

MORE QUESTIONS ON DETERMINANTS

MULTIPLE CORRECT TYPE :-

(1.) If  $a+b+c=0$ , then the value(s) of  $x$  which makes

$$\begin{vmatrix} a-x & b & c \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 \text{ is/are}$$

say<sup>n</sup>! (a)  $x=0$  (b)  $x = \sqrt{\frac{3}{2}(a^2+b^2+c^2)}$  (c)  $x = -\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$  (d) NOT.

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Delta = (a+b+c-x) \begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix}$$

Applying,  $C_2 \rightarrow C_2 - C_1$  &  $C_3 \rightarrow C_3 - C_1$ , we obtain

$$\Delta = -x \begin{vmatrix} 1 & 0 & 0 \\ c & b-c-x & a-c \\ b & a-b & c-b-x \end{vmatrix}$$

$$\text{or, } \Delta = x [(a-b)(a-c) - (x+b-c)(x-b+c)]$$

$$= x [a^2 - ab - ac + bc - x^2 + b^2 + c^2 - 2bc]$$

$$= x [a^2 + b^2 + c^2 - bc - ca - ab - x^2]$$

$$\Delta = 0 \Rightarrow x = 0 \text{ or, } x^2 = a^2 + b^2 + c^2 - bc - ca - ab.$$

$$\text{Now, } x^2 = a^2 + b^2 + c^2 - \frac{1}{2} [(a+b+c)^2 - a^2 - b^2 - c^2] = \frac{3}{2} (a^2 + b^2 + c^2)$$

$$\text{So, } x = \pm \sqrt{\frac{3}{2} (a^2 + b^2 + c^2)}$$

i.e. option (a), (b), (c)

(2.) The determinant  $A = \begin{vmatrix} a & b & a^2+b \\ b & c & b^2+c \\ a^2+b & b^2+c & 0 \end{vmatrix}$

is equal to zero if

(a)  $a, b, c$  are in A.P. (b)  $a, b, c$  are in G.P. (c)  $a, b, c$  are in H.P.

(d)  $x$  is a root of  $a^2x^2 + 2bx + c = 0$ .

say<sup>n</sup>!:- Applying  $R_3 \rightarrow R_3 - xR_1 - R_2$ , we get

$$\Delta = \begin{vmatrix} a & b & a^2+b \\ b & c & b^2+c \\ 0 & 0 & -(a^2x^2 + 2bx + c) \end{vmatrix} = (b^2 - ac) (a^2x^2 + 2bx + c)$$

So, option (b), (d)

(3) If,  $f(x) = \begin{vmatrix} (ax)^n & \sin x & \cos x \\ n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ a & a^2 & a^3 \end{vmatrix}$

the value of  $\frac{d^n}{dx^n} [f(x)]$  at  $x=0$  is ...

- (a) -1      (b) 0      (c) 1      (d) ~~not~~ Independent of 'a'.

Sol<sup>n</sup>:- we know,  $\frac{d^n}{dx^n} (x^n) = n!$ ,  $\frac{d^n}{dx^n} (\sin x) = \sin(x + \frac{n\pi}{2})$  and

$\frac{d^n}{dx^n} (\cos x) = \cos(x + \frac{n\pi}{2})$

so,  $f^n(x) = \begin{vmatrix} n! & \sin(x + \frac{n\pi}{2}) & \cos(x + \frac{n\pi}{2}) \\ n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ a & a^2 & a^3 \end{vmatrix}$

$\therefore f^n(0) = \begin{vmatrix} n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ a & a^2 & a^3 \end{vmatrix} = 0$

so, options (b), (d).

(4) If  $p, q$  &  $r$  are positive integers, and  $\Delta = 12 \begin{vmatrix} pC_1 & pC_2 & pC_3 \\ qC_1 & qC_2 & qC_3 \\ rC_1 & rC_2 & rC_3 \end{vmatrix}$

then

- (a)  $\Delta$  is an even integer    (b)  $\Delta$  is divisible by 12    (c)  $\Delta$  is an odd-  
-val no.    (d) NOT.

Sol<sup>n</sup>:-  $pC_1 = p$ ,  $pC_2 = \frac{1}{2} p(p-1)$  and  $pC_3 = \frac{1}{6} p(p-1)(p-2)$

thus,  $\Delta = pqr \begin{vmatrix} 1 & p-1 & (p-1)(p-2) \\ 1 & q-1 & (q-1)(q-2) \\ 1 & r-1 & (r-1)(r-2) \end{vmatrix}$

$R_3 \rightarrow R_3 - R_2 \leftarrow R_2 \rightarrow R_2 - R_1$ , we get

$\Delta = pqr \begin{vmatrix} 1 & p-1 & (p-1)(p-2) \\ 0 & q-p & (q-p)(q+p-2) \\ 0 & r-q & (r-1)(r+q-2) \end{vmatrix}$

$= pqr (q-p)(r-1)(r-p)$

Now, use at least one of these six factors is even.

so, options (a), (c).

