MATRICES AND DETERMINANTS

BASIC CONCEPTS

1. *Matrix* A matrix is a rectangular array of numbers, real or complex. The numbers are called the elements of the matrix (sometimes entries in the matrix).

2. A matrix $A_{mn} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}$ is called matrix of order $m \times n$, m representing number of rows and n, the number of columns. It is also denoted by $A = (a_{ij})_{m \times n}$.

3. *Square matrix* When number of rows in a matrix equals number of columns in it, the matrix is called a square matrix.

4. *Row matrix* A matrix having one row and any number of columns is called a row matrix. In other words, matrix of order $1 \times n$ is always a row matrix.

5. *Column matrix* A matrix having any number of rows but only one column is called a column matrix. Thus a matrix of order $m \times 1$ is always a column matrix.

6. *Zero matrix* (Null matrix) A matrix is said to be a zero matrix or null matrix if all its elements are zeroes.

7. *Diagonal matrix* A square matrix $[a_{ij}]$ is said to be a diagonal matrix if $a_{ij} = 0$ for $i \neq j$.

8. *Identity matrix (Unit matrix)* A square matrix having the elements in main diagonal as 1 and rest are zeroes is called an identity or unit matrix.

9. *Scalar matrix* A square matrix whose all non-diagonal elements are zeroes and diagonal elements are equal, is called a scalar matrix. In other words, if $A = [a_{ij}]$ is a square matrix and $a_{ij} = \begin{cases} \alpha & i = j \\ 0 & i \neq j \end{cases}$, then A is a scalar matrix.

10. $A = [a_{ij}] = [b_{ij}] = B$ if
    (*i*) A and B are of the same order
    (*ii*) $a_{ij} = b_{ij}$ for all i and j.

11. If A is an $m \times n$ matrix and k is a scalar, then
$$kA = k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$$

12. $-A = (-1)\, A$.

13. *Addition of two matrices* Let A and B be two $m \times n$ matrices given by $A = [a_{ij}]$ and $B = [b_{ij}]$, then their sum A + B is given by
$$A + B = [a_{ij} + b_{ij}]_{m \times n}.$$

14. $A - B = A + (-1)\, B$ where A and B are matrices of the same order.

15. $A + B = B + A$, $A + (B + C) = (A + B) + C$ where A, B and C are matrices of the same order.

16. $k\,(A + B) = k\,A + k\,B$ where k is a constant and A and B are two matrices of the same order.

17. $(k + l)\, A = k\, A + l\, A$ where k and l are constants.

18. *Product of matrices* Let $A = [a_{ij}]_{m \times n}$, $B = [b_{jk}]_{n \times p}$, then
$$AB = C = [c_{ik}]_{m \times p} \text{ where } c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}.$$

19. A (BC) = (AB) C, A (B + C) = AB + AC
(A + B) C = AC + BC.

20. If A and B are two matrices, then AB ≠ BA. In some cases AB may be equal to BA but not always.

21. *Transpose of a matrix* The matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of matrix A and is denoted by A′ or A^T.
In other words, if $A = [a_{ij}]_{m \times n}$, then $A' = [a_{ji}]_{n \times m}$.

22. *Properties of transpose of a matrix*
(a) $(A')' = A$ (b) $(A + B)' = A' + B'$ (c) $(AB)' = B'A'$ (c) $(kA)' = kA'$

23. *Symmetric matrix* A square matrix A is said to be *symmetric* if A′ = A, *i.e.*, $[a_{ji}] = [a_{ij}]$

24. *Skew symmetric matrix* A square matrix A is said to be *skew symmetric* if A′ = – A, *i.e.*, if A = $[a_{ij}]$, then $[a_{ji}] = [-a_{ij}]$.

25. Elements in the main diagonal of a skew symmetric matrix are all zeroes.

26. A matrix which is symmetric as well as skew symmetric is a zero square matrix.

27. For any square matrix A, A + A′ is symmetric and A – A′ is a skew symmetric matrix.

28. Any square matrix A can be expressed as the sum of a symmetric and skew symmetric matrix, *i.e.*,
$$A = \frac{A + A'}{2} + \frac{A - A'}{2}.$$

29. *Elementary operations of a matrix*
(i) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
(ii) $R_i \to k R_i$ or $C_i \leftrightarrow k C_i$
(iii) $R_i \to R_i + k R_j$ or $C_i \to C_i + k C_j$.

