HALF YEARLY EXAMINATION, 2017-18
MATHEMATICS

Time : 3 hrs.   **Class - XI**   **M.M. : 100**

Name of the student _____ Section _____ Date – 14.09.2017 (Thursday)

General Instructions :

- All questions are **compulsory**.
- This question paper is divided into four sections, **Section A** contains **4 questions** each carrying **1 mark**, **Section B** contains **8 questions** each carrying **2 marks,** **Section C** contains **11 questions** each carrying **4 marks** and **Section D** contains **6 questions** each carrying **6 marks**.
- **Question No. 18 & 28** must be attempted in **graph paper**.
- Graph paper will be provided to you.
- Use of **calculator** or any other **electronic devices** is **not allowed**.
- **Please check that this question paper contains 02 printed pages.**

SECTION-A

Q.1 Find the power set of $\{1, \{2, 3\}, 3\}$.

Q.2 If $A = \{1, -1\}$, find $A \times A \times A$.

Q.3 The angles of a triangle are in the ratio 3:4:5. Find the greatest angle in radians.

Q.4 Find all possible real values of 'a'. If $\displaystyle \lim_{x \to a} \frac{x^9 - a^9}{x - a} = \lim_{x \to 5} x + 4$

SECTION - B

Q.5 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 8\}$.

Verify that $(A \cup B)' = A' \cap B'$.

Q.6 Find the domain and range of the relation $R = \{ (x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

Q.7 Find the value of $\tan \dfrac{\pi}{8}$.

Q.8 If $\sin A = 3/5$ and $\cos B = -9/41$, where $A, B \in (\pi/2, \pi)$. Find $\sin (A+B)$.

Q.9 In any ΔABC, prove that : $(b+c) \cos \dfrac{B + C}{2} = a \cos \dfrac{B - C}{2}$.

Q.10 Solve : $\dfrac{2x - 3}{4} + 8 \geq 2 + \dfrac{4x}{3}$

Q.11 Find $\displaystyle \lim_{x \to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$

Q.12 $y = \dfrac{2 - 3 \cos x}{\sin x}$, find $\dfrac{dy}{dx}$

SECTION-C

Q.13 Let A and B be any two sets. If for some set X, $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ then prove that $A = B$.

Q.14 Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5\}$ and R be a relation from A to B defined by $a \, R \, b \Leftrightarrow a$ divides b. Represent it in tabular form and lattice form also find its domain and range.

Q.15 Prove that $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \dfrac{3}{16}$.

Q.16 Let f ={ (1, 1), (2, 3), (0, -1), (-1, -3)} be a linear function and g(x)=x. Find (f+g) (2) and f.g(1).

<p style="text-align:center">**OR**</p>

If f(x) = [x], g(x) = | x | find (f+g) (-1.5), (f-g) (3.1), f.g(-2) and $\left(\dfrac{f}{g}\right)$ (2.5)

Q.17 Prove that

$$\cos \alpha + \cos \beta + \cos \gamma + \cos (\alpha+\beta+\gamma) = 4 \cos \frac{(\alpha + \beta)}{2} \cdot \cos \frac{(\beta + \gamma)}{2} \cdot \cos \frac{(\gamma + \alpha)}{2}$$

<p style="text-align:center">**OR**</p>

Prove that $\dfrac{\sin(\theta + \phi) - 2 \sin \theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2 \cos \theta + \cos(\theta - \phi)} = \tan \theta$

Q.18 Draw the graph of $y = \tan x$, $x \in [-2\pi, 2\pi]$

Q.19 Solve : $\sin x + \sqrt{3} \cos x = \sqrt{2}$.

Q.20 In any $\triangle ABC$, prove that

$a\cos A + b\cos B + c\cos C = 2a \sin B \sin C$.

Q.21 A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to have more than 4% but less than 6% boric acid. If we have 640 litres of 8% solution, how many litres of 2% solution will have to be added?

Q.22 Evaluate : $\displaystyle \lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$

Q.23 If $f(x) = \dfrac{x^{100}}{100} + \dfrac{x^{99}}{99} + \dots + \dfrac{x^2}{2} + x + 1$

Prove that $f'(1) = 100 \times f'(0)$.

SECTION-D

Q.24 In a survey of 100 students, the numbers of students studying the various languages were found to be :

English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find :

    i) How many students were studying Hindi?

    ii) How many students were studying English and Hindi?

Q.25 Let $f=\{(x, \dfrac{x^2}{1 + x^2}) : x \in R\}$ be a function from R→R. Determine the domain and range.

Q.26 Prove that : $\cos^2 \theta + \cos^2 \left(\dfrac{2\pi}{3} - \theta \right) + \cos^2 \left(\dfrac{2\pi}{3} + \theta \right) = \dfrac{3}{2}$

<p style="text-align:center">**OR**</p>

Prove that : $\cos^4 \dfrac{\pi}{8} + \cos^4 \dfrac{3\pi}{8} + \cos^4 \dfrac{5\pi}{8} + \cos^4 \dfrac{7\pi}{8} = \dfrac{3}{2}$

Q.27 Solve : $\cos\theta \cos2\theta \cos3\theta = 1/4$

Q.28 Solve the following system of inequalities graphically :

$x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$.

Q.29 Using the first principle differentiate

<p style="text-align:center">$x \sin x$ **OR** $\sin x + \cos x$</p>