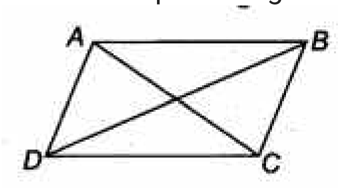
Class Notes

Class: IX	Topic: CH-8 Quadrilaterals Ex.8.1
Subject: Mathematics	

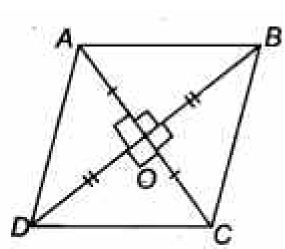
Q.1 The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.
Solution. Let the angles of the quadrilateral be 3x, 5x, 9x and 13x.
∴ 3x + 5x + 9x + 13x = 360° [Angle sum property of a quadrilateral]
⇒ 30x = 360°
⇒ $x = \frac{360^0}{30} = 12^0$
∴ 3x = 3 x 12° = 36°
5x = 5 x 12° = 60°
9x = 9 x 12° = 108°
13x = 13 x 12° = 156°
⇒ The required angles of the quadrilateral are 36°, 60°, 108° and 156°.

Q.2 If the diagonals of a parallelogram are equal, then show that it is a rectangle.
Solution. Let ABCD is a parallelogram such that AC = BD.



In ΔABC and ΔDCB,
AC = DB [Given]
AB = DC [Opposite sides of a parallelogram]
BC = CB [Common]
∴ ΔABC ≅ ΔDCB [By SSS congruency]
⇒ ∠ABC = ∠DCB [By C.P.C.T.] … (1)
Now, AB || DC and BC is a transversal. [∵ ABCD is a parallelogram]
∴ ∠ABC + ∠DCB = 180° … (2) [Co-interior angles]
From (1) and (2), we have
∠ABC = ∠DCB = 90°
i.e., ABCD is a parallelogram having an angle equal to 90°.
∴ ABCD is a rectangle.

Q.3 Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.
Solution. Let ABCD be a quadrilateral such that the diagonals AC and BD bisect each other at right angles at O.

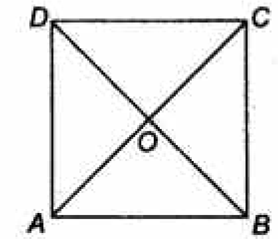


∴ In ΔAOB and ΔAOD, we have
AO = AO [Common]
OB = OD [O is the mid-point of BD]
∠AOB = ∠AOD [Each 90]
∴ ΔAQB ≅ ΔAOD [By, SAS congruency

∴ AB = AD [By C.P.C.T.] ……. (1)
Similarly, AB = BC …. (2)
BC = CD …..(3)
CD = DA ……(4)
∴ From (1), (2), (3) and (4), we have
AB = BC = CD = DA
Thus, the quadrilateral ABCD is a rhombus.

Q.4 Show that the diagonals of a square are equal and bisect each other at right angles.

Solution. Let ABCD be a square such that its diagonals AC and BD intersect at O.



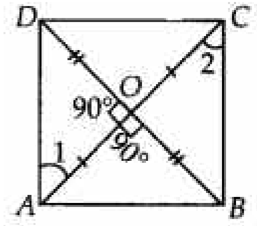
(i) To prove that the diagonals are equal, we need to prove AC = BD.
In ΔABC and ΔBAD, we have
AB = BA [Common]
BC = AD [Sides of a square ABCD]
∠ABC = ∠BAD [Each angle is 90°]
∴ ΔABC ≅ ΔBAD [By SAS congruency]
AC = BD [By C.P.C.T.] …(1)

(ii) AD || BC and AC is a transversal. [∵ A square is a parallelogram]
∴ ∠1 = ∠3
[Alternate interior angles are equal]
Similarly, ∠2 = ∠4
Now, in ΔOAD and ΔOCB, we have
AD = CB [Sides of a square ABCD]
∠1 = ∠3 [Proved above]
∠2 = ∠4 [Proved above]
∴ ΔOAD ≅ ΔOCB [By ASA congruency]
⇒ OA = OC and OD = OB [By C.P.C.T.]
i.e., the diagonals AC and BD bisect each other at O. ……. (2)

(iii) In ΔOBA and ΔODA, we have
OB = OD [Proved]
BA = DA [Sides of a square ABCD]
OA = OA [Common]
∴ ΔOBA ≅ ΔODA [By SSS congruency]
⇒ ∠AOB = ∠AOD [By C.P.C.T.] …(3)
∵ ∠AOB and ∠AOD form a linear pair.
∴∠AOB + ∠AOD = 180°
∴∠AOB = ∠AOD = 90° [By(3)]
⇒ AC ⊥ BD …(4)
From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles.

