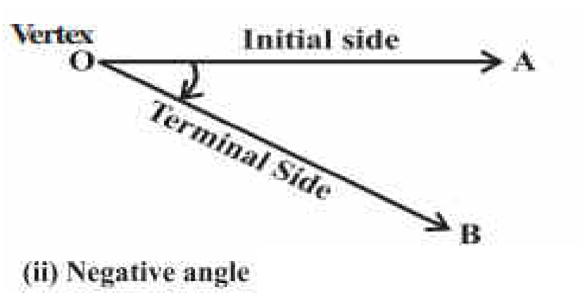
Class Notes

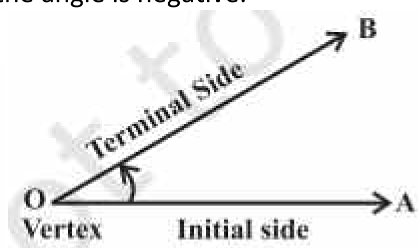
Class: 11	Topic: TRIGONOMETRIC FUNCTIONS
Subject: MATHEMATICS	

Definition: The word 'trigonometry' is derived from the Greek words 'trigon' and 'metron' and it means 'measuring the sides of a triangle'.

Angles: Angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.



(ii) Negative angle



(i)Positive angle

MEASUREMENT OF AN ANGLE

Measurement of an angle is done in three different systems

1. Sexagesimal System – In this system one right angle is divided into 90 equal parts called degree. It is also called English system of measurement. Each degree is divided into 60 equal parts called minutes and each minutes is divided into 60 equal parts called seconds .

Thus

$$1\ right\ angle = 90\ degree(90^\circ)$$

$$1\ degree = 60\ minutes(60')$$

$$1\ minutes = 60\ seconds(60'')$$

2. Centesimal System – In this system one right angle is divided into 100 equal parts called grades. It is also called French system of measurement. Each grades is divided into 100 equal parts called minutes and each minutes is divided into 100 equal parts called seconds

Thus

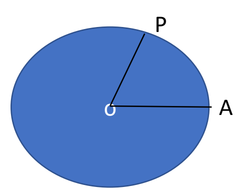
$$1\, right\ angle = 100\, grades(100^g)$$

$$1\, grade = 100\, minutes(100')$$

$$1\, minutes = 100\, seconds(100'')$$

Circular System – In this system the unit of measurement is called radian

Radian- One radian is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle. Denoted as 1^c



Let us consider a circle with centre O. Now let A and P be two point on the circle. Then AP is the arc length .If this arc length becomes equal to radius the we say that it is one radian. That means $\angle AOP = 1^c$

RELATION BETWEEN DEGREE, RADIAN AND GRADES

Relation between degree and radian -

$$360^\circ = 2\pi \; radians \quad ....(1)$$

Also

$$1 \; right \; angle = 90 \; degree(90^\circ)$$

$$4 \times 1 \; right \; angle = 4 \times 90 \; degree(90^\circ)$$

$$4 \; right \; angle = 360 \; degree(360^\circ) .....(2)$$

$$1 \; right \; angle = 100 \; grades(100^g)$$

$$4 \; right \; angle = 400 \; grades(400^g) ...(3)$$

From (1) and (2) we get

$$400 \; grades(400^g) = 360 \; degree(360^\circ) ....(4)$$

From (1) and (4) we get

$$400 \; grades(400^g) = 360 \; degree(360^\circ) = 2\pi \; radians$$

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Example 1 Convert $40^\circ 20'$ into radian measure.

Solution We know that $180^\circ = \pi$ radian.

Hence $40^\circ 20' = 40\frac{1}{3}$ degree $= \frac{\pi}{180} \times \frac{121}{3}$ radian $= \frac{121\pi}{540}$ radian.

Therefore $40^\circ 20' = \frac{121\pi}{540}$ radian.

Example 2 Convert 6 radians into degree measure.

Solution We know that π radian $= 180^\circ$.

Hence 6 radians $= \frac{180}{\pi} \times 6$ degree $= \frac{1080 \times 7}{22}$ degree

$= 343\frac{7}{11}$ degree $= 343^\circ + \frac{7 \times 60}{11}$ minute [as $1^\circ = 60'$]

$= 343^\circ + 38' + \frac{2}{11}$ minute [as $1' = 60''$]

$= 343^\circ + 38' + 10.9''$ $= 343^\circ 38' 11''$ approximately.

Hence 6 radians $= 343^\circ 38' 11''$ approximately.

Trigonometric Functions

In earlier classes, we have studied trigonometric ratios for acute angles as the ratio of sides of a right angled triangle. We will now extend the definition of trigonometric ratios to any angle in terms of radian measure and study them as trigonometric functions.