30. If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.

31. Inverse of a square matrix, if it exists, is unique.

32. Determinant of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by
$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{12} a_{22}$$
and determinant of a matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is given by
$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

33. The value of the determinant $\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix} = a\,(qn - mr) - b\,(pn - lr) + c\,(pm - lq)$.

34. *Minor* of an element a_{ij} in |A| is the value of the determinant obtained by deleting the *i*th row and *j*th column of |A| in which the element a_{ij} lies and is denoted by M_{ij}.

35. *Cofactor* of an element a_{ij} in |A| is given by
$$C_{ij} = (-1)^{i+j} M_{ij}$$

36. *Properties of determinants*
(a) If the rows and columns of a determinant are interchanged, its value remains unaltered.
(b) If two rows (columns) of a determinant are identical, its value is zero.
(c) If any two rows (columns) of a determinant are interchanged, the value of the determinant is (–1) times the original determinant.
(d) If all the elements of one row (column) of a determinant are multiplied by *k*, the value of the new determinant is *k* times the original determinant.
(e) If, to the elements of a row (or column) of a determinant are added *k* times the corresponding elements of another row (or column), the value of the determinant remains unchanged.
(f) The sum of the products of the elements of any row (column) with their corresponding cofactors is equal to the value of the determinant.
(g) The sum of the products of the elements of any row (column) with the cofactors of the corresponding elements of any other row (column) is zero.

37. *Area of a triangle* whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by
$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

38. *Adjoint of a matrix* Let $A = (a_{ij})_{m \times n}$ be a square matrix and c_{ij} be cofactor of a_{ij} in |A|. Then adj $A = [c_{ij}]'$.

39. *Properties of adjoint of a matrix*
(i) A.(adj A) = (adj A). A = | A | I (ii) adj (AB) = (adj B).(adj A)

40. *Singular and non-singular matrices* A matrix A is singular if | A | = 0 and it is non-singular if | A | ≠ 0.

41. *Inverse of a matrix* A square matrix A is said to be invertible if there exists a square matrix B of the same order such that AB = BA = I and we write
$$A^{-1} = B.$$
A^{-1} exists only if | A | ≠ 0
$$\text{and } A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{|A|} \begin{vmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{vmatrix}$$

42. $(AB)^{-1} = B^{-1} A^{-1}$.

43. Solution of system of equations by matrix method is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} B$, where A is matrix of coefficients of *x*, *y* and *z* and B is matrix of the constants.

44. In case | A | = 0, then equations are either *dependent* or *inconsistent* according as the product (adj. A) B is a *zero matrix* or *non-zero matrix.*

45. *Solution of a system of homogeneous equations*
$$a_1 x + b_1 y + c_1 z = 0$$
$$a_2 x + b_2 y + c_2 z = 0$$
$$a_3 x + b_3 y + c_3 z = 0$$
There is only a *trivial solution* $x = 0, y = 0, z = 0$
$$\text{if } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$
otherwise there are an infinite number of solutions, *i.e.*, system of equations is dependent.

46. *Product of two non-zero matrices can be a zero matrix*
$$i.e., \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\text{and } \begin{pmatrix} 0 & 4 \\ 0 & 7 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Group A (1 mark)

1. Find the values of x and y if

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$ (C.B.S.E. 2008)

2. Find the cofactor of a_{12} in the following :

$$\begin{vmatrix}2 & -3 & 5\\6 & 0 & 4\\1 & 5 & -7\end{vmatrix}$$ (C.B.S.E. 2008)

Evaluate each of the following (3 – 4):

3. $\begin{vmatrix}a+ib & c+id\\-c+id & a-ib\end{vmatrix}$ (C.B.S.E. 2008)

4. $\begin{vmatrix}\sin 30^\circ & \cos 30^\circ\\-\sin 60^\circ & \cos 60^\circ\end{vmatrix}$ (C.B.S.E. 2008)

5. For what value of x, is the following matrix singular?

$$\begin{bmatrix}3-2x & x+1\\2 & 4\end{bmatrix}$$ (C.B.S.E. 2008)

6. A matrix A of order 3×3 has determinant 4. Find the value of $|3A|$. (C.B.S.E. 2008)

7. If $\begin{vmatrix}2x+5 & 3\\5x+2 & 9\end{vmatrix}=0$, find x. (C.B.S.E. 2008)

8. If $\begin{bmatrix}x+3 & 4\\y-4 & x+y\end{bmatrix}=\begin{bmatrix}5 & 4\\3 & 9\end{bmatrix}$, find x and y. (C.B.S.E. 2008)

