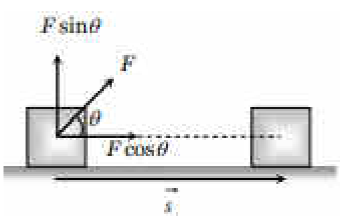
Class Notes	
Class: XI	**Topic: Work , power and Energy**
Subject: Physics	

Work Done by a Constant Force:

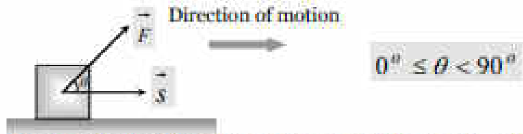
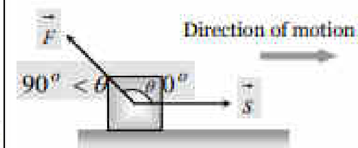
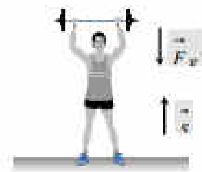
Let a constant force F be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance. By resolving force F into two components: (i) F cosθ in the direction of displacement of the body. (ii) F sinθ in the perpendicular direction of displacement of the body. Since body is being displaced in the direction of Fcosθ , therefore work done by the force in displacing the body through a distance s is given by W = (Fcos θ)s = Fs cos θ or

$$W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar or dot product of the force and the displacement of the body.



Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement	Negative work means that force (or its component) is opposite to displacement *i.e.*

 The positive work signifies that the external force favours the motion of the body.	 The negative work signifies that the external force opposes the motion of the body.
Example: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive 	*Example*: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative. 

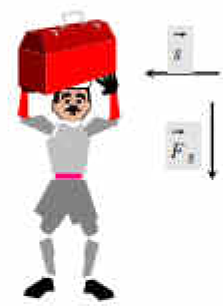
ZERO WORK:

(1) If the force is perpendicular to the displacement $[\vec{F} \perp \vec{s}]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done against gravity by the coolie is zero.(Work done against friction is +ve)

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

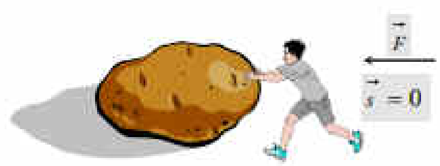
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) If there is no displacement [*s* = 0]

Example: (i) When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



(3) If there is no force acting on the body [*F* = 0]

Example: Motion of an isolated body in free space.

Work Done by a Variable Force:

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.d\vec{s}$

The total work done in going from *A* to *B* as shown in the figure is

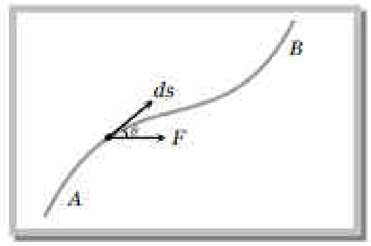
$$W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)ds$$

In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\text{or } W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$



Work Done Calculation by Force Displacement Graph:

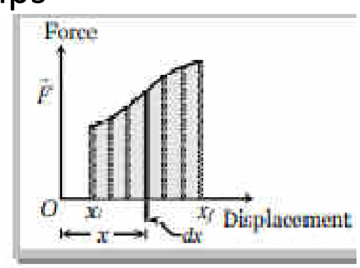
Let a body, whose initial position is i x, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position f x. Let F be the average value of variable force within the interval dx from position x to (x + dx) i.e. for small displacement dx. The work done will be the area of the shaded strip of width dx. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips

$$dW = \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve Between } x_i \text{ and } x_f$$



Kinetic Energy:

The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples: (i) Flowing water possesses kinetic energy which is used to run the water mills. (ii) Moving vehicle possesses kinetic energy. (iii) Moving air (i.e. wind) possesses kinetic energy which is used to run wind mills. (iv) The hammer possesses kinetic energy which is used to drive the nails in wood. (v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

Calculus method : Let a body is initially at rest and force F is applied on the body to displace it through $d\vec{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

$\Rightarrow \quad dW = m\,a\,ds \qquad$ [As $F = ma$]

$\Rightarrow \quad dW = m\dfrac{dv}{dt}ds \qquad \left[As\ a = \dfrac{dv}{dt}\right]$

$\Rightarrow \quad dW = m\,dv.\dfrac{ds}{dt}$

$\Rightarrow \quad dW = m\,v\,dv \qquad$ ....(i) $\left[As\ \dfrac{ds}{dt} = v\right]$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = \frac{1}{2}mv^2$.

In vector form $KE = \frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar *i.e.* kinetic energy can never be negative.

