BASICS OF MATHEMATICS

Mathematics may not teach us how to <u>add</u> <u>Love</u> or <u>subtract</u> <u>Hate</u> but it gives us hope that <u>every problem</u> has a <u>Solution</u>.

The essence of mathematics is not to make simple things complicated but to make complicated things simpler.

In O. P. Jindal School, Raigarh Mathematics is an integral and highly valued component of curriculum. While learning about Maths students recognize that there are particular ways of working with basic concepts of Mathematics. They also recognize that there are particular facts and procedures required for knowing and understanding in Mathematics. Though our school has achieved greater heights in all Board Examinations & External Examinations related to Mathematics still some students are lagging behind due to lack of basic knowledge and practice. So to bring them up we have planned out a CMP for basics of Mathematics for middle and high school children.

Hope that by the help of this plan and our Maths fraternity we can overcome this problem in future.

BOOKLET OF CLASSES VI&VII

MATHEMATICS

TABLES – 1 to 20

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

11	12	13	14	15	16	17	18	19	20
22	24	26	28	30	32	34	36	38	40
33	36	39	42	45	48	51	54	57	60
44	48	52	56	60	64	68	72	76	80
55	60	65	70	75	80	85	90	95	100
66	72	78	84	90	96	102	108	114	120
77	84	91	98	105	112	119	126	133	140
88	96	104	112	120	128	136	144	152	160
99	108	117	126	135	144	153	162	171	180
100	120	130	140	150	160	170	180	190	200

FOUR BASIC FUNDAMENTAL OPERATIONS

There are situations in day to day life when we are required to add or subtract large numbers.

Addition :-

Arrange the given nos. in columns, ones under ones, tens under tens and so on. Starting from ones place add the digits in each column & write the sum under the corresponding columns.

Eg. Add – 3,67,52,896 and 1,51,62,905

$$\begin{array}{r} 3,67,52,\ 896 \\ +\ 1,51,62,\ 905 \\ \hline 5,19,15\ \ 801 \\ \hline \end{array}$$

Subtraction :-

Do the same arrangement of numbers like addition. Beginning with ones, go on subtracting, column wise borrowing from the next column to the left.

Eg. Sub – 6,14,78,165 – 2,81,65,236

$$
\begin{array}{rl}
 & 5\ 11\quad\quad 7\ \ 11\ 5\ 15 \\
 & \cancel{6},\ \cancel{1}\ 4,\ 7\ \cancel{8},\ \cancel{1}\ \cancel{6}\ \cancel{5} \\
+ & 2,\ 8\ 1,\ 6\ 5,\ 2\ 3\ 6 \\
\hline
 & 3,\ 3\ 3,\ 1\ 2,\ 9\ 2\ 9 \\
\hline
\end{array}
$$

Multiplication :-

Whenever you are encountered with a situation where one is given & many are required to be ascertained it certainly involves the use of multiplication.

Eg.

$$
\begin{array}{r}
2198 \\
\text{x}125 \\
\hline
10990 \\
43960 \\
219800 \\
\hline
274750 \\
\hline
\end{array}
$$

Division :-

Division is an inverse process of multiplication.

Eg. Divide 51615 by 55

```
           938
        ________
  55 )  51615
      - 495
       _______
        211
        165
       _______
         465
         440
       _______
          25
```

Thus,

Quotient 938

& Remainder 25

For the Verification of division, we can use the given formula

Dividend = Quotient x Divisior + Remainder.

Exercise

1) Add 61,55,44,444 and 3,83,91,121
2) Subtract 1,61,90,988 from 2,35,42,699
3) Simplify 6,12,35,262 + 1,28,618 – 6,12,34,567

4) Multiply 7576 by 7314

5) Which is greater – 1238 x 485 or 1535 x 335

6) Divide and check 39476 ÷ 69

7) Evaluate 55 ÷ 5 x 75 + 25 – 25

Number system –

1) **Natural Numbers** – The counting numbers 1,2,3,4 ------ are called natural numbers.

2) **Whole Numbers** – The natural numbers along with 0 are called whole numbers.

3) **Even Numbers –** Any number which is divisible by 2 is called an even number or if the unit's place digit of a given number is 0, 2, 4, 6, 8, then the number is called an even no.

4) **Odd Numbers –** If the unit's place digit of a given number is either 1,3,5,7,9 then the number is called an odd number.

5) **Integers –** The negative numbers ----------- -4, -3, -2, -1 along with whole numbers are called integers. In other words ------ -4, -3, -2, -1, 0, 1,2,3,4 ---- are integers.

- The numbers --------- -4, -3, -2, -1 are called negative integers.
- The numbers 1, 2, 3, 4 ---- are called positive integers.
- The number zero is an integer but it is neither positive nor negative.

6) **Fraction –** Fraction is a number representing a part of a whole.

7) **Decimals –** Decimals are another way of writing parts of a whole number. Every decimal number has two parts. The whole or integral part and the decimal part.

Eg. 13.25 → Decimal part

↓

Whole part

8) **Factor –** A Factor of a number is an exact divisor of that number, i.e. it divides the number completely without leaving any remainder.

Eg Factors of 12 ⟶ 1 x 12

2 x 6

3 x 4

⟹ 1, 2, 3, 4, 6, 12

9) **Multiple –** A multiple of a number is obtained by multiplying it with a natural number. Multiple of a number is always equal to or greater than given number.

10) **Perfect Number –** If the sum of all the factors of a number is equal to twice the number then it is called a perfect number.

Eg- 6 is a perfect number

Factors of 6 - 1, 2, 3, 6

Sum of the factors = 1 + 2 + 3 + 6

= 12

∴6 is a perfect no.

Exercise

1) Write all the factors of the following number :-

a) 729 b) 144 c) 108

2) Write the first five multiples of the following number :-

a) 27 b) 35 c) 13

3) Match the following :-

a)	42	i)	Multiple of 8
b)	15	ii)	Multiple of 7
c)	16	iii)	Multiple of 9
d)	20	iv)	Factor of 30
e)	25	v)	Factor of 50
f)	18	vi)	Factor of 20

Prime Numbers – The numbers other than 1 whose only factors are 1 and the number itself are called prime numbers.

Examples of prime numbers are 2, 3, 5, 7, 11, 13, ----- as these do not have factors other than 1 and the number itself.

- The smallest prime no. is 2
- 2 is the only even no which is prime.

Composite Numbers – The numbers having more than two factors are called composite numbers. These numbers have atleast one factor other than 1 and the number itself.

Eg. 4 is a composite number as the factors of 4 are 1, 2, 4.

- 1 is neither prime nor composite.

Twin Primes – If the difference between two prime numbers is 2, then such pair of prime numbers are called twin primes eg. (3, 5), (5, 7), (11, 13), (17, 19)

Exercise

1) Write all the prime numbers between 1 & 50
2) Write all the even numbers between 60 and 80
3) Write all the odd numbers between 50 & 70
4) Write all the twin primes between 1 & 100.

Common Factors – A number which exactly divides two or more given numbers is a common factor of the given numbers.

Eg. Find common factors of 15 and 18.

- Factors of 15 ⟶ 1, 3, 5, 15
- Factors of 18 ⟶ 1, 2, 3, 6, 9, 18
- Common Factors ⟶ 1, 3

Common Multiples – A number which is exactly divisible by two or more given numbers is the common multiple of the given numbers.

Eg. Find the common multiples of 12 & 16

- Multiples of 12 ⟶ 12, 24, 36, 48, 60, 72, 84, 96, 108 ------
- Multiples of 16 ⟶ 16, 32, 48, 64, 80, 96, 112, 128, 144 ------
- Common multiples ⟶ 48, 96.

HCF – Highest Common Factor of two or more given numbers is the highest of their common factors.

Eg. Find the HCF of 15 & 18

Factors of 15 ⟶ 1, 3, 5, 15

Factors of 18 ⟶ 1, 2, 3, 6, 9, 18

Common factors ⟶ 1, 3

∴ HCF ⟶ 3

LCM – Lowest Common Multiple of two or more numbers is the lowest of their common multiples.

- Multiples of 8 ⟶ 8, 16, 24, 32, 40, 48, 56 ------
- Multiples of 12 ⟶ 12, 24, 36, 48, 60, 72, 84 ------
- Common multiples ⟶ 24, 48, 96 -----

∴LCM ⟶ 24

Exercise

1) Find the HCF of – (a) 72 and 144 (b) 24, 36, 90
2) Find the LCM of – (a) 12 and 8 (b) 15,9

Length , Mass & Capacity

Measurement

Metric **Impiral**

Metric Measurements :-

Units of Measurement :-

Length	mm	m	Km
Mass	mg	g	Kg
Capacity	ml	l	Kl

a) Convert in Kg

Eg. 8 kg 7g

$8kg + \frac{7}{1000}$ kg

8Kg + 0.007 Kg

= 8.007 Kg

b) Convert l into ml

1.6 l = 1.6 x1000
= 1600 ml

Exercise

Q1. Change into gram
a. 6.5 Kg b. 2.36 Kg c. 2 Kg 6 g

Q2. Change into Litre
a. 6 ml b. 5l 8ml c. 1254 ml

Q3. Change into Kg.
a. 6mg b. 8.2 g c. 8Kg 2g d. 82 mg

PROFIT AND LOSS

The words “Profit” and “Loss” are used very frequently in the field of business. In our day to day life, we purchase goods from various shops. The shop-keepers purchase these goods either from wholesalers or directly from the manufacturers by paying a certain price.