9. Construct a 2×2 matrix whose element a_{ij} is given by $a_{ij} = i + 2j$. (C.B.S.E. 2008)

10. If $\begin{bmatrix}x+3y & y\\7-x & 4\end{bmatrix}=\begin{bmatrix}4 & -1\\0 & 4\end{bmatrix}$, find the values of x and y. (C.B.S.E. 2008)

11. If $\begin{vmatrix}x+2 & 3\\x+5 & 4\end{vmatrix}=3$, find the value of x. (C.B.S.E. 2008)

12. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$ (C.B.S.E. 2008 C)

13. If $\begin{bmatrix}x+2y & -y\\3x & 4\end{bmatrix}=\begin{bmatrix}-4 & 3\\6 & 4\end{bmatrix}$, find the values of x and y. (C.B.S.E. 2008 C)

14. Evaluate : $\begin{vmatrix}2\cos\theta & -2\sin\theta\\\sin\theta & \cos\theta\end{vmatrix}$ (C.B.S.E. 2008 C)

15. If $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$, find $A - B$. (C.B.S.E. 2008 C)

16. If $\begin{bmatrix}x+2y & 3y\\4x & 2\end{bmatrix}=\begin{bmatrix}0 & -3\\8 & 2\end{bmatrix}$, find the value of x and y. (C.B.S.E. 2008 C)

17. If $\begin{vmatrix}x & x\\1 & x\end{vmatrix}=\begin{vmatrix}3 & 4\\1 & 2\end{vmatrix}$, find the value of x. (C.B.S.E. 2008 C)

18. If matrix $A = [1\ 2\ 3]$, write AA' where A' is the transpose of matrix A. (C.B.S.E. 2009)

19. Write the value of the determinant

$$\begin{vmatrix}2 & 3 & 4\\5 & 6 & 8\\6x & 9x & 12x\end{vmatrix}$$ (C.B.S.E. 2009)

20. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|\text{adj } A|$. (C.B.S.E. 2009)

21. Find the value of x if

$$\begin{bmatrix}3x+y & -y\\2y-x & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\-5 & 3\end{bmatrix}$$ (C.B.S.E. 2009)

22. Find the value of x from the following :

$$\begin{vmatrix}x & 4\\2 & 2x\end{vmatrix}=0$$ (C.B.S.E. 2009)

23. If $A=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$, then find k if $|2A| = k|A|$. (C.B.S.E. 2009)

24. Write the value of the determinant $\begin{vmatrix}a-b & b-c & c-a\\b-c & c-a & a-b\\c-a & a-b & b-c\end{vmatrix}$. (C.B.S.E. 2009)

25. If $A=(a_{ij})=\begin{bmatrix}2 & 3 & -5\\1 & 4 & 9\\0 & 7 & -2\end{bmatrix}$

and $B=(b_{ij})=\begin{bmatrix}2 & 1 & -1\\-3 & 4 & 4\\1 & 5 & 2\end{bmatrix}$, find $a_{22}+b_{21}$. (C.B.S.E. 2009)

Group B (3 or 4 marks)

26. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$ (C.B.S.E 1992, 2003)

27. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (C.B.S.E. 1992, 1998 C, 2009)

28. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$ (C.B.S.E. 1992)

29. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'.A = I$ (C.B.S.E. 1992)

30. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 = 8A + kI$. (C.B.S.E. 1993)

31. Without expanding the determinant show that $a + b + c$ is a factor of the determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$ (C.B.S.E 1993)

32. If $A = \begin{bmatrix} 0 & -1 & 5 \\ 6 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 1 & 0 \\ 7 & -6 \end{bmatrix}$ show that $(AB)' = B'A'$. (C.B.S.E. 1993)

33. Without expanding the determinant, prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$. (C.B.S.E. 1993, 2009)

34. If $A = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$ and $B = [3 \quad 1 \quad -2]$, verify that $(AB)' = B'A'$. (C.B.S.E. 1994)

35. Find a 2×2 matrix B such that $B\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$. (C.B.S.E. 1994, 2009)

36. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$. (C.B.S.E. 1995)

37. Using determinants, find the area of the triangle whose vertices are $(-2, 4)$, $(2, -6)$ and $(5, 4)$. Are the given points collinear? (C.B.S.E. 1995)