(6.) If  $f(x) = \begin{vmatrix} \sec^2 x & 1 & 1 \\ \cos^2 x & \cos^2 x & \cos^2 x \\ 1 & \cos^2 x & \cot^2 x \end{vmatrix}$  then

(a)  $\int_{-\pi/4}^{\pi/4} f(x) dx = 1/2 (3\pi + 8)$  (b)  $f'(\pi/2) = 0$

(c) Max<sup>m</sup> value of  $f(x)$  is 1 (d) min<sup>m</sup> value of  $f(x)$  is 0.

Sol<sup>n</sup>:- Applying  $C_2 \rightarrow C_2 - \cos^2 x C_1$ , we get.

$$f(x) = \begin{vmatrix} \sec^2 x & 0 & 1 \\ \cos^2 x & \cos^2 x - \cos^2 x & \cos^2 x \\ 1 & 0 & \cot^2 x \end{vmatrix}$$

$$\begin{aligned} &= (\cos^2 x - \cos^2 x) (\sec^2 x \cot^2 x - 1) \\ &= \cos^2 x \sin^2 x (\sec^2 x - 1) \\ &= \cos^2 x \sin^2 x \cot^2 x \\ &= \cos^4 x \\ &= 1/4 (3 + 4 \cos 2x + \cos 4x). \end{aligned}$$

Thus,  $\int_{-\pi/4}^{\pi/4} f(x) dx = \left[ 1/4 (2x + 2 \sin 2x + 1/4 \sin 4x) \right]_0^{\pi/4} = 1/2 (3\pi + 8)$

So, options (a), (b), (c), (d)

PARAGRAPH TYPE :-

Let  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  be three points. Area of  $\Delta$  with vertices  $A, B$  and  $C$  is given by  $1/2 |\Delta|$ .

where,  $\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

(6.) points  $A, B, C$  are collinear if and only if  
(a)  $\Delta = 0$  (b)  $\Delta > 0$  (c)  $\Delta < 0$  (d)  $\Delta \leq 0$

(7.) If  $a = BC, b = CA, c = AB$  and  $2s = a + b + c$ , then  $\Delta^2$  equals  
(a)  $abc$  (b)  $s(s-a)(s-b)(s-c)$  (c)  $abc/4$  (d)  $4s(s-a)(s-b)(s-c)$

(8.) If  $z_k = x_k + iy_k$  for  $k = 1, 2, 3$  and  $\Delta_1 = \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$

then area of  $\Delta ABC$  is ...

- (a)  $1/4 |\Delta_1|$  (b)  $1/4i |\Delta_1|$
- (c)  $1/2i |\Delta_1|$  (d)  $1/2 |\Delta_1|$

Sol<sup>n</sup>:-

(6.) points A, B, C are collinear if and only if area of  $\Delta ABC$  is '0'.  
So, option (A)

(7.) use Heron's formula, and find.  
So, option 'd'

(8.) use, if  $z = x + iy$  then  $z + \bar{z} = 2x$  &  $z - \bar{z} = 2iy$

So, option 'a'

Comp: 2 :-

$$(9.) \Delta(x) = \begin{vmatrix} 2x^3 - 3x^2 & 5x + 7 & 2 \\ 4x^3 - 7x & 3x + 2 & 1 \\ 7x^3 - 8x^2 & x - 1 & 3 \end{vmatrix} = a_0 + a_1x + \dots + a_4x^4$$

To evaluate  $a_i$ , we differentiate  $\Delta(x)$   $i$  times w.r.t.  $x$  and put  $x=0$   
or, divide  $\Delta(x)$  by  $x^4$  put  $1/x = t$ , differentiate  $(4-i)$  time w.r.t.  $t$   
and put  $t=0$ .

(9.)  $a_0 = \dots$

- (a) 0                      (b) 1                      (c) 2                      (d) 3

(10.)  $a_1 = \dots$

- (a) 0                      (b) 61                      (c) 161                      (d) 191.

(11.)  $a_4 = \dots$

- (a) 41                      (b) -43                      (c) -41                      (d) 43.

