

SYSTEM OF LINEAR EQUATIONS:-

for 3 eqns in 3 variables

$$a_1x + a_2y + a_3z = d_1$$

$$b_1x + b_2y + b_3z = d_2$$

$$c_1x + c_2y + c_3z = d_3$$

Can be transformed into,

$$\therefore \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\text{Let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & a_2 & a_3 \\ d_2 & b_2 & b_3 \\ d_3 & c_2 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & a_3 \\ a_2 & d_2 & b_3 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\& \Delta_3 = \begin{vmatrix} a_1 & a_2 & d_1 \\ b_1 & b_2 & d_2 \\ c_1 & c_2 & d_3 \end{vmatrix}$$

Solutions under different conditions:

(i). If $\Delta \neq 0$, then given system of eqns is consistent and it has unique (one) solution which is given by $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$ (Cramer's Rule).

(ii). If $\Delta = 0$ & at least one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero, then given system of eqns is inconsistent and it will have no solution.

(iii). If all of Δ_1, Δ_2 & Δ_3 are zero, then given system of eqns is consistent and has infinitely many solutions.

Conditions for consistency of three linear equations in two unknowns:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$a_3x + b_3y + c_3 = 0$$

will be consistent if the values of x & y obtained from any two eqns satisfy the third eqn.

$$\text{or, } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

This is reqd. condⁿ for consistency of three linear eqns in two unknowns. If such system of eqns is consistent then no. of solutions is one.

System of Homogenous Linear eqⁿs :-

A system of linear eqⁿs is said to be homogenous if the sum of powers of variable in each term is one.

Let, the three homogenous linear eqⁿs in three unknowns x, y & z are

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0$$

$$a_3x + b_3y + c_3z = 0$$

Clearly, $x = y = z = 0$ is a solution of the system of eqⁿs. This solution is called TRIVIAL solution. Any other solution is called non-trivial solution. Now consider

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(i) if $\Delta \neq 0$, then the given system of eqⁿs has only-trivial solutions and the no. of solutions in this case is one.

(ii) if $\Delta = 0$, then the given system of equations has non-trivial solⁿ as well as trivial solⁿ and number of solutions in this case is infinite.

Q.1. The number of values of 'k' for which the system of equations

$$(k+1)x + 8y = 4k, \quad kx + (k+3)y = 2k-1$$

has no solutions is ---

- (a) 0
- (b) 3
- (c) 2
- (d) Infinite.

solⁿ:- for the systems of eqⁿs to have no solutions, we must have

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{2k-1}$$

$$\Rightarrow (k+1)(k+3) = 8k \text{ \& \ } 8(2k-1) \neq 4k(k+3)$$

$$\text{But, } (k+1)(k+3) = 8k \Rightarrow (k-1)(k-3) = 0 \Rightarrow k=1, k=3$$

$$\text{for } k=1, \quad 8(2k-1) = 16 \text{ and } 4k(k+3) = 16$$

$$\text{for } k=3, \quad 8(2k-1) = 64 \text{ and } 4k(k+3) = 72$$

Thus, for $k=2, \quad 8(2k-1) \neq 4k(k+3)$

Q.2. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ and $|A| \neq 0$, then the system of equations

$$a_1x + b_1y + c_1z = 0, \quad a_2x + b_2y + c_2z = 0 \text{ and } a_3x + b_3y + c_3z = 0 \text{ has}$$

- (a) only one solution
- (b) Infinite number of solutions
- (c) no solution
- (d) more than one but finite number of solutions.

Solⁿ:- $|A| \neq 0$

$$\text{Now, } AX = 0 \Rightarrow A^{-1}(AX) = 0$$

$$\Rightarrow (A^{-1}A)X = 0 \text{ or, } IX = 0 \text{ or, } X = 0$$

∴ $x = y = z = 0$ is only solution of the system of eqⁿs.

Q.3. The system of equations

$$(a+b)x + ay + bz = 0$$

$$(bx+tc)x + by + cz = 0$$

$$(ax+tb)y + (bx+tc)z = 0$$

has a non-trivial solution, if

(a) a, b, c are in A.P. (b) a, b, c are in G.P. (c) a, b, c are in H.P.

(d) x is a root of $ax^2 + 2bx + tc = 0$

Solⁿ:- for non-trivial solution,

$$\Delta = \begin{vmatrix} a+b & a & b \\ bx+tc & b & c \\ 0 & ax+tb & bx+tc \end{vmatrix} = 0$$

Applying $R_3 \rightarrow R_3 - xR_1 - R_2$, we get

$$\Delta = \begin{vmatrix} a+b & a & b \\ bx+tc & b & c \\ -(ax^2+2bx+tc) & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -(ax^2+2bx+tc)(ac-b^2) = 0$$

$$\Rightarrow ax^2+2bx+tc = 0 \text{ or, } ac = b^2$$

$\Rightarrow x$ is the root of $ax^2+2bx+tc = 0$ or a, b, c are in G.P.