The price at which the shopkeeper purchases the goods is called the Cost Price (C.P). The price at which the shopkeeper sells the goods is called the selling Price (S.P)

Profit: If the selling price of an article is more than its cost price; then the shop-keeper earns a gain or profit. Thus, if (S.P > C.P), then Profit = Selling Price – Cost Price

Loss: If the selling price of an article is less than its cost price, then the shopkeeper suffers a loss. Thus, if (C.P > SP), then Loss = Cost Price – Selling Price

Exercise Questions:

a) Find the profit or loss in each of the following:

i) C.P = Rs. 482 S.P = Rs. 500

ii) C.P = Rs. 1500 S.P = Rs. 1200

b) Find the unknown in the following:

i) S.P =Rs. 750.50 Profit =Rs. 100 C.P = ?

ii) C.P =Rs. 1002 Loss = Rs. 87 S.P = ?

MENSURATION

PERIMETER

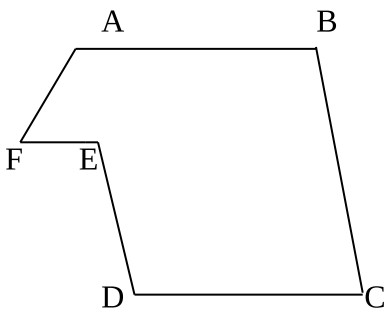
Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.

So, how will you find perimeter of any closed figure made up entirely of line segments?

Simply, find the sum of the lengths of all the sides.

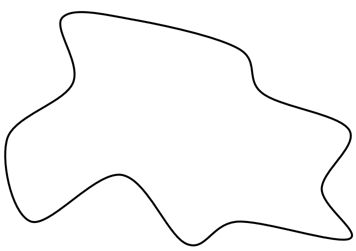
AB=100m, BC=120m, CD=90m, DE=45m, EF=60m, FA=80m

Perimeter=AB+BC+CD+DE+EF+FA=_____+______+______+______+______+______+____.



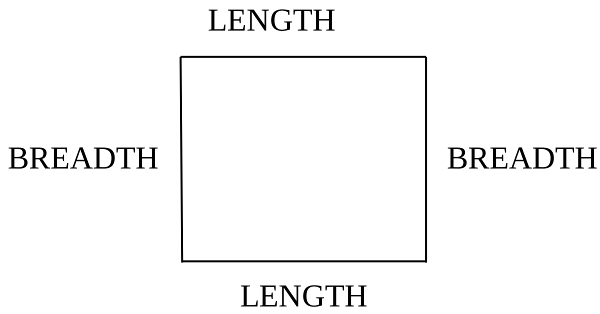
How to find the perimeter of any closed figure if it is not made up of line segments?

Simply by moving a string along the boundary of the closed figure starting from the point and reaching the point .The length of the string is the perimeter of the figure.



Deduction of formula for perimeter.

I. RECTANGLE :-

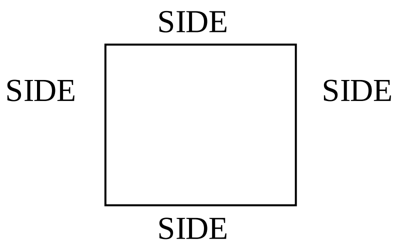


Perimeter of a rectangle=length +breadth +length +breadth

=2(length +breadth).

II.REGULAR POLYGONS

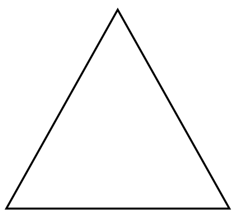
A. SQUARE :-



Perimeter=side +side +side +side

=4 xSide.

B.EQUILATERAL TRIANGLE:-



Perimeter= side +side +side

= 3 x Side.

C. n- Sided Polygon

Perimeter= side +side +side+……..

= n x Side.

EXERCISE

Q1. Find the perimeter of:

(a) A Triangle of side 3cm,4cm and 5cm

(b) A regular hexagon with each side measuring 8m.

Q2. Find the side of the square whose perimeter is 20m.

Q3. Two sides of triangle are 12cm and 14cm the perimeter of triangle is 36cm what is the third side?

AREA

The amount of surface enclosed by a closed figure is called its area.

So, we can find the area of a closed figure by counting the no. of centimeter squares that are needed to cover it

For that we have to follow the following conventions:-

→ The area of one full square is taken as 1 sq.unit

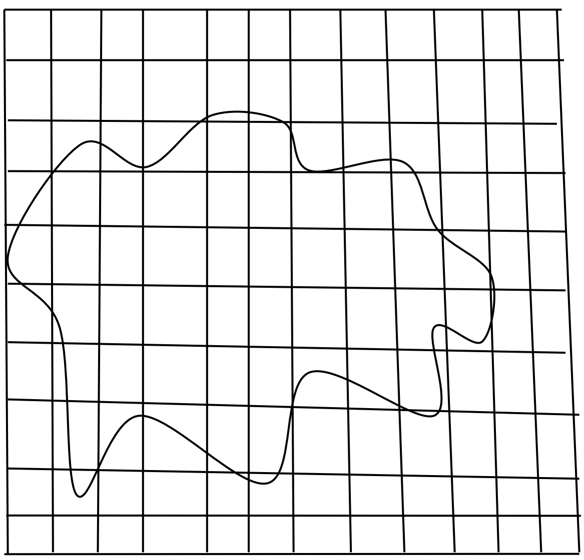
→ Ignore portions of the area that are lesser than half a square.

→ If more than half of a square is in a region count it as one square.

→ If exactly half the square is counted, take its area as $\frac{1}{2}$ sq.unit.

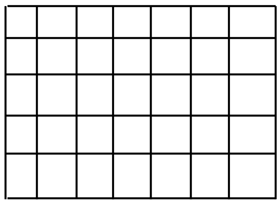
COVERED AREA	NUMBERESTIMATE	AREA(sq.units)
Fully filled squares	11	11
Half filled squares	3	$3\times \frac{1}{2} = \frac{3}{2}$
More than Half filled squares	7	7
Less than Half filled Squares	5	0

Total area= $11 + \frac{3}{2} + 7 = 19\frac{1}{2}$ sq.units



Deduction of formula for area of :-

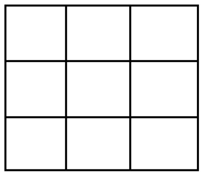
I.RECTANGLE :-   7 UNITS



5 UNITS

No. of squares of 1 unit side= no. of columns x no. of rows
= 7 x 5
= length x breadth

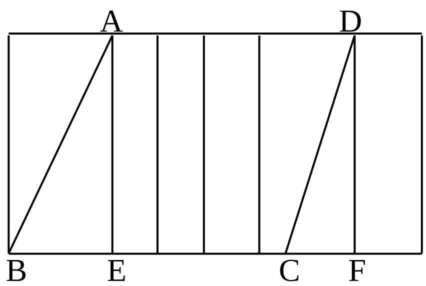
II.SQUARE :-   3 UNITS



3 UNITS

No. of 1 unit square endorsed = no. of columns x no. of rows
= 3 x 3
= side x side.

III. PARALLELOGRAM :-



Area of parallelogram ABCD = Area of rectangle AEFD

= Length x breadth

= AD x AE

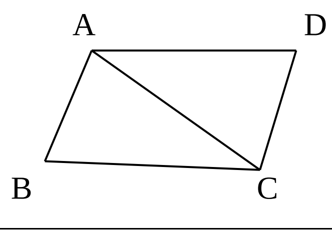
= BC x AE   (as AD = BC)

= BASE x HEIGHT.

IV. TRIANGLE :- If we draw a diagonal of a parallelogram, it is divided into two triangle.

The triangle superpose on each other

→area of parallelogram = 2 x area of △ ABC

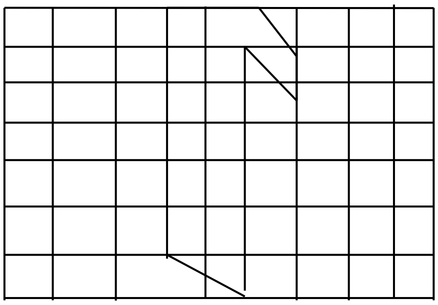


OR, area of △ ABC = $\frac{1}{2}$ x area of parallelogram ABCD

= $\frac{1}{2}$ x base x height.

EXERCISE

Q1. Find the area of the shape shown in the figure.



Q2.Find the area of rectangles whose sides are

(a) 3cm and 4 cm (b) 2m and 70 cm.

Q3. Find the area of square whose sides are

(a) 10cm (b) 5m.

Q4. The area of rectangular garden 50m long is 300 sq m. Find the width of the garden.

