

PRACTICE PAPER
SUB.-MATHEMATICS
CLASS-XII

Time allowed: 3hrs.

Total marks: 100

General Instructions:

- i) All the questions are compulsory.
- ii) The question paper is divided into three sections. Section A consists of 10 questions 1 mark each, Section B consists of 12 questions 4 marks each and Section C consists of 7 questions 6 marks each.
- iii) Use of calculator is not permitted.

SECTION: A

1. Find the projection of \vec{a} on \vec{b} if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$
2. Write a unit vector in the direction of $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$
3. Find the direction cosines of the line passing through the points (-2,4,-5) and (1,2,3)
4. If matrix $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, write AA' , where A' is the transpose of matrix A
5. Write the value of the determinant: $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$
6. Using principal value, evaluate: $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$
7. Evaluate: $\int \frac{\sec^2 x}{3 + \tan x} dx$
8. If $\int_0^1 (3x^2 + 2x + k) dx = 0$, find the value of 'k'.
9. If the binary operation '*' on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value of $2 * 4$
10. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|adjA|$

SECTION: B

11. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$
12. Prove that: $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

OR

Solve for 'x': $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

13. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

14. Solve the following differential equation: $\frac{dy}{dx} + y = \cos x - \sin x$

15. Find the particular solution, satisfying the given condition, for the following differential equation: $\frac{dy}{dx} - \frac{y}{x} + \cos ec\left(\frac{y}{x}\right) = 0$, $y = 0$ when $x = 1$

16. By using properties of determinants, prove that:
$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

17. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

18. Differentiate the following function w.r.t. x : $x^{\sin x} + (\sin x)^{\cos x}$

19. Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x - e^{2x}}} dx$

OR

Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3} dx$

20. Prove that the relation R on the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

21. Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

OR

If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

22. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$

OR

Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$ is (i) increasing (ii) decreasing.

SECTION: C

23. Find the volume of the largest cylinder that can be inscribed in a sphere of radius 'r'.

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 cubic meters. If building of tank costs Rs.70 per sq. meter for the base and Rs. 45 per sq. meter for sides, what is the cost of least expensive tank?

24. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a LPP and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

25. Three bags contain balls as shown in the table below:

Bag	Number of White balls	No. of Black balls	No. of Red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen randomly and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the bag III?

26. Using matrices, solve the following system of equations:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

27. Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

Evaluate: $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$

28. Using method of integration, find the area of the region bounded by the lines:

$$2x + y = 4, \quad 3x - 2y = 6 \quad \text{and} \quad x - 3y + 5 = 0$$

29. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.