Q.4. The system of homogeneous eqⁿs

$$tx + (t+1)y + (t-1)z = 0$$

$$(t+1)x + ty + (t+2)z = 0$$

$$(t-1)x + (t+2)y + tz = 0$$

has non-trivial solutions for

(a) exactly three real values of t (b) exactly two real values of t

(c) exactly one real value of t (d) infinite no. of values of t .

Solⁿ:- for, non-trivial solⁿs

$$\Delta = \begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix} = 0$$

$$\Rightarrow (2t+1)(-4) = 0$$

$$\Rightarrow t = -\frac{1}{2}$$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\Delta = \begin{vmatrix} t & 1 & -1 \\ t+1 & -1 & 1 \\ t-1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2t+1 & 0 & 0 \\ t+1 & -1 & 1 \\ t-1 & 2 & 1 \end{vmatrix}$$

∴ option (c)

$$R_1 \rightarrow R_1 + R_2$$

Q.5. Suppose a_1, a_2, \dots are real numbers, with $a_1 \neq 0$. If a_1, a_2, a_3, \dots are in A.P then,

(a) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ is singular

(b) The system of eqⁿs $a_1x + a_2y + a_3z = 0, a_4x + a_5y + a_6z = 0, a_7x + a_8y + a_9z = 0$ has infinite no. of solⁿs

(c) $B = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix}$ is non-singular.

(d) none of these.

Solⁿ:- Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1,$

$$|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ d & d & d \\ d & d & d \end{vmatrix} = 0 \quad (d \rightarrow \text{C.D. of an A.P.})$$

\therefore the given system of eqⁿs has infinite number of solⁿs

Also, $|B| = a_1^2 + a_2^2 \neq 0$.

So, options (a), (b), (c).

Q.6. The system of eqⁿs.

$$-2x + y + z = a$$

$$x - 2y + z = b$$

$$x + y + 2z = c \quad \text{has}$$

(a) no solⁿ if $a+b+c \neq 0$ (b) unique solⁿ if $a+b+c = 0$ (c) infinite solⁿ if $a+b+c = 0$ (d) none

\Rightarrow Students must try this. (Correct options are (a), (c)).

Q.7. The system of eqⁿs

$$ax + by + cz = q - r$$

$$bx + cy + az = r - p$$

$$cx + ay + bz = p - q \quad \text{if}$$

(a) consistent if $p = q = r$

(b) inconsistent if $a = b = c$, and p, q, r are distinct

(c) consistent if a, b, c are distinct and $a+b+c \neq 0$

(d) None.

Solⁿ:- If $p = q = r$ then $x = 0, y = 0, z = 0$ is a solution of system.

If a, b, c are distinct, then determinant of the co-eff. matrix is --

$$\frac{1}{2} (a+b+c) [(b-c)^2 + (c-a)^2 + (a-b)^2] \neq 0$$

\therefore hence the system has a unique solⁿ.

So, correct options (a), (c).

Q.7. Let 'S' denote the set of all values of λ for which the system of eqn.

$$\lambda x_1 + x_2 + x_3 = 1$$

$$x_1 + \lambda x_2 + x_3 = 1$$

$$x_1 + x_2 + \lambda x_3 = 1$$

is inconsistent, find $\sum_{\lambda \in S} |\lambda|$.

Solⁿ:- Let, A denote co-eff matrix, then $|A| = \lambda^2 - 2 - 2\lambda = (\lambda - 1)^2 (\lambda + 2)$

for $\lambda = 1$, the system has infinite no. of solutions.

for $\lambda = -2$, the system has no solutions.

$$\therefore \sum_{\lambda \in S} |\lambda| = 2$$

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Q.8. Let 'A' be the set of all 2×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

(i) the no. of matrices in A is

- (A) 12 (B) 6 (C) 9 (D) 3.

(ii) the no. of matrices A for which the system of linear eqns $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solⁿ, is

- (A) less than 4 (B) at least 4 but less than 7 (C) at least 7 but less than 10 (D) at least 10.

(iii) the no. of matrices A for which the system of linear eqns $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent is --

- (A) 0 (B) more than 2 (C) 2 (D) 1

Solⁿ:- (i) If two zero's are the entries in the diagonal then, ${}^3C_2 \times {}^3C_1$,
if all the entries in the principle diagonal is 1, then 3C_1 ,
 \Rightarrow Total = 12 (option A).

(ii) $\begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{bmatrix} \rightarrow$ either $b=0$ or, $c=0 \Rightarrow |A| \neq 0 \Rightarrow$ Two matrices.

$\begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix} \rightarrow$ either $a=0$ or, $c=0 \Rightarrow |A| \neq 0 \Rightarrow$ Two matrices.

$\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix} \rightarrow$ either $a=0$ or, $b=0 \Rightarrow |A| \neq 0 \Rightarrow$ Two matrices.

$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix} \rightarrow$ if $a=b=0 \Rightarrow |A| \neq 0$
if $a=c=0 \Rightarrow |A| = 0$
if $b=c=0 \Rightarrow |A| = 0$
 \therefore there will be only 6 matrices. (option B).

(iii) the 2×3 matrix A for which $|A| \neq 0$ are..

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow \text{infinite sol}^n$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \text{infinite sol}^n$$

(option B)