The values of trigonometric functions for these angles are same as that of trigonometric ratios studied in earlier classes. Thus, we have the following table

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	– 1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	– 1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0

Sign of trigonometric functions

We can find the signs of trigonometric functions in different quadrants which is given in following table.

	I	II	III	IV
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosec x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

Domain and range of trigonometric functions

From the definition of sine and cosine functions, we observe that they are defined for all real numbers.

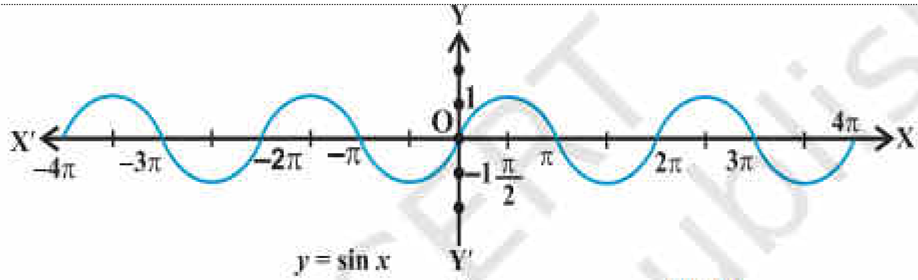
Further, we observe that for each real number x, $-1 \le \sin x \le 1$ and $-1 \le \cos x \le 1$

Thus, domain of y = sin x and y = cos x is the set of all real numbers and range is the interval [–1, 1], i.e., $-1 \le y \le 1$.

Similarly, we can discuss the behavior of other trigonometric functions. In fact, we have the following table:

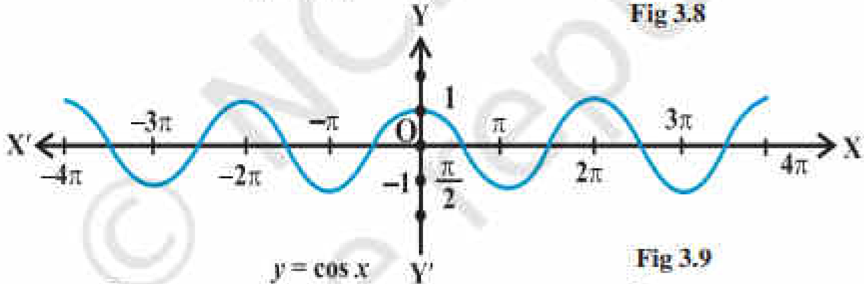
	I quadrant	II quadrant	III quadrant	IV quadrant
sin	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0
cos	decreases from 1 to 0	decreases from 0 to – 1	increases from –1 to 0	increases from 0 to 1
tan	increases from 0 to ∞	increases from –∞to 0	increases from 0 to ∞	increases from –∞to 0
cot	decreases from ∞ to 0	decreases from 0 to–∞	decreases from ∞ to 0	decreases from 0to –∞
sec	increases from 1 to ∞	increases from –∞to–1	decreases from –1to–∞	decreases from ∞ to 1
cosec	decreases from ∞ to 1	increases from 1 to ∞	increases from –∞to–1	decreases from–1to–∞