Sol<sup>n</sup>:-

$$(10.) \Delta'(x) = \begin{vmatrix} 6x^2 - 6x & 5x + 7 & 2 \\ 12x^2 - 7 & 3x + 2 & 1 \\ 21x^2 - 16x & x - 1 & 3 \end{vmatrix} + \begin{vmatrix} 2x^3 - 3x^2 & 5 & 2 \\ 4x^3 - 7x & 3 & 1 \\ 7x^3 - 8x^2 & 1 & 3 \end{vmatrix}$$

$$\therefore a_1 = \Delta'(0) = 161.$$

So, option 'c'

(9.) putting  $x=0$ ,  $a_0 = 0$

So, option (a)

$$(11.) \frac{\Delta(x)}{x^4} = \begin{vmatrix} 2 - 3/x & 5 + 7/x & 2 \\ 4 - 7/x^2 & 3 + 2/x & 1 \\ 7 - 8/x & 1 - 1/x & 3 \end{vmatrix}$$

Taking limit as  $x \rightarrow \infty$ , we get

$$a_4 = \begin{vmatrix} 2 & 5 & 2 \\ 4 & 3 & 1 \\ 7 & 1 & 3 \end{vmatrix} = -43.$$

So, option 'b'

NUMERICAL VALUE TYPE :-

(12.) If  $\Delta = \begin{vmatrix} e^{-2iA} & e^{iC} & e^{iB} \\ e^{iC} & e^{-2iB} & e^{iA} \\ e^{iB} & e^{iA} & e^{-2iC} \end{vmatrix}$  & A, B, C are angles of  $\Delta$ , and then

find  $|\Delta|$ .

Sol<sup>n</sup>:- Let  $\alpha = e^{iA}$ ,  $\beta = e^{iB}$  and  $\gamma = e^{iC}$  then  $\alpha\beta\gamma = e^{i(A+B+C)} = e^{i\pi} = -1$ .

$$\Delta = \begin{vmatrix} 1/\alpha^2 & \gamma & \beta \\ \gamma & 1/\beta^2 & \alpha \\ \beta & \gamma & 1/\gamma^2 \end{vmatrix} = \frac{1}{\alpha^2\beta^2\gamma^2} \begin{vmatrix} 1 & \alpha\gamma & \beta\alpha^2 \\ \beta^2\gamma & 1 & \beta^2\alpha \\ \gamma^2\beta & \gamma^2\alpha & 1 \end{vmatrix} = \begin{vmatrix} 1 & -\alpha/\beta & -\alpha/\gamma \\ -\beta/\alpha & 1 & -\beta/\gamma \\ -\gamma/\alpha & -\gamma/\beta & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{\alpha\beta\gamma} \begin{vmatrix} \alpha & -\alpha & -\alpha \\ -\beta & \beta & -\beta \\ -\gamma & -\gamma & \gamma \end{vmatrix} = \frac{\alpha\beta\gamma}{\alpha\beta\gamma} \begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = \boxed{-4}$$

(13.) Let  $f(x) = \begin{vmatrix} \cos x & -x & 1 \\ 2\sin x & -x^2 & 2x \\ \tan x & -x & 1 \end{vmatrix}$  find  $\lim_{x \rightarrow 0} \frac{f'(x)}{x} = \dots$

Sol<sup>n</sup>:- Differentiating w.r.t 'x',

$$-f'(x) = \begin{vmatrix} -\sin x & x & 1 \\ 2\cos x & 2x & 2x \\ \sec^2 x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x & 2x & 2x \\ \tan x & 1 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 0 \\ 2\sin x & x^2 & 2 \\ \tan x & x & 0 \end{vmatrix}$$

$$\Rightarrow -\frac{f'(x)}{x} = \begin{vmatrix} -\sin x & 1 & 1 \\ 2\cos x & x & 2x \\ \sec^2 x & 1 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 0 \\ 2\sin x & x & 2 \\ \tan x & 1 & 0 \end{vmatrix}$$

$$\Rightarrow -\lim_{x \rightarrow 0} \frac{f'(x)}{x} = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} = -2$$

So,  $\boxed{\lim_{x \rightarrow 0} \frac{f'(x)}{x} = 2}$

(14.) Let  $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$ . Find  $\frac{1}{\pi} \int_0^{\pi/2} [f(x) + f'(x)] dx$

Sol<sup>n</sup>:- Applying  $C_1 \rightarrow C_1 - 2\sin x C_2$  and  $C_2 \rightarrow C_2 + 2\cos x C_1$ , we get

$$f(x) = \begin{vmatrix} 2 & 0 & -\sin x \\ 0 & 2 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix} = 2\sin^2 x + 2\cos^2 x = 2$$

$\Rightarrow f'(x) = 0$

$\therefore \int_0^{\pi/2} [f(x) + f'(x)] dx = \int_0^{\pi/2} 2 dx = 2 \left(\frac{\pi}{2}\right) = \boxed{\pi}$