LINEAR EQUATION

The branch of mathematics in which we use letters for unknown quantities is called algebra. The variable allow us to express relations in any practical situation.

For example, the rule that the sum of two numbers remains the same if the order in which the numbers are taken is reversed can be expressed as a +b=b +a, where variable a and b stands for any number.

EQUATION- An equation is a condition on variable .It is expressed by saying that an expression with a variable is equal to a fixed no. e.g., x-3=0

It is satisfied only for a definite value of x i.e. 13.

Frame equation for the following :-

The sum of three times x and 11 is 32

Three times x is 3x

Sum of 3x and 11 and is 3x +11. The sum is 32

The equation 3x +11=32

SOLVING AN EQUATION

I. Consider the equation x+3=8

Subtracting 3 from both sides of the equation will reduce the LHS to x

Thus, x+3-3+8-3

Or, x=5

II. Consider the equation 5y=35

We shall divide both the sides by 5 . This will give us just y on LHS

Thus, $\frac{5y}{5} = \frac{35}{5}$

Or, y=7

Thus, by solving an equation our attempt should be to get the variable in the equation separated.

EXERCISE

Q1. State which of the following are equations (with a variable). Give reasons for your answer.

(a) 17 = x+17 (b) (t-7) > 5 (c) $\frac{4}{2}$=2

Q2. Write equations for following statements:-

(1) The sum of a number x and 4 is 9.

(2) Ten times a is 70.

(3) One-fourth of a no x minus 4 gives 4.

(4) If you take away 6 from 6 times y, you get 60.

Q3. Solve the following.

(1) 10p=100 (2) 10p +10=100 (3) $-\frac{p}{3}$=5 (4) 2q-6=0 (5) $\frac{3p}{4}$=6

<u>**Introduction to the word -Exponents and Powers:**</u>

Let's see few examples:

i) 2 x 2 x 2 x 2 x 2 x 2 can be written as 2^6
ii) 3 x 3 x 3 x 3 can be written as 3^4
iii) 5 x 5 can be written as 5^2

Here i) 2^6 — Exponent/Power Base ii) 3^4 — Exponent/Power Base iii) 5^2 — Exponent/Power Base

Similarly, 2^2 means a base of two raised to the second power and is read as 'two squared'. Sometimes the 'exponent' is also referred to as 'index'

Exercise

Q1. Write in exponential form:

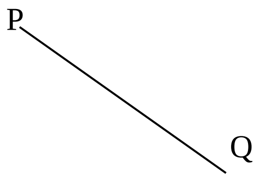
i) a x a x a x a = ______
ii) 7 x 7 x 7 x 7 x 7 x 7 = _______
iii) (-1) x (-1) x (-1) = ________

Q2. Write the base and the exponents of each of the following:

i) $(-6)^3$ ii) $(3.5)^6$ iii) $(7)^{11}$

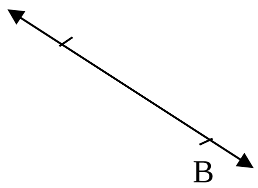
BASIC GEOMETRICAL IDEAS

Line Segment - A geometrical figure obtained by joining any two points in a straight way, is called a line segment.

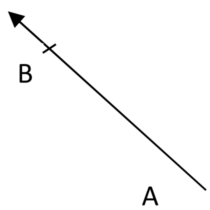


It can be denoted by $\overleftrightarrow{PQ}$

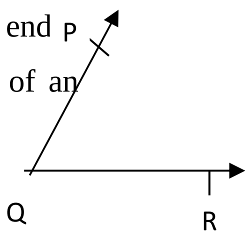
Line- If a line segment is extended infinitely in both directions, we get a line. A line has no end points and is denoted by $\overleftrightarrow{AB}$



Ray- If a line segment is extended infinitely in one direction the geometrical figure hence obtained is called a ray. A ray has only one direction. The geometrical figure hence obtained is called a ray. A ray has only one end point. In the given fig. $\overrightarrow{AB}$ represents a ray. A is called the initial point.

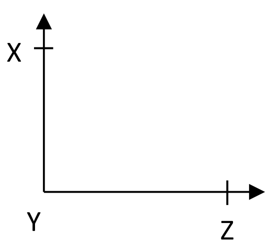


Angles- Two rays with common end point form an angle.Common end point Q is called the vertex of an angle. PQ & QR are the sides of an angle. The name of the angle is $\angle$ PQR.



Types of Angles- Angles can be classified into different categories according to their measures.

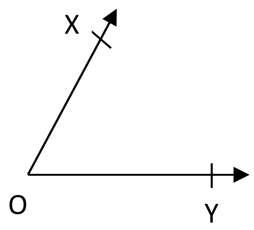
a) Right Angle :-



An angle having a measure of 90^0 is called a right angle. The measure of $\angle XYZ = 90^0$.

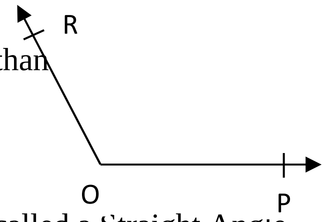
b) **Acute Angle**-

An angle having its measure as more than 0^0 but less than 90^0 is called an Acute Angle.

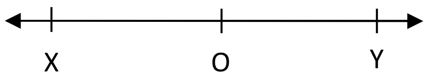


c) **Obtuse Angle** **:-**

An angle having a measure as more than 90^0 but less than 180^0 is called as Obtuse Angle.



d) **Straight Angle** **:-** An angle whose measure is 180^0 is called a Straight Angle.



Polygons **-**

A simple closed figure made up of line segments is called as Polygons.

Types of Polygon

Sr.No.	No. of Sides	Name
1	3	Triangle
2	4	Quadrilateral
3	5	Pentagon
4	6	Hexagon
5	7	Heptagon / Septagon
6	8	Octagon
7	9	Nonagon
8	10	Decagon

Pairs of Angles -

1) **Complementary Angles** **:-** If the sum of measures of two angle is 90^0, they are known as the complementary angles.

Ex. 40^0& 50^0 are Complementary because $40^0 + 50^0 = 90^0$.

2) **Supplementary Angles** **:-** If the sum of the measures of two angles is 180^0, they are called the supplementary angles.

Ex. 60^0& 120^0 are supplementary because $60^0 + 120^0 = 180^0$.

Exercise

1) Classify the following angles as acute, obtuse, right and straight angle.

a) 73^0 b) 180^0 c) 40^0 d) 65^0 e) 105^0

f) 60^0 g) 135^0 h) 89^0 i) 90^0 j) 135^0

2) Find the supplementary angles of the following :-

a) 110^0 b) 75^0 c) 130^0 d) 140^0 e) 115^0

3) Are the following angles Complementary →

a) $20^0, 70^0$ b) $40^0, 45^0$ c) $10^0, 60^0$

d) $30^0, 60^0$ e) $10^0, 80^0$

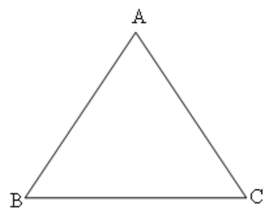
4) Draw (represent) the following on Paper

a) Ray AB b) Seg MN and Line EF

5) Differentiate between line, line segment& ray (write at least 3 points)

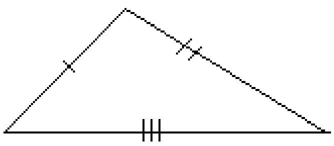
<u>TRIANGLE</u>

A triangle is a closed figure made of three line segments. It is the first polygon. Triangle ABC has six elements, the three sides AB, BC and CA and three angles angle A, angle B and angle C.

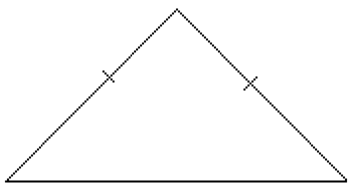


<u>Classification of triangle on the basis of:</u>

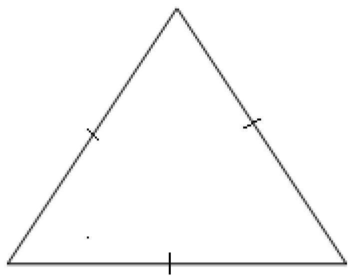
i) **Side –** Scalene, Isosceles and Equilateral triangle.



Scalene triangle
(all the sides are of
different length)

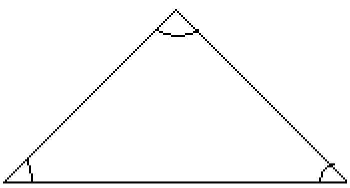


Isosceles triangle
(Any two of its sides
are equal in length)

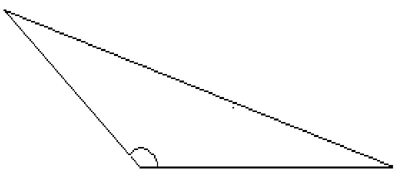


Equilateral triangle
(All the three sides
are equal)

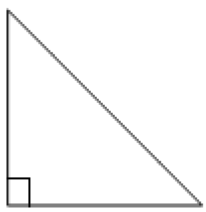
ii) **Angles –** Acute angled, Obtuse-angled and Right-angled triangle



(All the three angles
are acute)



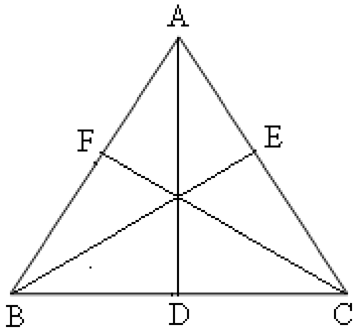
(One angle is
Obtuse)



(One angle is
right angle)

Median of a Triangle:-

A line segment joining the vertex of a triangle to the midpoint of opposite side is called the median. Three median can be drawn in a triangle.

