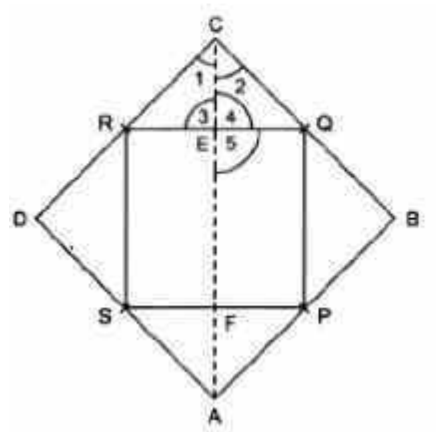
Class Notes

Class: IX	**Topic:** Ch – 8, "QUADRILATERALS"
Subject: MATHEMATICS	**E.X -8.2**

Q.2 ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Solution-

Given P is the mid-point of AB, Q is the mid-point of BC, R is the mid-point of CD, S is the mid-point of DS.

To prove - PQRS is a rectangle.

Construction: Join AC.

Proof- ∵ In ΔABC, P and Q are the mid-points of AB and BC.

$\therefore PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ ...(1)

Also in ΔADC, R and S are the mid-points of CD and DA.

$\therefore SR = \frac{1}{2}AC$ and $SR \parallel AC$ ...(2)

From (1) and (2), we get

$PQ = \frac{1}{2}AC = SR$ and $PQ \parallel AC \parallel SR$

⇒ PQ = SR and PQ || SR

i.e. One pair of opposite sides of quadrilateral PQRS is equal and parallel.

∴ PQRS is a parallelogram.

Now, in ΔERC and ΔEQC,

(i) ∠1 = ∠2 [∵The diagonal of a rhombus bisects the opposite angles]

(ii) CR = CQ [DC=BC ⇒ $\frac{1}{2}$ DC= $\frac{1}{2}$ BC => CR=CQ]

(iii) CE = CE [Common]

ΔERC ≅ ΔEQC [SAS criteria]

∠3 = ∠4 [cpct]

But ∠3 + ∠4 = 180° [Linear pair]

⇒ ∠3 + ∠3 = 180^0

⇒ 2∠3=180^0 ⇒ ∠3=90^0

But ∠5 = ∠3 [Vertically opposite angles]

∠5 = 90°

PQ || AC ⇒ PQ || EF

∴PQEF is a quadrilateral having a pair of opposite sides parallel and one of the angles is 90°.

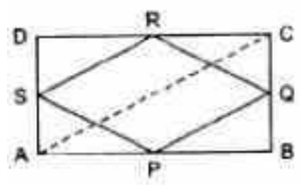
∴PQEF is a rectangle.

⇒∠RQP = 90°

∴One angle of parallelogram PQRS is 90°.

Thus, PQRS is a rectangle.

Q.3 ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.



Solution-

In a rectangle ABCD, P is the mid-point of AB, Q is the mid-point of BC, R is the mid-point of CD, S is the mid-point of DA AC is the diagonal.

Construction: Join AC.

Now, in ΔABC,

$$PQ = \frac{1}{2} AC \text{ and } PQ \parallel AC$$

Similarly, in ΔACD, [Mid-point theorem] ...(1)

$$SR = \frac{1}{2} AC \text{ and } SR \parallel AC$$

From (1) and (2), we get

PQ = SR and PQ || SR

Similarly, by joining BD, we have

PS = QR and PS || QR

i.e. Both pairs of opposite sides of quadrilateral PQRS are equal and parallel.

∴ PQRS is a parallelogram.

Now, in ΔPAS and ΔPBQ

	$\angle A = \angle B$	[Each = 90°]
	AP = BP	[Each = $\frac{1}{2}$ AB]
	AS = BQ	[Each = $\frac{1}{2}$ of opposite sides of a rectangle]
∴	$\Delta PAS \cong \Delta PBQ$	[SAS criteria]

.

⇒PS = PQ [by cpct]

Also, PS = QR [Proved above]

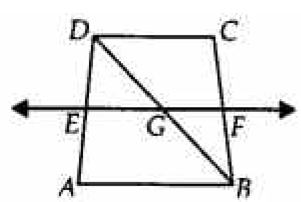
And PQ = SR [Proved]

PQ = QR = RS = SP

i.e. PQRS is a parallelogram having all of its sides equal.