38. Define a symmetric matrix. Prove that for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A + A'$ is a symmetric matrix where A' is transpose of A. (C.B.S.E. 1996)

39. Using determinants, find the area of the triangle with vertices $(-3, 5)$, $(3 - 6)$ and $(7, 2)$. (C.B.S.E. 1997)

40. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (C.B.S.E. 1998)

41. Compute the adjoint of the matrix

$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify that $A.(\text{adj } A) = |A| I$. (C.B.S.E. 1998)

42. For matrix $A = \begin{bmatrix} -3 & 6 & 0 \\ 4 & -5 & 8 \\ 0 & -7 & -2 \end{bmatrix}$, find $\frac{1}{2}(A - A')$, where A' is the transpose of the matrix A. (C.B.S.E. 1998 C)

43. If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, find a matrix C such that $A + B + C$ is a zero matrix. (C.B.S.E. 1999)

44. Construct a 2×3 matrix whose elements in the ith row and jth column are given by $a_{ij} = \frac{3i - j}{2}$. (C.B.S.E. 1999)

45. Construct a 2×3 matrix whose elements in the ith row and jth column are given by $a_{ij} = \frac{2i + 3j}{2}$. (C.B.S.E. 1999)

46. Without expanding the determinant, prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$. (C.B.S.E. 1999 C)

47. If $f(x) = x^2 - 4x + 1$, find $f(A)$ when $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. (C.B.S.E. 1999 C)

48. Without expanding the determinant, prove that $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = 0$. (C.B.S.E. 1999 C)

49. Find a matrix X such that $2A + B + X = O$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$. (C.B.S.E. 2000,2002 C)

50. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, show that $AB \neq BA$. (C.B.S.E. 2000 C)

51. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix where A^T denotes the transpose of A. (C.B.S.E. 2001)

52. If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that $A + A^T$ is a symmetric matrix where A^T denotes the transpose of matrix A. (C.B.S.E. 2001)

53. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2 \quad -1 \quad -4]$, verify that $(AB)' = B'A'$. (C.B.S.E. 2002)

54. Construct a 2×3 marix A, whose elements are given by $a_{ij} = \frac{(i-2j)^2}{2}$. (C.B.S.E. 2002)

55. From the following equation, find the values of x and y :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$ (C.B.S.E. 2002 C)

56. Using the properties of determinants, evaluate the following : $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$. (C.B.S.E. 2003)

Using the properties of determinants, show that (57 – 58) :

57. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$. (C.B.S.E. 2003)

58. $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$. (C.B.S.E. 2003, 2005, 2008)

59. Find X such that X. $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$. (C.B.S.E. 2003)

60. Solve for x and y given that

$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (C.B.S.E. 2003 C)

61. Find the values of a and b for which the following holds

$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ (C.B.S.E. 2003 C)

62. Express $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (C.B.S.E. 2004)

63. Using the properties of determinants, solve for x

$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$. (C.B.S.E. 2004, 2005)

64. If $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$, find $f(a)$ where $f(x) = x^2 - 5x + 7$. (C.B.S.E. 2004 C)

65. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that

$A^3 - 4A^2 + A = O$ (C.B.S.E 2005)

66. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a, b and c are in A.P. (C.B.S.E. 2005)

67. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = [-2 \ \ -1 \ \ -4]$, verify that $(AB)' = B'A'$. (C.B.S.E. 2005)

68. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + kI$. (C.B.S.E. 2005 C)

69. Using the properties of determinants, prove that

$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ (C.B.S.E. 2005 C)

70. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (C.B.S.E. 2006)

71. Using the properties of determinants, prove that

$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$ (C.B.S.E. 2006)

72. If a, b and c are in A.P., show that

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ (C.B.S.E. 2006)

73. Find the value of x if

$[1 \ \ x \ \ 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$ (C.B.S.E. 2006 C)

74. Using the properties of determinants, prove that

$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ (C.B.S.E. 2006 C)

75. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = O$. (C.B.S.E. 2007)

Hence find A^{-1}.