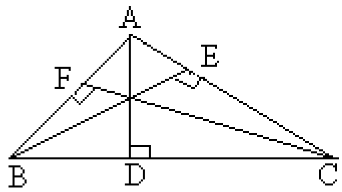


AD,BE,CF are the medians.

Altitude of a Triangle:-

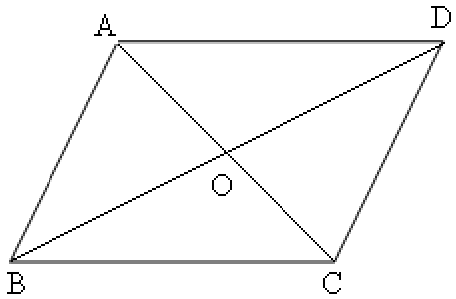
A line segment drawn from a vertex and is perpendicular to the opposite side of a triangle is called its attitude. Three attitudes can be drawn in a triangle.

AD $\perp$ BC, BE $\perp$ AC, CF $\perp$ AB

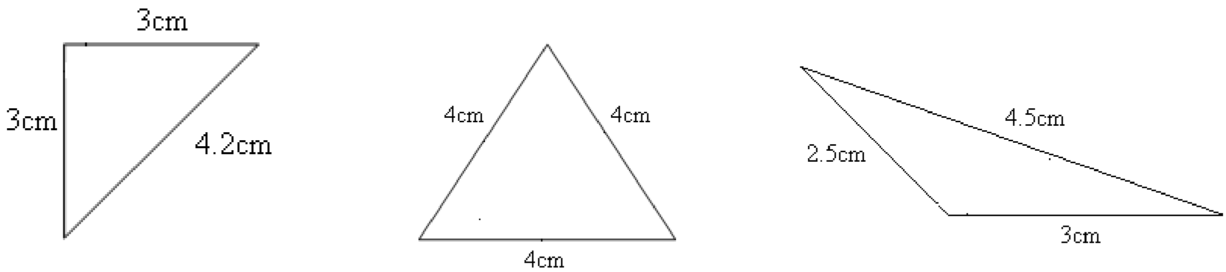


<u>Exercise</u>

1. Name all the triangle in the given figure:



2. Classify the triangles shown as scalene, isosceles or equilateral.

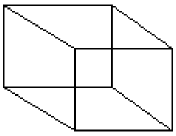
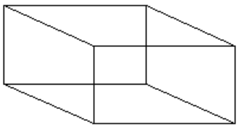
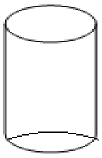
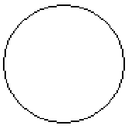
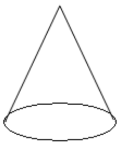
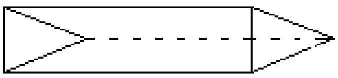
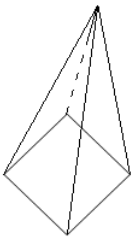
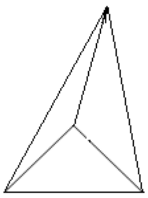


3. Can a triangle have:
   i) More than one right angle?
   ii) An obtuse angle and a right angle?
   iii) More than one obtuse angle?
   iv) Two sides?
   v) A right angle and an acute angle?

vi) An obtuse angle and an acute angle?
vii) More than one acute angle?
viii) All angles more than 60^0 ?

Solid Shapes:

An object with length, breadth and height is called a solid.

Name of Solid	No. of Faces	No. of Vertices	No. of Edges
 Cube	6	8	12
 Cuboid	6	8	12
 Cylinder	3	None	2
 Sphere	1	None	None
 Cone	2	1	1
 Prism	5	6	9
 Square Pyramid	5	5	8
 TriangularPyramid	4	4	6

1. **Multiple Choice Questions**:

1. The distance between the two end faces of the cylinder is called its:

   i) Curved Surface ii) Length iii) Area iv) Breadth

2. Clown's cap is an example of

   i) Cone ii) Cuboid iii) Pyramid iv) Sphere

3. A solid with no edges, no vertices is called a

   i) Cylinder ii) Sphere iii) Prism iv) Cube

2. **Fill in the blanks**:

1. All the faces of a _____________ are identical.
2. A triangular prism has ___________ faces.
3. A ____________ has 6 square faces, 12 edges and 8 vertices.

3. **Draw the following solids and write the number of vertices, edges and faces.**

i) A cube ii) a triangular pyramid iii) a triangular prism

Try these:

Draw a net for the following

a) A cube b) a cone c) a square pyramid

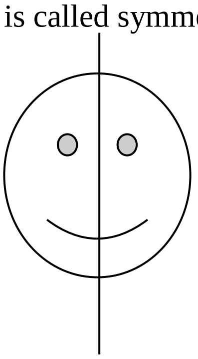
<u>CONGURENCE:</u> Suppose there are two figures F1 &F2

Take a trace-copy of one of them and place it over the other. If figures cover each other completely, they are congruent. The relation of object being congruent is called congruence.

If figure F1 is congruent to figure F2 we write F1≅ F2

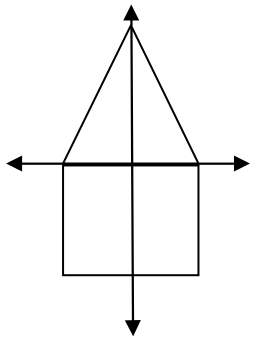
SYMMETRY: A figure through which a line can be drawn such that it divides the figure into two identical parts is called symmetrical figures and the line is called line of symmetry

As,



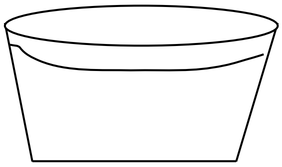
EXERCISE

Q1.For the given figure, which one is the line of symmetry, l1 or l2

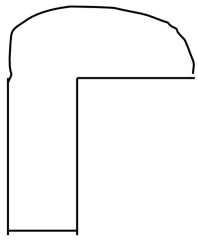


Q2. Identify the shapes given below. Check whether they are symmetric or not .Draw the line of symmetry as well.

(a)

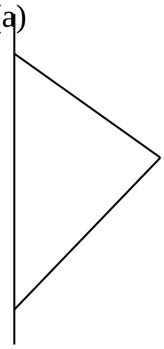


(b)

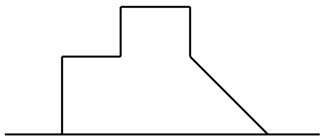


Q3.Complete the shapes such that the dotted line is the line of symmetry

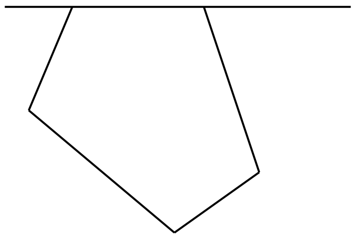
(a)



(b)

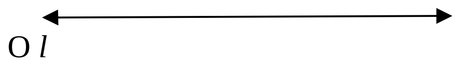


(c)



CONSTRUCTION OF BASIC ANGLES

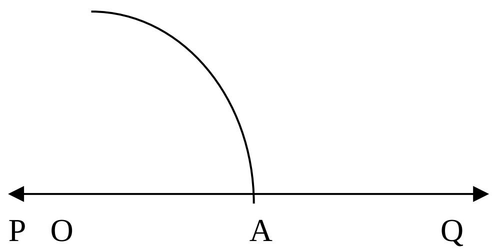
STEP 1- Draw a line l and mark a point o on it



STEP 2- Place the pointer of the compasses at O

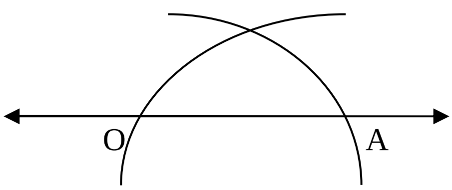
and draw an arc of convenient radius

which cuts the line $\overleftrightarrow{PQ}$ at a point say , A



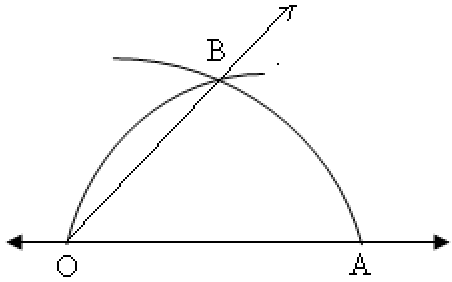
STEP 3- With A as centre,

Now draw an arc through O.