Graphical representation of trigonometric function



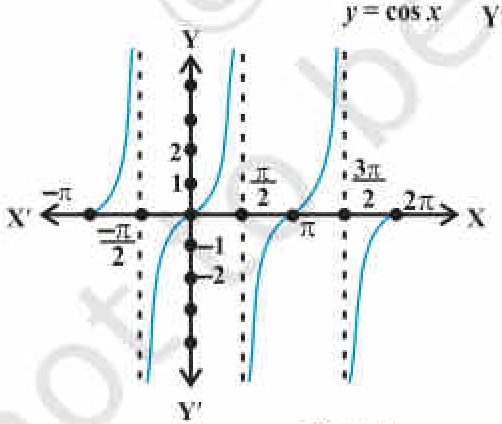
y = sin x

Fig 3.8



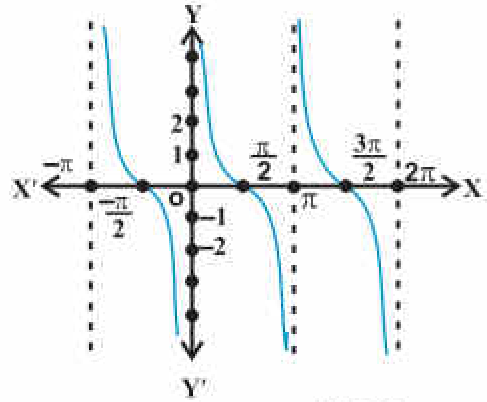
y = cos x

Fig 3.9



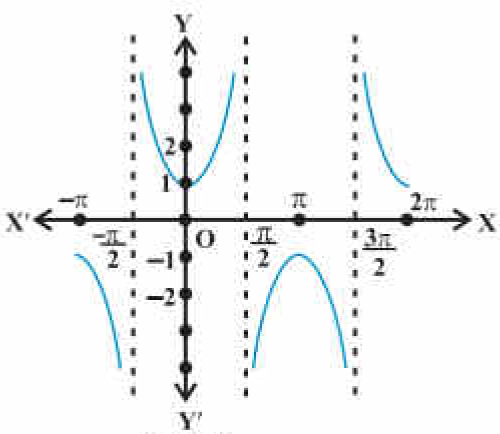
y = tan x

Fig 3.10

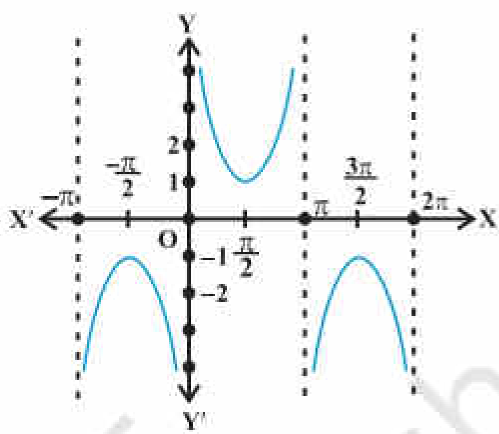


y = cot x

Fig 3.11



y = sec x



y = cosec x

Trigonometric Functions of Sum and Difference of Two Angles

cos (A + B) = cos A cos B − sin A sin B

cos (A − B) = cos A cos B + sin A sin B

sin (A+B) = sin A cos B + cos A sin B

sin (A -B) = sin A cos B − cos A sin B

- tan(A+B) = [(tan A + tan B)/(1 − tan A tan B)]
- tan(A-B) = [(tan A − tan B)/(1 + tan A tan B)]

Sum to Product Formulas
sin x + sin y = 2 sin [(x+y)/2] cos [(x-y)/2]
sin x − sin y = 2 cos [(x+y)/2] sin [(x-y)/2]
cos x + cos y = 2 cos [(x+y)/2] cos [(x-y)/2]
cos x − cos y = -2 sin [(x+y)/2] sin [(x-y)/2]

Product to Sum Formulas
sin x sin y = 1/2 [cos(x−y) − cos(x+y)]
cos x cos y = 1/2[cos(x−y) + cos(x+y)]
sin x cos y = 1/2[sin(x+y) + sin(x−y)]
cos x sin y = 1/2[sin(x+y) − sin(x−y)]

Some additional formulas for sum and product of angles:

- $\cos(A+B)\cos(A-B)=\cos^2A-\sin^2B=\cos^2B-\sin^2A$
- $\sin(A+B)\sin(A-B) = \sin^2A-\sin^2B=\cos^2B-\cos^2A$
- $\sin A+\sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

Formulas for twice of the angles:

- $\sin 2A = 2\sin A\cos A = [2\tan A /(1+\tan^2A)]$
- $\cos 2A = \cos^2A-\sin^2A = 1-2\sin^2A = 2\cos^2A-1= [(1-\tan^2A)/(1+\tan^2A)]$
- $\tan 2A = (2\tan A)/(1-\tan^2A)$

Formulas for thrice of the angles:

- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = [3\tan A - \tan^3 A]/[1 - 3\tan^2 A]$

Trigonometric Equations

Equations involving trigonometric functions of a variable are called trigonometric equations. In this , we shall find the solutions of such equations. We have already learnt that the values of sin x and cos x repeat after an interval of 2π and the values of tan x repeat after an interval of π. The solutions of a trigonometric equation for which $0 \le x < 2\pi$ are called principal solutions. The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution. We shall use 'Z' to denote the set of integers.

Example 18 Find the principal solutions of the equation $\sin x = \frac{\sqrt{3}}{2}$.

Solution We know that, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Therefore, principal solutions are $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$.

General solutions is given in table form

Equations	Solutions
sin x = 0	x = nπ
cos x = 0	x = (nπ + π/2)
tan x = 0	x = nπ
sin x = 1	x = (2nπ + π/2) = (4n+1)π/2
cos x = 1	x = 2nπ
sin x = sin θ	x = nπ + (-1)nθ, where θ ∈ [-π/2, π/2]
cos x = cos θ	x = 2nπ ± θ, where θ ∈ (0, π]
tan x = tan θ	x = nπ + θ, where θ ∈ (-π/2 , π/2]
sin2 x = sin2 θ	x = nπ ± θ
cos2 x = cos2 θ	x = nπ ± θ
tan2 x = tan2 θ	x = nπ ± θ

Using all formulae solve the questions of NCERT

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