76. Using the properties of determinants, prove that

$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$ (C.B.S.E. 2007)

77. Using the properties of determinants, prove that

$\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0$. (C.B.S.E. 2007)

Group C (4 marks)

78. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = O$. Thus find A^{-1}. (C.B.S.E. 1994)

Using the properties of determinants, show that (79 – 80):

79. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$. (C.B.S.E. 1995)

80. $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4).$ *(C.B.S.E. 1996)*

81. If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, verify $A^2 - 4A - I = O$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Hence find A^{-1}. *(C.B.S.E. 1996)*

Using the properties of determinants, prove each of the following (82 – 90):

82. $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$ *(C.B.S.E. 1997, 2000)*

83. $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$ *(C.B.S.E. 1998)*

84. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$ *(C.B.S.E. 1998, 1999, 2008)*

85. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$ *(C.B.S.E. 1999, 2001, 2007, 2008 C)*

86. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$ *(C.B.S.E. 2000 C, 2007)*

87. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0.$ *(C.B.S.E. 2001 C)*

88. $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$ *(C.B.S.E. 2002)*

89. $\begin{vmatrix} 1 & x+y & x^2+y^2 \\ 1 & y+z & y^2+z^2 \\ 1 & z+x & z^2+x^2 \end{vmatrix} = (x-y)(y-z)(z-x).$ *(C.B.S.E. 2002)*

90. $\begin{vmatrix} 43 & 3 & 6 \\ 35 & 21 & 4 \\ 17 & 9 & 2 \end{vmatrix} = 0.$ *(C.B.S.E. 2002 C)*

91. Evaluate, using the properties of determinants : $\begin{vmatrix} 9 & 9 & 12 \\ 1 & 3 & -4 \\ 1 & 9 & 12 \end{vmatrix}.$ *(C.B.S.E. 2002 C)*

92. Using the properties of determinants, prove that

$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz.$ *(C.B.S.E. 2004 C*

93. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric. *(C.B.S.E. 2008*

94. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify $A^2 - 4A - 5I = O$. *(C.B.S.E. 2008*

95. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that $xyz = -1$. *(C.B.S.E. 2008, 2008 C*

Using the properties of determinants, prove each of the following (96 – 99) :

96. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$ *(C.B.S.E. 2008*

97. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$ *(C.B.S.E. 2008*

98. $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2).$ *(C.B.S.E. 2008*

99. $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b).$ *(C.B.S.E. 200*

100. If a, b and c are all positive and distinct, show that $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value. *(C.B.S.E. 200*

101. Solve for x : $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0.$ *(C.B.S.E. 200*

Using the properties of determinants, prove each of the following (102 – 108) :

102. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$ *(C.B.S.E. 2008*

103. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ *(C.B.S.E. 2008*

104. $\begin{vmatrix} x & x^2 & 1+ax^3 \\ y & y^2 & 1+ay^3 \\ z & z^2 & 1+az^3 \end{vmatrix} = (1+axyz)(x-y)(y-z)(z-x)$ (C.B.S.E. 2008 C)

105. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$ (C.B.S.E. 2009)

106. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$ (C.B.S.E. 2009)

107. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$ (C.B.S.E. 2009)

108. $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$ (C.B.S.E. 2009)

Group D (6 marks)

109. Solve the following system of equations by matrix method:

$$\begin{aligned} 3x+4y+7z &= 14 \\ 2x-y+3z &= 4 \\ x+2y-3z &= 0 \end{aligned}$$ (C.B.S.E. 1992)

110. Find A^{-1} if $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

Hence solve the following system of linear equations:

$$\begin{aligned} x+2y+5z &= 10 \\ x-y-z &= -2 \\ 2x+3y-z &= -11 \end{aligned}$$ (C.B.S.E. 1993)

Solve each of the following system of equations, using matrix method (111 – 113):

111. $6x-12y+25z=4$
$4x+15y-20z=3$
$2x+18y+15z=10$ (C.B.S.E. 1994)

112. $x+2y+z=7$
$x+3z=11$
$2x-3y=1$ (C.B.S.E. 1995, 2002, 2003)

113. $3x-4y+2z=-1$
$2x+3y+5z=7$
$x+z=2$ (C.B.S.E. 1996)

114. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of linear equations

$$\begin{aligned} x-2y &= 10 \\ 2x+y+3z &= 8 \\ -2y+z &= 7 \end{aligned}$$ (C.B.S.E. 1997)

115. If $A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equations :

$$\begin{aligned} 8x-4y+z &= 5 \\ 10x+6z &= 4 \\ 8x+y+6z &= \frac{5}{2} \end{aligned}$$ (C.B.S.E. 1998 C)

116. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2-4A-5I=O$. (C.B.S.E. 1999)

117. Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \dfrac{A^2-3I}{2}$. (C.B.S.E. 1999)