STEP 4- Let the two arcs intersect at B .Join OB

We get∠ BOA whose measure is 60°



CONSTRUCTING 120º ANGLE

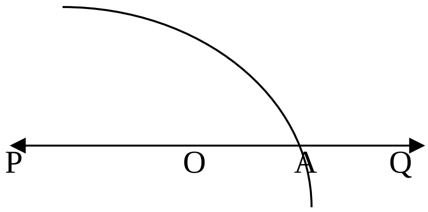
An angle of 120° is nothing but twice of an angle of 60º therefore, it can be constructed as follows:

STEP 1- Draw any line PQ and take any point O

STEP 2- Place the pointer of the compasses at O

And draw an arc of convenient radius

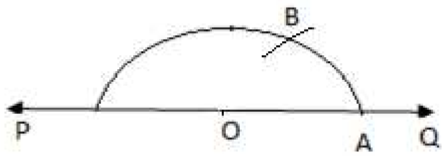
Which cuts the line at A



STEP 3- Without disturbing the radius on the

Compasses, draw an arc with A as centre

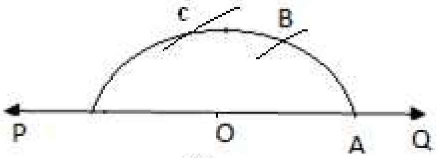
Which cuts first arc at B



STEP 4- Again without disturbing the radius on

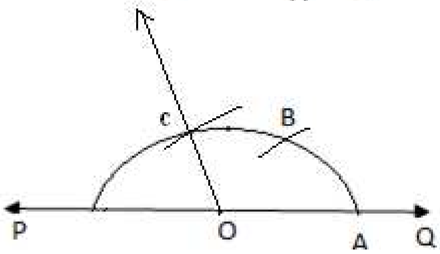
The compasses and with B as centre

Draw an arc which cuts first arc at C



STEP5- Join OC, ∠COA is the required angle whose

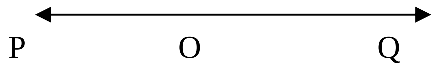
Measure is120°



CONSTRUCTING 90º ANGLE

Construct a perpendicular to line from a point lying on it

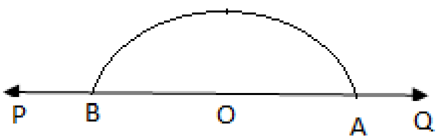
STEP 1- Draw a line PQ and take a point O



STEP 2- Place the points of the compass at O

and draw a semicircle of convenient radius which

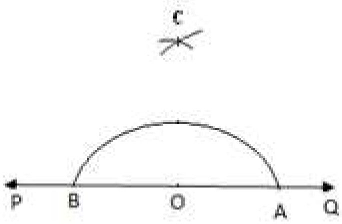
Cuts the line at A &B.



STEP 3- Increase the radius on the compass

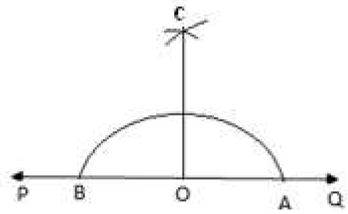
and with A as well as B as centre, draw

two arcs which cuts each other at C.



STEP 4- Join OC, $\angle$COA is the required angle

whose measure is 90°



EXERCISE

Q1. CONSTRUCT WITH RULER AND COMPASS , ANGLES OF FOLLOWING MEASURES:

(A) 60º (B) 30º (C) 120º (D) 90º (E) 45º (F) 135º

Statistics

Statistics – deals with the studies of collection , organisation , interpretation & representation of data .

*** Collection of data :-**

Data :- Collection of numbers gathered to give some information is called data .

Data
World Cup 2006

Spain beat Junisia by	3-1
Switzerland beat Jogo by	2-0
Ukraine beat Saudi Arabia by	4-0

Survey :-

Collect information regarding the number of family members of your classmates & represents it n the form of a table .

Organising Data :-

Organising data in a frequency distribution table :-

When we collect data , we have to record & organize it .

Eg.- Sahil is asked to collect data for size of shoes of students in his class . His finding are record in the manner shown below :-

5	4	7	5	6	7	6	5	6	6	5
4	5	6	8	7	4	6	5	6	4	6
7	5	7	6	5	7	6	4	8	7	

Sahil Prepared a table using tally marks

Shoe size	Tally Marks	No. of Students
4	𝍸	5
5	𝍸 𝍷𝍷𝍷	8
6	𝍸 𝍸	10
7	𝍸 𝍷𝍷	7
8	𝍷𝍷	2

Interpretation of Data

* Arithmatic Mean – $\frac{\text{Sum of all Observations}}{\text{No. of Observations}}$

* Range - Highest Observation - Lowest Observation

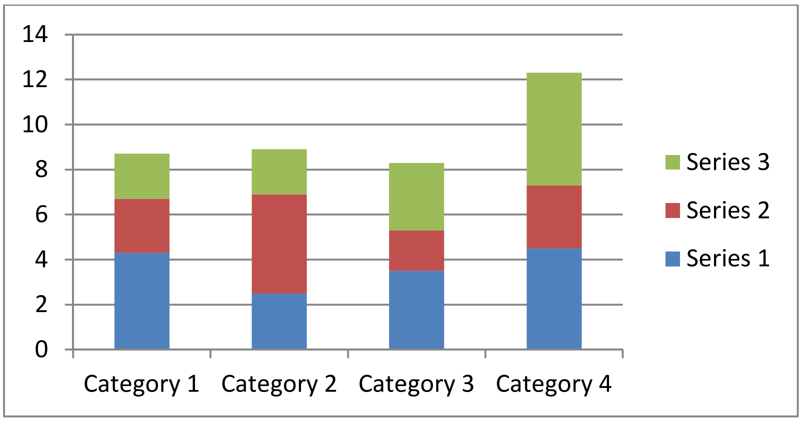
* Mode - Observation that occurs most often

* Median - Middle Observation (Data must be in ascending / descending Order)

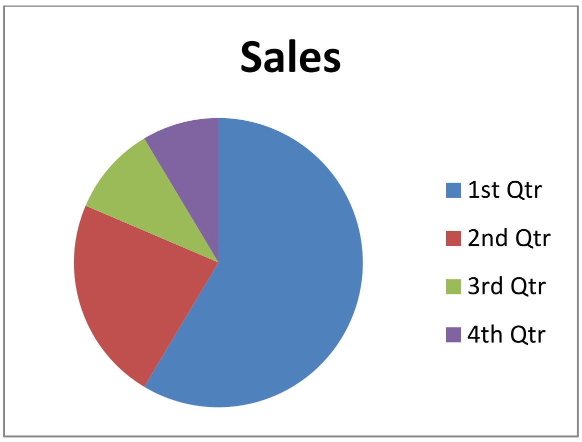
<u>Representation of Data</u> :-

Representation of data can be done by using:

Bar Graph :- Visual representation in the form of bars.



Pie Chart :- Visual representation in the form of circle.



Pictograph :- Visual representation in the form of picture.

Exercise

Q1. Record the age in years of all your classmates. Tabulate the data and find the mode.

Q2. The records of runs in a cricket match by 11 players is as follows :-

9	18	125	52	100	99	10	18	8	10	18

Find mean , mode & median of this data.Are the three same ?

Q3. Two hundred Students of 6^{th}& 7^{th}class are asked to name their favourite colour so as to decide upon what should be the colour of their school building . The results are shown in the following table.

Favourite colours	Red	Green	Blue	Yellow	Orange
No. of the Students	43	19	55	49	34

Represent the given data on

1. a bar graph 2. pictograph 3. Piechart

Answer the following questions with the help of the bar graph :-

(i) Which is the most preferred colour & which is the least preferred colour ?

(ii) How many colours are there in all ? What are they ?

Q4. Weigh (in Kg) atleast 25 students of your Class. Organise the data & answer the following questions using the data.

(i) Who is the heaviest of all ?

(ii) What is the mode of the data obtained?

(iii) Find the Range.

*****

BOOKLET OF CLASSES VIII & IX

MATHEMATICS

Fractions

Following questions are consisting four sub questions in each. Illustration for one from each is given here. Students are supposed to solve other three from each by own. They are further supposed to get them checked/corrected by the concerned teachers.