⇒ PQRS is a rhombus.

Q.4 ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the midpoint of BC.



Solution:

In ΔDAB, we know that E is the mid-point of

AD and EG || AB [∵ EF || AB]

Using the converse of mid-point theorem, we get, G is the mid-point of BD.

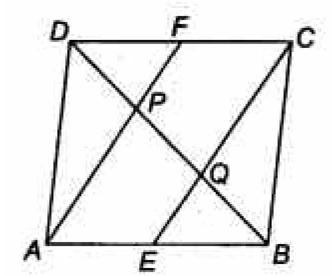
Again in ΔBDC, we have G is the midpoint of BD and GF || DC.

[∵ AB || DC and EF || AB and GF is a part of EF]

Using the converse of the mid-point theorem, we get, F is the mid-point of BC

Q.5 In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.

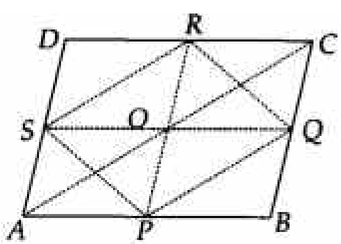
Solution :



Since, the opposite sides of a parallelogram are parallel and equal.
∴ AB || DC
⇒ AE || FC ...(1)
and AB = DC
⇒ 1/2AB = 1/2DC
⇒ AE = FC ...(2)
From (1) and (2), we have
AE || PC and AE = PC
∴ ΔECF is a parallelogram.
Now, in ΔDQC, we have F is the mid-point of DC and FP || CQ
[∵ AF || CE]
⇒ DP = PQ ...(3)
[By converse of mid-point theorem]
Similarly, in Δ BAP, E is the mid-point of AB and EQ || AP [∵AF || CE]
⇒ BQ = PQ ...(4)
[By converse of mid-point theorem]
∴ From (3) and (4), we have
DP = PQ = BQ
So, the line segments AF and EC trisect the diagonal BD.

Q.6 Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

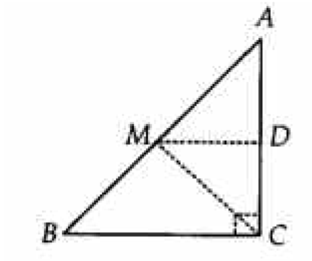
Solution: Let ABCD be a quadrilateral, where P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Join PQ, QR, RS and SP.
Let us also join PR, SQ and AC.



Now, in ΔABC, we have P and Q are the mid-points of its sides AB and BC respectively.
∴ PQ || AC and PQ = 1/2 AC ...(1)
[By mid-point theorem]
Similarly, RS || AC and RS = 1/2AC ...(2)
∴ By (1) and (2), we get
PQ || RS, PQ = RS
∴ PQRS is a parallelogram.
And the diagonals of a parallelogram bisect each other, i.e., PR and SQ bisect each other. Thus, the line segments joining the midpoints of opposite sides of a quadrilateral ABCD bisect each other.

Q.7 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) MD ⊥ AC

(iii) CM = MA = $1/2$AB



Solution: (i) In ΔACB, We have

M is the mid-point of AB. [Given]

MD || BC , [Given]

∴ Using the converse of mid-point theorem,

D is the mid-point of AC.

(ii) Since, MD || BC and AC is a transversal.

∠MDA = ∠BCA

[∵ Corresponding angles are equal] As

∠BCA = 90° [Given]

∠MDA = 90°

(iii) In ΔADM and ΔCDM, we have

∠ADM = ∠CDM [Each equal to 90°]

MD = MD [Common]

AD = CD [∵ D is the mid-point of AC]

∴ ΔADM ≅ ΔCDM [By SAS congruency]

⇒ MA = MC [By C.P.C.T.] .. .(1)

∵ M is the mid-point of AB [Given]

MA = $1/2$AB ...(2)

From (1) and (2), we have

CM = MA = $1/2$AB

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NOTE: The students are advised to write this content in their Maths notebook.

ASSIGNMENT: Related questions can be done from Reference book R.D.Sharma for practice.

"CONTENT ABSOLUTELY PREPERED AT HOME "