Work-energy theorem: From equation (i) $dW = mv\,dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$\Rightarrow \quad W = \dfrac{1}{2}m[v^2 - u^2]$

Work done = change in kinetic energy

$$W = \Delta E$$

This is work energy theorem, it states that work done by a force acting on a body is equal to change produced in the kinetic energy of the body.

Relation of kinetic energy with linear momentum: As we know

$E = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\left[\dfrac{P}{v}\right]v^2 \qquad$ [As $P = mv$]

$\therefore \quad E = \dfrac{1}{2}Pv$

or $\quad E = \dfrac{P^2}{2m} \qquad \left[\text{As } v = \dfrac{P}{m}\right]$

So we can say that kinetic energy $E = \dfrac{1}{2}mv^2 = \dfrac{1}{2}Pv = \dfrac{p^2}{2m}$

and $\qquad$ Momentum $P = \dfrac{2E}{v} = \sqrt{2mE}$.

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

Change in potential energy :

Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W$$

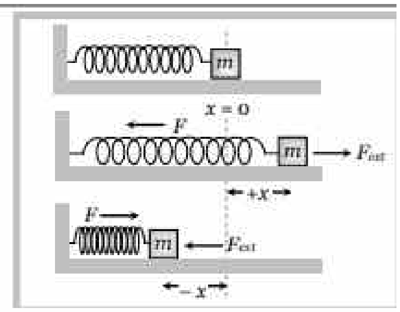
We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there,

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from reference position to given position. This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive i.e. potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative i.e. potential energy will decrease.

Elastic Potential Energy:

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position. According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\,\vec{x}$ .....(i)

where k is called spring constant.

If $x = 1$, $F = k$ (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \dfrac{F}{x}$ $\quad \therefore [k] = \dfrac{[F]}{[x]} = \dfrac{[MLT^{-2}]}{L} = [MT^{-2}]$

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force. $\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.\vec{dx} = F_{ext}.dx\cos 0^o = kx\,dx \qquad [\text{As } \cos 0^o = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

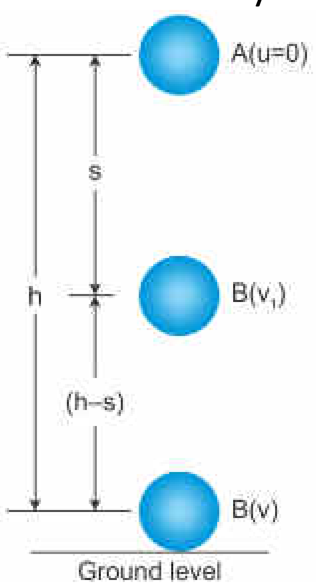
$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

CONSERVATION OF MECHANICAL ENERGY:

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

Example:

Consider a freely falling body.



Consider a body of mass m placed at A.

h = AB is the height of the body above the ground

u = 0 is initial velocity at A

v_1 = velocity of body at C

v = velocity of body at B, i.e., just above the ground

(i) At point A

$PE_A = mgh$

$KE_A = 0$

Total mechanical energy at A, $E_A = PE_A + KE_A = mgh + 0 = mgh$

(ii) At point C

$v_1^2 - 0 = 2gs$

$v_1^2 = 2gs$

$$KE_C = \frac{1}{2}mv_1^2 = \frac{1}{2}m(2gs) = mgs$$

$PE_C = mg(h-s)$

Total mechanical energy at C, $E_C = PE_C + KE_C = mg(h-s) + mgs = mgh$

(iii) At point B

$v^2 - 0^2 = 2gh$

$v^2 = 2gh$

$$KE_B = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$$

$PE_B = 0$

Total mechanical energy at B, $E_B = PE_B + KE_B = 0 + mgh = mgh$

The total mechanical energy of the body at A, B and C (also at any other point in the path AB) is the same. So, the total mechanical energy of the body throughout the free fall is conserved.

POWER:

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$$P_{inst} = \vec{F}.\vec{v} \qquad [\text{As } \vec{v} = \frac{d\vec{s}}{dt}]$$

i.e. power is equal to the scalar product of force with velocity.

(1) Dimension : $[P] = [F][v] = [MLT^{-2}][LT^{-1}]$

$\therefore \quad [P] = [ML^2T^{-3}]$

(2) Units : *Watt* or *Joule/sec* [S.I.]

Erg/sec [C.G.S.]

Practical units : *Kilowatt* (*kW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units : $1\,watt = 1\,Joule/\sec = 10^7\,erg/\sec$

$$1hp = 746\ Watt$$

N.B.-This note has been prepared from home.