Q (1) Find HCF and LCM of the following pairs of numbers

(i) 12,18 (ii) 24,36 (iii) 36,54 (iv) 48,72 (by prime factorization method)

Illustration (i) 12,18

For LCM
$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

For HCF
$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$12 = 2 \times 2 \times 3$ $\quad 18 = 2 \times 3 \times 3$

LCM $= 2 \times 2 \times 3 \times 3 = 36$

HCF $= 2 \times 3 = 6$

Q (2) Find the answers in simplest form /mixed fractions :-

i) $\frac{5}{12} + \frac{7}{18}$ (ii) $\frac{7}{24} + \frac{5}{36}$ (iii) $\frac{5}{36} - \frac{7}{54}$ (4) $\frac{7}{48} - \frac{5}{72}$

Illustration (1) $\frac{5}{12} + \frac{7}{18}$

LCM of 12 & 18 = 36

$$= \frac{5 \times 3}{12 \times 3} + \frac{7 \times 2}{18 \times 2}$$

$$= \frac{15 + 14}{36}$$

$$= \frac{29}{36}$$

Q (3) Find the answers in simplest form /mixed fractions :-

(i) $\frac{6}{8}$ X $\frac{12}{15}$ (ii) $\frac{16}{12}$ X $\frac{25}{20}$ (iii) $\frac{10}{8}$ X $\frac{18}{15}$ (iv) $\frac{18}{15}$ X $\frac{28}{24}$

Illustration (i) $\frac{6}{8} \times \frac{12}{15}$

$$= \frac{6 \times 12}{8 \times 15}$$

$= \frac{3}{5}$

Q (4) Find the answers in simplest form /mixed fractions :-

(i) $\frac{12}{16} \div \frac{30}{24}$ (ii) $\frac{16}{12} \div \frac{40}{50}$ (iii) $\frac{10}{8} \div \frac{45}{54}$ (iv) $\frac{18}{15} \div \frac{96}{112}$

Illustration (i) $\frac{12}{16} \div \frac{30}{24}$

$= \frac{12}{16} \times \frac{24}{30}$

$= \frac{12 \times 24}{16 \times 30}$

$= \frac{6}{10}$

$= \frac{3}{5}$

Linear Equations

Solve the following equations and find the value of x :-

Q (1) i) 2x + 3 = 5 ii) 3x + 4 = 10 iii) 4x + 5 = 17 iv) 5x + 6 = 26

i) 2x + 3 = 5

Subtracting 3 from both sides,

Or, 2x + 3 – 3 = 5 – 3

Or, 2x = 2

Dividing both sides by 2 we get,

Or, x = 1

Q (2) i) 8x - 9 = 63 ii) 9x - 10 = 80 iii) 10x -11 = 99 iv) 11x-12= 120

i) 8x – 9 = 63

Adding 9 to both sides,

Or, 8x – 9 + 9 = 63 + 9

Or, 8x = 72

Dividing both sides by 8

Or, x = 9

Q (3) i) x/5 + 2 = 6 ii) x/3 + 5 = 12 iii) x/4 + 7 = 10 iv) x/2 – 3 = 5

i) $\frac{x}{5} + 2 = 6$

Subtracting 2 from both sides we get,

Or, $\frac{x}{5} + 2 – 2 = 6 – 2$

Or, $\frac{x}{5} = 4$

Multiplying 5 on both sides,

Or, $(\frac{x}{5}) \times 5 = 4 \times 5$

Or, $x = 20$

Integers

Q (1) Perform the following operations on Integers :-

1. (+5) + (+2) =	2. (-5) + (-2) =	3. (+5) + (-2) =
4. (-5) + (+2) =	5. (+7) + (+3) =	6. (-7) + (-3) =
7. (+7) + (-3) =	8. (-7) + (+3) =	9. (+9) + (+4) =
10. (-9) + (-4) =	11. (+9) + (-4) =	12. (-9) + (+4) =
13. (-7) +(-2) =	14. (-7) +(+2) =	15. (+7) +(-2) =

Q (2) Perform the following operations on Integers :-

1. (+5) - (+2) =	2. (-5) - (-2) =	3. (+5) - (-2) =
4. (-5) - (+2) =	5. (+7) - (+3) =	6. (-7) - (-3) =
7. (+7) - (-3) =	8. (-7) - (+3) =	9. (+9) - (+4) =
10. (-9) - (-4) =	11. (+9) - (-4) =	12. (-9) - (+4) =
13. (-7) - (-2) =	14. (-7) - (+2) =	15. (+7) -(-2) =

Q (3) Perform the following operations on Integers :-

1. (+5) × (+2) =	2. (-5) × (-2) =	3. (+5) × (-2) =
4. (-5) × (+2) =	5. (+7) × (+3) =	6. (-7) × (-3) =
7. (+7) × (-3) =	8. (-7) × (+3) =	9. (+9) × (+4) =
10. (-9) × (-4) =	11. (+9) × (-4) =	12. (-9) × (+4) =
13. (-7) × (-2) =	14. (-7) × (+2) =	15. (+7) × (-2) =

Quadrilaterals

Q (1) Fill up the blanks :-

i) Opposite sides of parallelogram are ________ and __________.

ii) Opposite angles of parallelogram are _________.

iii) Adjacent angles of parallelogram are __________.

iv) Diagonals of parallelogram _________ each other.

v) Diagonals of rectangle are _________.

vi) Each angle of rectangle measures _________.

vii) All sides of rhombus are __________.

viii) Diagonals of rhombus _________ each other at _________.

ix) Diagonals of rhombus __________ opposite angles.

x) Diagonals of square are _________.

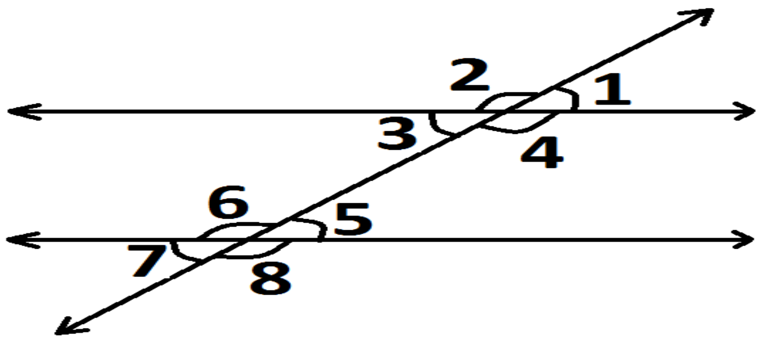
xi) Each angle of square measures _________.

xii) All sides of square are __________.

xiii) Diagonals of square _________ each other at _________.

xiv) Diagonals of square __________ opposite angles.

Parallel lines



Q (1) When two parallel lines are cut by a transversal then :-

i) Vertically opposite angles so formed are ___________.

$\angle$ 1= ____, $\angle$2= ____, $\angle$ 5= ____, $\angle$ 6= _____

ii) Corresponding angles formed are ___________.

$\angle$ 1= ____, $\angle$2= ____, $\angle$ 3= ____, $\angle$4= _____

iii) Alternate interior angles are ___________.

$\angle$ 3= ____, $\angle$4= ____

iv) Alternate exterior angles are ___________.

$\angle$ 1= ____, $\angle$2= ____

v) Co-interior angles are ___________.

$\angle 3+\angle$ __= 180°, $\angle 4+\angle$ __= 180°

vi) Co-exterior angles are ___________.

$\angle 2+\angle$ __= 180°, $\angle 1+\angle$ __= 180°

vii) Linear pairs are ___________.

$\angle 1+\angle 2$= _____, $\angle 2+\angle 3$= _____, $\angle 3+\angle 4$= _____, $\angle 4+\angle 1$= _____,

$\angle 5+\angle 6$= _____, $\angle 6+\angle 7$= _____, $\angle 7+\angle 8$= _____, $\angle 8+\angle 5$= _____,

Constructions of Triangles

Q (1) Construct triangle ABC if AB=4 cm, BC=5 cm, CA=6 cm

Q (2) Construct triangle ABC if AB=5 cm, BC=6 cm, CA=7 cm

Q (3) Construct triangle ABC if AB=4 cm, BC=5 cm, <B=60˚

Q (4) Construct triangle ABC if AB=5 cm, BC=6 cm, <B=30˚

Q (5) Construct triangle ABC if AB=4.5 cm, BC=5.5 cm, <B=45˚

Q (6) Construct triangle ABC if AB=3 cm, BC=4 cm, <B=90˚

Q (7) Construct triangle ABC if AB=4 cm, BC= 6 cm, <B=120˚

Q (8) Construct triangle ABC if <B=30˚, BC= 6 cm, <C=60˚

Q (9) Construct triangle ABC if <B=90˚, BC= 6 cm, <C=45˚

Q (10) Construct triangle ABC if <B=90˚, BC= 4 cm, AC= 5 cm

Q (11) Construct triangle ABC if <B=90˚, BC= 6 cm, AC= 7.5 cm

Profit-Loss

(1) Profit = SP-CP

(2) Loss = CP-SP

(3) Profit % = Profit/CP X 100

(4) Loss % = Loss/CP X 100

(5) SP = CP (1+ P%)

(6) SP = CP (1-L%)

Q (1) Find Unknown in the following :-

S.N.	CP (Rs)	SP(Rs)	Profit (Rs)	Loss (Rs)	Profit %	Loss %
1	400				10%	
2	800				20%	
3	400					10%
4	800					20%
5		880			10%	
6		1920			20%	
7		720				10%
8		1280				20%

