

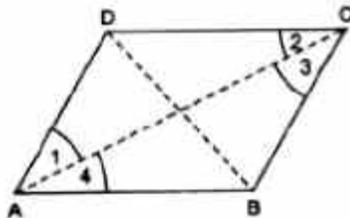
Class: IX

Topic: CH-8, QUADRILATERALS

E.X-8.1

Subject: MATHEMATICS

Q.7 ABCD is a rhombus. Show that diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .



**Solution.** ABCD is a rhombus.

$$\therefore AB = BC = CD = AD$$

Also,  $AD \parallel BC$  and  $AB \parallel CD$

Now,  $AD = CD$

$$\Rightarrow \angle 1 = \angle 2 \dots (1) \text{ [Angles opposite to equal sides are equal]}$$

Also,  $AD \parallel BC$  [Opposite sides of the parallelogram]

$$\therefore \angle 1 = \angle 3 \dots (2) \text{ [alternate interior angles]}$$

From (1) and (2), we have

$$\angle 2 = \angle 3 \text{ and } \angle 1 = \angle 4$$

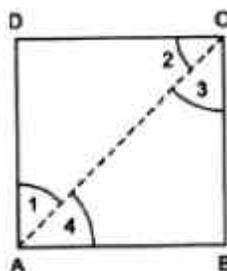
$\Rightarrow$  AC bisects  $\angle C$  as well as  $\angle A$ .

Similarly, we prove that BD bisects  $\angle B$  as well as  $\angle D$ .

Q.8 ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as  $\angle C$ . Show that:

(i) ABCD is a square

(ii) diagonal BD bisects  $\angle B$  as well as  $\angle D$ .



**Solution.** We have a rectangle ABCD such that AC bisects  $\angle A$  as  $\angle C$   
i.e.  $\angle 1 = \angle 4$  and  $\angle 2 = \angle 3 \dots (1)$

(i) Since, rectangle is a parallelogram.

$\therefore$  ABCD is a parallelogram.

$\Rightarrow AB \parallel CD$  and AC is a transversal.

$$\therefore \angle 2 = \angle 4 \text{ [Alternate interior angles] } \dots (2)$$

From (1) and (2), we have

$$\angle 3 = \angle 4$$

$\Rightarrow AB = BC$  [  $\because$  Sides opposite to equal angles in  $\triangle ABC$  are equal.]

$$\therefore AB = BC = CD = AD$$

$\Rightarrow$  ABCD is a rectangle having all of its sides equal.

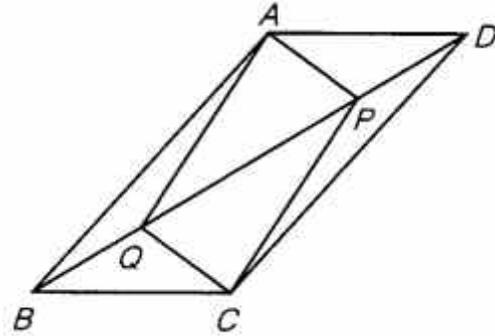
$\therefore$  ABCD is a square.

(ii) Since, ABCD is a square, and diagonals of a square bisect the opposite angles.

$\therefore$  BD bisects  $\angle B$  as well as  $\angle D$ .

**Q.9** In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that:

- (i)  $\triangle APD \cong \triangle CQB$
- (ii)  $AP = CQ$
- (iii)  $\triangle AQB \cong \triangle CPD$
- (iv)  $AQ = CP$
- (v) APCQ is a parallelogram.



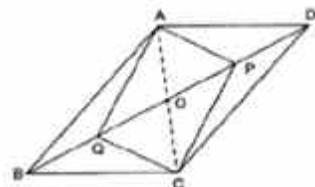
**Solution.** We have parallelogram ABCD. BD is a diagonal and 'P' and 'Q' are such that  $PD = QB$

- (i) To prove that  $\triangle APD \cong \triangle CQB$   
 $\because AD \parallel BC$  and BD is a transversal. [ $\because$  ABCD is a parallelogram.]  
 $\therefore \angle ADB = \angle CBD$  [Interior alternate angles]  
 $\Rightarrow \angle ADP = \angle CBQ$   
 $\triangle APD$  and  $\triangle CQB$ , we have  
  - (i)  $AD = CB$  [Opposite side of the parallelogram]
  - (ii)  $PD = QB$  [Given]
  - (iii)  $\angle CBQ = \angle ADP$  [Proved above] $\triangle APD \cong \triangle CQB$  (by SAS)
- (ii) To prove that  $AP = CQ$   
 Since  $\triangle APD \cong \triangle CQB$  [Proved above ]  
 $\Rightarrow AP = CQ$  (by C.P.C.T)
- (iii) To prove that  $\triangle AQB \cong \triangle CPD$ .  
 $AB \parallel CD$  and BD is a transversal. [ $\because$  ABCD is a parallelogram.]  
 $\therefore \angle ABD = \angle CDB$   
 $\Rightarrow \angle ABQ = \angle CDP$   
 Now, in  $\triangle AQB$  and  $\triangle CPD$ , we have  
  - (i)  $QB = PD$  [Given]
  - (ii)  $\angle ABQ = \angle CDP$  [Proved]
  - (iii)  $AB = CD$  [Opposite sides of parallelogram ABCD] $\therefore \triangle AQB \cong \triangle CPD$  [SAS criteria]
- (iv) To prove that  $AQ = CP$ .  
 Since  $\triangle AQB \cong \triangle CPD$  [Proved]  
 $\therefore$  Their corresponding parts are equal.  
 $\Rightarrow AQ = CP$ .

(v) To prove that APCQ is a parallelogram.  
 join AC.

Since, the diagonals of a  $\parallel$  gm bisect each other  
 $\therefore AO = CO$  and  $BO = DO$   
 $\Rightarrow (BO - BQ) = (DO - DP)$  [ $\because BQ = DP$  (Given)]  
 $\Rightarrow QO = PO$ .....(2)

Now, in quadrilateral APCQ, we have  
 $AO = CO$  and  $QO = PO$   
 i.e. AC and QP bisect each other at O.  
 $\Rightarrow$  APCQ is a parallelogram.



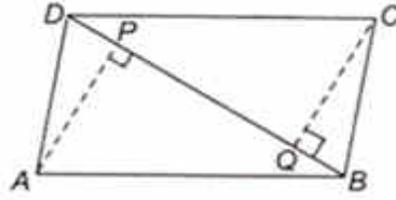
Q.10 ABCD is a parallelogram and AP and perpendiculars from vertices A and C on diagonal BD (see Show that

- (i)  $\triangle APB \cong \triangle CQD$
- (ii)  $AP = CQ$

**Solution.** (i) In  $\triangle APB$  and  $\triangle CQD$ , we have

- 1.  $\angle APB = \angle CQD$  [ $90^\circ$  each]
  - 2.  $AB = CD$  [Opposite sides of parallelogram ABCD]
  - 3.  $\angle ABP = \angle CDQ$
- $\Rightarrow$  Using AAS criteria, we have  
 $\angle APB \cong \triangle CQD$

- (ii) Since,  $\triangle APB \cong \triangle CQD$   
 $\therefore$  Their corresponding parts are equal.  
 $\Rightarrow AP = CQ$



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**NOTE: THE STUDENTS ARE ADVISED TO WRITE THIS CONTENT IN THEIR MATHS NOTEBOOK.**

**ASSIGNMENT: PRACTICE THESE SUMS AND RELATED TOPICS FROM R.D.SHARMA.**

**“ CONTENT PREPARED ABSOLUTELY AT HOME “**