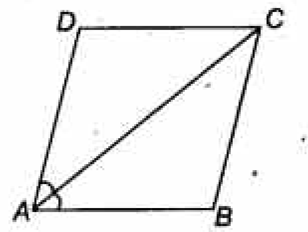
Q.5 Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution. Let ABCD be a quadrilateral such that diagonals AC and BD are equal and bisect each other at right angles.

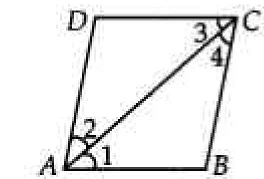


Now, in ΔAOD and ΔAOB, we have
∠AOD = ∠AOB [Each 90°]
AO = AO [Common]
OD = OB [∵ O is the midpoint of BD]
∴ ΔAOD ≅ ΔAOB [By SAS congruency]
⇒ AD = AB [By C.P.C.T.] ...(1)
Similarly, we have
AB = BC ... (2)
BC = CD ...(3)
CD = DA ...(4)
From (1), (2), (3) and (4), we have
AB = BC = CD = DA
∴ Quadrilateral ABCD have all sides equal.
In ΔAOD and ΔCOB, we have
AO = CO [Given]
OD = OB [Given]
∠AOD = ∠COB [Vertically opposite angles]
So, ΔAOD ≅ ΔCOB [By SAS congruency]
∴∠1 = ∠2 [By C.P.C.T.]
But, they form a pair of alternate interior angles.
∴ AD || BC
Similarly, AB || DC
∴ ABCD is a parallelogram.
∴ Parallelogram having all its sides equal is a rhombus.
∴ ABCD is a rhombus.
Now, in ΔABC and ΔBAD, we have
AC = BD [Given]
BC = AD [Proved]
AB = BA [Common]
∴ ΔABC ≅ ΔBAD [By SSS congruency]
∴ ∠ABC = ∠BAD [By C.P.C.T.] ...... (5)
Since, AD || BC and AB is a transversal.
∴∠ABC + ∠BAD = 180° .. .(6) [Co – interior angles]
⇒ ∠ABC = ∠BAD = 90° [By(5) & (6)]
So, rhombus ABCD is having one angle equal to 90°.
Thus, ABCD is a square.

Q.6 Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that
(i) it bisects ∠C also,
(ii) ABCD is a rhombus.



Solution: We have a parallelogram ABCD in which diagonal AC bisects ∠A
⇒ ∠DAC = ∠BAC



(i) Since, ABCD is a parallelogram.
∴ AB || DC and AC is a transversal.
∴ ∠1 = ∠3 … (1)
[∵ Alternate interior angles are equal]
Also, BC || AD and AC is a transversal.
∴ ∠2 = ∠4 … (2)
[v Alternate interior angles are equal]
Also, ∠1 = ∠2 … (3)
[∵ AC bisects ∠A]
From (1), (2) and (3), we have
∠3 = ∠4
⇒ AC bisects ∠C.

(ii) In ΔABC, we have
∠1 = ∠4 [From (2) and (3)]
⇒ BC = AB … (4)
[∵ Sides opposite to equal angles of a Δ are equal]
Similarly, AD = DC ……… (5)
But, ABCD is a parallelogram. [Given]
∴ AB = DC …(6)
From (4), (5) and (6), we have
AB = BC = CD = DA
Thus, ABCD is a rhombus.

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NOTE: THE STUDENTS ARE ADVISED TO WRITE THIS CONTENT IN THEIR MATHS NOTEBOOK.

ASSIGNMENT: PRACTICE THESE SUMS AND RELATED TOPICS FROM R.D. SHARMA.

"CONTENT PREPARED ABSOLUTELY AT HOME "