove: $\angle OEA = \angle OED$

Construction: Join OA and OD

Proof: In Δ OEA and Δ OED

OE = OE |Common

OA = OD | Radius of a circle

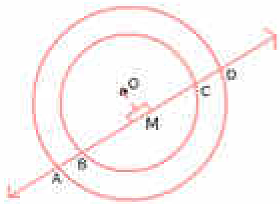
AE = DE | Proved in

$\therefore$ D OEA $\cong$ D OED | SSS Rule

$\therefore \angle OEA = \angle OED$ | CPCT

QUESTION-4 **If a line interesects two concentric cirles (circles with the same centre) with centre O at A,B,C and D prove that AB=CD.**

Solution:



Given: A line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D.

To Prove: AB = CD

Construction: Draw OM ⊥ BC.

Proof: The perpendicular drawn from the centre of a circle to a chord bisects the chord.

∴ AM = DM ....(1)

and BM = CM ...(2)

Subtracting (2) from (1), we get

AM − BM = DM − CM

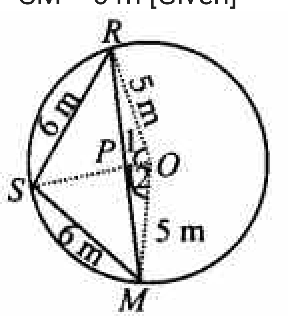
AB = CD (HP)

QUESTION-5 Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Solution:

Let the three girls Reshma, Salma and Mandip be positioned at R, S and M respectively on the circle with centre O and radius 5 m such that

RS = SM = 6 m [Given]



Equal chords of a circle subtend equal angles at the center.

∴ ∠1 = ∠2

In ΔPOR and ΔPOM, we have

OP = OP [Common]

OR = OM [Radii of the same circle]

∠1 = ∠2 [Proved above]

∴ ΔPOR ≅ ΔPOM [By SAS congruence criteria]

∴ PR = PM and

∠OPR = ∠OPM [C.P.C.T.]

∵∠OPR + ∠OPM = 180° [Linear pair]

∴∠OPR = ∠OPM = 90°

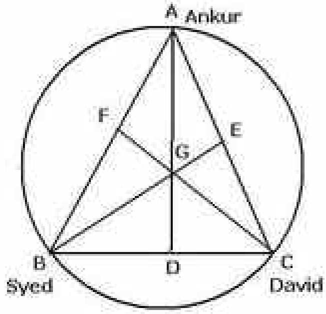
⇒ OP ⊥ RM

Now, in ΔRSP and ΔMSP, we have

RS = MS [Each 6 cm]
SP = SP [Common]
PR = PM [Proved above]
∴ ΔRSP ≅ ΔMSP [By SSS congruence criteria]
⇒ ∠RPS = ∠MPS [C.P.C.T.]
But ∠RPS + ∠MPS = 180° [Linear pair]
⇒ ∠RPS = ∠MPS = 90°
SP passes through O.
Let OP = x m
∴ SP = (5 − x)m
Now, in right ΔOPR, we have
$x^2 + RP^2 = 5^2$
$RP^2 = 5^2 - x^2$
In right ΔSPR, we have
$(5 - x)^2 + RP^2 = 6^2$
⇒ $RP^2 = 6^2 - (5 - x)^2$ ……..(ii)
From (i) and (ii), we get
⇒ $5^2 - x^2 = 6^2 - (5 - x)^2$
⇒ $25 - x^2 = 36 - [25 - 10x + x^2]$
⇒ − 10x + 14 = 0
⇒ 10x = 14 ⇒ x = 1410 = 1.4
Now, $RP^2 = 5^2 - x^2$
⇒ $RP^2 = 25 - (1.4)^2$
⇒ $RP^2 = 25 - 1.96 = 23.04$
∴ RP = $\sqrt{23.04}$= 4.8
∴ RM = 2RP = 2 x 4.8 = 9.6
Thus, distance between Reshma and Mandip is 9.6 m.

QUESTION-6 **A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary. Each boy has a toy telephone in his hand to talk with each other. Find the length of the string of each phone.**

Solution:



Given: $\overline{AB} = \overline{BC} = \overline{CA}$

i.e., Chord AB = Chord BC = Chord CA

⇒ ΔABC is equilateral

⇒ Centroid G of ΔABC coincides with its circumstance

⇒ AG = BG = CG = 20m

Since G divides AD in the ratio 2 : 1

$\frac{GA}{GD} = \frac{2}{1}$

$\Rightarrow \frac{20}{GD} = \frac{2}{1}$

GD = 10m

In ΔBDG, we have

$BG^2 = BD^2 + GD^2$

$20^2 = BD^2 + 10^2$

$BD = 10\sqrt{3}$ m

BC = 2BD

$BC = 20\sqrt{3}$ m.

∴ The length of the string of each phone = $20\sqrt{3}$ m.

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