Using matrix method, solve each of the following system of equations (118 – 126) :

118. $2x-z=3$
$5x+y=7$
$y+3z=-1$ (C.B.S.E 1999C)

119. $x+2y-3z=6$
$3x+2y-2z=3$
$2x-y+z=2$ (C.B.S.E. 2000, 2007)

120. $x+y+z=1$
$x-2y+3z=2$
$x-3y+5z=3$ (C.B.S.E. 2000 C)

121. $x-y+z=3$
$2x+y-z=2$
$-x-2y+2z=1$ (C.B.S.E. 2001)

122. $x+y+z=6$
$x+2y+3z=14$
$x+4y+7z=30$ (C.B.S.E. 2001 C)

123. $x+2y-3z=-4$
$2x+3y+2z=2$
$3x-3y-4z=11$ (C.B.S.E. 2002 C, 2008)

124. $5x+3y+z=16$
$2x+y+3z=19$
$x+2y+4z=25$ (C.B.S.E. 2002 C)

125. $\dfrac{2}{x}+\dfrac{3}{y}+\dfrac{10}{z}=4$
$\dfrac{4}{x}-\dfrac{6}{y}+\dfrac{5}{z}=1$
$\dfrac{6}{x}+\dfrac{9}{y}-\dfrac{20}{z}=2$ (C.B.S.E. 2002 C)

126. $2x+6y=2$
$3x-z=-8$
$2x-y+z=-3$ (C.B.S.E. 2003)

127. Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB.

Use this to solve the following system of equations :

$$\begin{aligned} x - y &= 3 \\ 2x + 3y + 4z &= 17 \\ y + 2z &= 7 \end{aligned}$$

(C.B.S.E. 2003 C)

Using matrices, solve each of the following system of equations (128–131):

128. $2x - 3y + z = -1$
$x - 2y + 3z = 6$
$-3y + 2z = 0$ *(C.B.S.E. 2004 C)*

129. $x + y + z = 4$
$2x - y + z = -1$
$2x + y - 3z = -9$ *(C.B.S.E. 2005)*

130. $x + y - z = 1$
$x - y - z = 1$
$3x + y - 2z = 3$ *(C.B.S.E. 2005)*

131. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Use it to solve the following system of equations :

$$\begin{aligned} 2x - 3y + 5z &= 16 \\ 3x + 2y - 4z &= -4 \\ x + y - 2z &= -3 \end{aligned}$$

(C.B.S.E. 2005 C)

Using matrices, solve the following system of equations (132 – 137) :

132. $3x - y + z = 5$
$2x - 2y + 3z = 7$
$x + y - z = -1$ *(C.B.S.E. 2006)*

133. $3x + 4y + 2z = 8$
$2y - 3z = 3$
$x - 2y + 6z = -2$ *(C.B.S.E. 2006 C)*

134. $x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$ *(C.B.S.E. 2007)*

135. $x + 2y + z = 1$
$x + 3z = 11$
$2x - 3y = 1$ *(C.B.S.E. 2007)*

136. $x + 3y + 4z = 8$
$2x + y + 2z = 5$
$5x + y + z = 7$ *(C.B.S.E. 2008)*

137. $2x - y + z = 3$
$-x + 2y - z = -4$
$x - y + 2z = 1$ *(C.B.S.E. 2008)*

138. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

(C.B.S.E. 2008, 2009)

Using elementary transformations, find the inverse of each of the following matrices (139 – 141):

139. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ *(C.B.S.E. 2008)*

140. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ *(C.B.S.E. 2008)*

141. $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ *(C.B.S.E. 2008)*

142. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

(C.B.S.E. 2008 C, 2009)

Using matrices solve each of the following system of equations (143 – 147) :

143. $x + 2y + z = 7$
$x + 3z = 11$
$2x - 3y = 1$ *(C.B.S.E. 2008 C)*

144. $2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y - 2z = 3$ *(C.B.S.E. 2008 C)*

145. $2x + y + z = 7$
$x - y - z = -4$
$3x + 2y + z = 10$ *(C.B.S.E. 2008 C)*

146. $x + y + z = 6$
$x + 2z = 7$
$3x + y + z = 12$ *(C.B.S.E. 2009)*

147. $3x - 2y + 3z = 8$
$2x + y - z = 1$
$4x - 3y + 2z = 4$ *(C.B.S.E. 2009)*

148. Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

(C.B.S.E. 2009)