1. C.P. = Rs. 400 , P% = 10 %

$$P\ \% = \left(\frac{Profit}{C.P.}\right)\times 100$$

$$\text{Or, } 10 = \left(\frac{Profit}{400}\right)\times 100$$

$$\text{Or, } \left(\frac{10\times 400}{100}\right) = Profit$$

Or, Profit = 40

S.P. = C.P. + Profit

= 400 + 40

S.P. = Rs. 440

Algebra

Learn the identities -

(i) $(a+b)^2 = a^2+2ab+b^2$

(ii) $(a-b)^2 = a^2-2ab+b^2$

(iii) $(a+b)(a-b) = a^2- b^2$

1. By multiplication show that - i) $(a+b)(a+b)= a^2+2ab+b^2$
   ii) $(a-b)(a-b)= a^2-2ab+b^2$
   iii) $(a+b)(a-b)= a^2- b^2$

2. Using suitable identities expand the following :-

   i) $(x+2y)^2$ ii) $(2x+y)^2$ iii) $(2x-3y)^2$

   iv) $(3x-4y)^2$ v) $(2x+3y)(2x-3y)$ vi) $(3x+4y)(3x-4y)$

   Solution 1: - $(x+2y)^2 = x^2+2x(2y)+(2y)^2 = x^2+4xy+4y^2$

3. Using suitable identities evaluate the following :-

   i) 101^2 ii) 102^2 iii) 99^2 iv) 98^2 v) 101×99

   vi) 102 X 98 vii) 103^2 viii) 96^2 ix) 103×97

   Solution 1:

$$101^2 = (100+1)^2 = (100)^2 + 2\times 100\times 1 + (1)^2 = 10000+200+1 = 10201$$

4. Using suitable identities evaluate the following :-

   i) 10.1^2 ii) 10.2^2 iii) 9.9^2

   iv) 9.8^2 v) 1.01 X 9.9 vi) 1.02 X 9.8

   vii) 10.3^2 8) 9.6^2 ix) 1.0302×9.7

   Solution 1:

$$(10+0.1)^2 = (10)^2 + 2\times 10\times 0.1 + (0.1)^2 = 100+2+0.01 = 102.01$$

5. Show that: -

i) $(a+b)^2 - 4ab = (a-b)^2$

ii) $(x+2y)^2 - 8xy = (x-2y)^2$

iii) $(2x+3y)^2 - 24xy = (2x-3y)^2$

Solution 1: - $(a+b)^2 - 4ab = a^2 + 2ab + b^2 - 4ab = a^2 + 2ab + b^2 = (a+b)^2$

6 Add: $7\,xy + 5\,yz - 3\,zx,\ 4\,yz + 9\,zx - 4\,y,\ -3\,xz + 5\,x - 2\,xy.$

Solution :- $(7\,xy + 5\,yz - 3\,zx) + (4\,yz + 9\,zx - 4\,y) + (-3\,xz + 5\,x - 2\,xy)$

$= 7\,xy + 5\,yz - 3\,zx + 4\,yz + 9\,zx - 4\,y - 3\,xz + 5\,x - 2\,xy$

$= 7xy - 2xy + 5yz + 4yz - 3zx + 9zx - 3zx + 5x - 4y = 5xy + 9yz + 3zx + 5x - 4y$

7 Q Subtract $5\,x^2 - 4\,y^2 + 6\,y - 3$ from $7\,x^2 - 4\,xy + 8\,y^2 + 5\,x - 3\,y$.

Solution: - $7\,x^2 - 4\,xy + 8\,y^2 + 5\,x - 3\,y - (5\,x^2 - 4\,y^2 + 6\,y - 3)$

$= 7\,x^2 - 4\,xy + 8\,y^2 + 5\,x - 3\,y - 5\,x^2 + 4\,y^2 - 6\,y + 3$

$= 7\,x^2 - 5\,x^2 - 4xy + 8\,y^2 + 4\,y^2 + 5x - 3y - 6y + 3$

$= 2x^2 - 4xy + 12y^2 + 5x - 9y + 3$

8 Si Simplify the expression and evaluate them as directed

(a) $x(x - 3) + 2$ for $x = 1$

(b) $3y(2y - 7) - 3(y - 4) - 63$ for $y = -2$

Solution (b):

$3y(2y - 7) - 3(y - 4) - 63 = 6y^2 - 21y - 3y + 12 = 6y^2 - 24y + 12$

Put $y = -2,\ 6y^2 - 24y + 12 = 6(-2)^2 - 24 \times (-2) + 12 = 6 \times 4 + 48 + 12 = 84$

9 Find- (i) $5\,m(3 - m)$ and $6\,m^2 - 13\,m$

(ii) $4\,y\,(3y^2 + 5\,y - 7)$ and $2\,(y^3 - 4\,y^2 + 5)$

10 Subtract 3 pq (p-q) from 2 pq (p+q).

11 Multiply

(i) (x-4) and (2x +3) (ii) (x-y) and (3x + 5y)

12 Multiply

(i) (a+7) and (b-5) (b) $(a^2 + 2b^2)$ and (5a-3b)

13 Simplify (a+b)(2a-3b+c) –(2a -3c)c

14 find-

(i) $\left[\frac{3}{2}m + \frac{2}{3}n\right]\left[\frac{3}{2}m - \frac{2}{3}n\right]$ (ii) $983^2 - 17^2$ (iii) 194 X 206

15 Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

16 Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

17 4P Subtract $P^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

18 Simplify $3x(4x-5) + 3$ and find its values for (i) $x=3$ (ii) $x=\frac{1}{2}$

19 Simplify $a(a^2 + a + 1) + 5$ and find its values for (i) $a=0$ (ii) $a=1$

20 Add $p(p-q)$, $q(q-r)$ and $r(r-p)$

21 Add $2x(z-x-y)$ and $2y(z-y-x)$

22 Subtract $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

23 Subtract $3a(a+b+c) - 2b(a-b+c)$ from $4c(-a + b + c)$

24 Using $a^2 - b^2 = (a+b)(a-b)$, find:

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$ (iv) $12.1^2 - 7.9^2$

25 Using $(x+a)(x-b) = x^2 + (a+b)x + ab$ find:

(i) 103 X 104 (ii) 5.1 X 5.2

(iii) 103 X 98 (iv) 9.7 X 9.8

26 Using identities, evaluate

(i) 71^2 (ii) 99^2 (iii) 102^2

(iv) 998^2 (v) 5.2^2 (vi) 297 X 303

(vii) 78 X 82 (viii) 8.9^2 (ix) 1.05 x 9.5

Show that :-

(1) $(a-1)(b-c) + (b-1)(c-a) + (c-1)(a-b) = 0$

(2) $(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q) = 0$

(3) $(x-1)(y-z) + (y-1)(z-x) + (z-1)(x-y) = 0$

Euler's formula - F+V-E = 2

Q (1) Find unknown in the following :-

S.N.	F	V	E
1	6	8	
2	9	12	
3		16	24
4		20	30
5	18		36
6	21		42
7	24		48

Solution 1: $F + V = E + 2$

Here, $F = 6$ and $V = 8$

$6 + 8 = E + 2 \Rightarrow 14 = E + 2 \Rightarrow E = 12$

Perimeter and area of plane figures

Q (1) Complete the following formula table :-

S.N.	Figure	Perimeter	Area
1	Square		
2	Rectangle		
3	Parallelogram		
4	Triangle		
5	Rhombus		
6	Trapezium		
7	Circle		

Q (2) Find unknown for a square :-

S.N.	Side(in cm)	Perimeter(in cm)	Area(in cm^2)
1	2		
2	5		
3		40	
4		80	
5			36
6			144
7			49

Solution 1: - Here, side= 2 cm

$Perimeter\ of\ Square = 4 \times side = 4 \times 2 = 8\ cm$

$Area\ of\ Square = Side \times Side = 2 \times 2 = 4\ cm^2$

Q (3) Find unknown for a rectangle :-

S.N.	Length (in cm)	Breadth (in cm)	Perimeter (in cm)	Area (in cm^2)
1	10	5		

2	20	10		
3	30		90	
4		20	120	
5	50			1250
6		30		1800
7	70		210	

Solution 1: - Here, length = 10 cm and breadth = 5 cm

$$Perimeter\ of\ Rectangle = 2(length + breadth) = 2(10+5) = 2 \times 15 = 30\ cm$$

$$Area\ of\ Rectangle = length \times breadth = 10 \times 5 = 50\ cm^2$$

Q (4) Find unknown for a parallelogram:-

S.N.	Base (in cm)	Height (in cm)	Area (in cm^2)
1	10	5	
2	20	10	
3	40		800
4	80		3200
5		15	300
6		25	1000
7	40		1200

Solution 1:- Here, base = 10 cm and height = 5 cm

$$Area\ of\ Parallelogram = base \times height = 10 \times 5 = 50\ cm^2$$

Q (5) Find unknown for a triangle :-

S.N.	Base (in cm)	Height (in cm)	Area (in cm^2)
1	10	5	
2	20	10	
3	40		800
4	80		3200
5		15	300
6		25	1000
7	40		1200

Solution 1:- Here, base = 10 cm and height = 5 cm

$Area\ of\ Triangle = \frac{1}{2} \times base \times height = \frac{1}{2} \times 10 \times 5 = \frac{1}{2} \times 50 = 25\ cm^2$

Q (6) Find unknown for a rhombus :-

S.N.	Length of each side or base (in cm)	Perpendicular distance between two opposite parallel sides or height/altitude (in cm)	Perimeter of Rhombus (in cm)	Area of Rhombus (in cm^2)	Length of first diagonal (in cm)	Length of second diagonal (in cm)
1					6	8
2					10	24
3					16	30
4					14	48
5					18	80
6					12	16
7					20	48

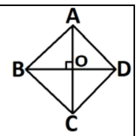
Solution 1:- Here, d_1 = 6 cm and d_2 = 8 cm

$Area\ of\ Rhombus = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 48 = 24\ cm^2$

Length of each side or base = $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5\ cm$

{By Pythagoras Theorem, as we know that the diagonals of Rhombus are perpendicular bisector of each other}

Perpendicular distance between two opposite parallel sides or height = $\frac{Area}{base} = \frac{24}{5} = 4.8\ cm$



Q (7) Find unknown for a trapezium :-

S.N.	Length of $base_1$ (in cm)	Length of $base_2$ (in cm)	Height or the distance between two parallel lines (in cm)	Area (in cm^2)
1	5	15	4	
2	10	30	8	
3	20	60		800

4	40	120		3200
5	10		5	75
6	20		10	300
7		80	20	1200

Q (8) Find unknown for a circle :-

S.N.	Radius (in cm)	Diameter (in cm)	Perimeter (in cm)	Area (in cm^2)
1			132	
2			264	
3			220	
4				154
5				616
6				2464
7				5544

Q (9) Find unknown for a cylinder :-

S.N.	Radius of base (in cm)	Height (in cm)	Area of base (in cm^2)	CSA (in cm^2)	TSA (in cm^2)	Volume (in cm^3)
1				20π	28π	
2				80π	112π	
3				320π	448π	
4				1280π	1792π	
5				440	748	
6				1760	2992	
7				220	528	

Solution 1 : - Total Surface Area − Curved Surface Area (CSA) = $28\pi - 20\pi$

$$\Rightarrow \quad 2\pi r(r+h) - 2\pi rh = 8\pi$$

$$\Rightarrow \quad 2\pi r^2 + 2\pi rh - 2\pi rh = 8\pi$$

$$\Rightarrow \quad 2\pi r^2 = 8\pi$$

$$\Rightarrow \quad r^2 = 4$$

$$\Rightarrow \quad r = 2 \text{ cm}$$

$$CSA = 20\pi$$

$$\Rightarrow \quad 2\pi rh = 20\pi$$
$$\Rightarrow \quad 2h = 10$$
$$\Rightarrow \quad h = 5\ cm$$

Area of base $= \pi r^2 = \pi(2)^2 = 4\pi$ cm^2

Laws of exponents

Q (1) Fill up the blanks :-

i) $a^m \times b^n =$ _______ ii) $a^m \div b^n =$ _______

iii) $(a^m)^n =$ _______ iv) $(a \times b)^m =$ _______

v) $(\frac{a}{b})^m =$ ________ vi) $a^{-m} = \frac{1}{\ }$

vii) $a^m = \frac{1}{\ }$ vii) $a^0 =$ _____

ix) $\sqrt[m]{a} = a^{1/}$

Q (2) $\boldsymbol{SIMPLIFY}$:

a)	$8^{\frac{2}{3}}$	b)	$16^{-\frac{3}{4}}$	c)	$32^{\frac{4}{5}}$	d	$64^{-\frac{5}{6}}$
e)	$128^{\frac{2}{7}}$	f)	$256^{-\frac{3}{8}}$	g)	$512^{\frac{2}{9}}$	h)	$1024^{-\frac{3}{10}}$
i)	$2048^{\frac{4}{11}}$	j)	$4096^{-\frac{5}{12}}$	k)	$8192^{\frac{2}{13}}$	l)	$27^{-\frac{2}{3}}$
m)	$81^{-\frac{3}{4}}$	n)	$243^{-\frac{4}{5}}$	o)	$729^{-\frac{5}{6}}$	p)	$2189^{-\frac{6}{7}}$
q)	$125^{-\frac{2}{3}}$	r)	$625^{-\frac{3}{4}}$	s)	$3125^{-\frac{4}{5}}$	t)	$343^{-\frac{2}{3}}$
u)	$2401^{-\frac{3}{4}}$	v)	$216^{-\frac{2}{3}}$	w)	$1296^{-\frac{3}{4}}$	x)	$1331^{-\frac{2}{3}}$
y)	$1728^{-\frac{2}{3}}$						

Solution 1: - $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3 \times \frac{2}{3}}$ {By using the Power Law $(a^m)^n = a^{mn}$}

$= 2^2 = 4$

Q (3) $\boldsymbol{FIND}$:

i) $64^{\frac{1}{2}}$ ii) $32^{\frac{1}{2}}$ iii) $125^{\frac{1}{3}}$

Q (4) $\boldsymbol{FIND}$:

i) $9^{\frac{3}{2}}$ ii) $32^{\frac{2}{5}}$ iii) $16^{\frac{3}{4}}$ iv) $125^{-\frac{1}{3}}$

Q (5) $\boldsymbol{SIMPLIFY}$:

i) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ ii) $7^{\frac{1}{2}}.\ 8^{\frac{1}{2}}$

Factorisation and Algebraic Identities

Q (1) Factorize using suitable identities:

i) $(x^2 + 4x + 4) - (y^2 - 6y + 9)$
ii) $(x^2 + 6x + 9) - (y^2 - 8y + 16)$
iii) $(x^2 - 8x + 16) - (y^2 + 10y + 25)$
iv) $(x^2 + 10x + 25) - (y^2 - 12y + 36)$
v) $9x^2 + 6xy + y^2$
vi) $4y^2 - 4y + 1$
vii) $x^2 - \frac{y^2}{100}$

Solution 1: - $(x^2 + 4x + 4) - (y^2 - 6y + 9)$

$$= (x^2 + 2 \times x \times 2 + 2^2) - (y^2 - 2 \times y \times 3 + 3^2)$$
$$= (x + 2)^2 - (y - 3)^2$$
$$= \{(x + 2) + (y - 3)\}\{(x + 2) - (y - 3)\}$$
$$= (x + y + 2 - 3)(x - y + 2 + 3) = (x + y - 1)(x - y + 5)$$

Q (2) $\boldsymbol{Simplify}$:

i) $(x^2 - 5x + 6) \div (x - 2)$
ii) $(x^2 - 9x + 20) \div (x - 4)$
iii) $(x^2 + 8x + 15) \div (x + 5)$
iv) $(x^2 + 6x + 5) \div (x + 1)$
v) $(x^2 - 5x + 6) \div (x - 3)$
vi) $(x^2 - 9x + 20) \div (x - 5)$
viii) $(x^2 - 6x + 5) \div (x - 5)$

Solution 1: - $(x^2 - 5x + 6) \div (x - 2) = \{x^2 - (3 + 2)x + 6\} \div (x - 2)$

$$= \{x^2 - 3x - 2x + 6\} \div (x - 2)$$
$$= \{x(x - 3) - 2(x - 3)\} \div (x - 2)$$
$$= (x - 3)(x - 2) \div (x - 2) = x - 3$$

Q (3) Factorize by middle term splitting method :-

i)	$2x^2 + 5x + 3$	ii)	$2x^2 + 7x + 5$	iii)	$6x^2 + 7x + 2$
iv)	$5x^2 – 7x + 2$	v)	$2x^2 – 9x + 7$	vi)	$3x^2 -7x + 4$
vii)	$2x^2 + 5x -7$	viii)	$3x^2 + 2x – 8$	ix)	$5x^2 + 2x – 3$
x)	$3x^2 – 2x-5$	xi)	$5x^2 -8x – 4$	xii)	$3x^2 -7x -10$
xiii)	$5x^2 -7x-6$	xiv)	$2x^2 + 7x-15$	xv)	$3x^2-8x + 4$
xvi)	$3x^2-8x +4$	xvii)	$3x^2 -10x +7= 0$	xviii)	$3x^2 –x-4$
xix)	$6x^2 – 7x-3$	xx)	$4x^2 – 4x + 1$	xxi)	$x^2 – 2x - 8$

Q (4) ***Using suitable identities find the value of the following:***

i)	103×102	ii)	103×104	iii)	96×97	iv)	98×91
v)	103×98	vi)	102×97	vii)	101×98	viii)	102×99
ix)	102×98	x)	103×97	xi)	104×96	xii)	108×98
xiii)	101×99	xiv)	103×107	xv)	95×96	xvi)	104×96

Solution 1:- $103 \times 102 = (100 + 3) \times (100 + 2)$

$$= (100)^2 + (3 + 2) \times 100 + 3 \times 2$$

$$\{By\ using\ the\ identity\ (x + a)(x + b) = x^2 + (a + b)x + ab\}$$

$$= 10000 + 5 \times 100 + 6 = 10000 + 500 + 6 = 10506$$

Bar Graphs

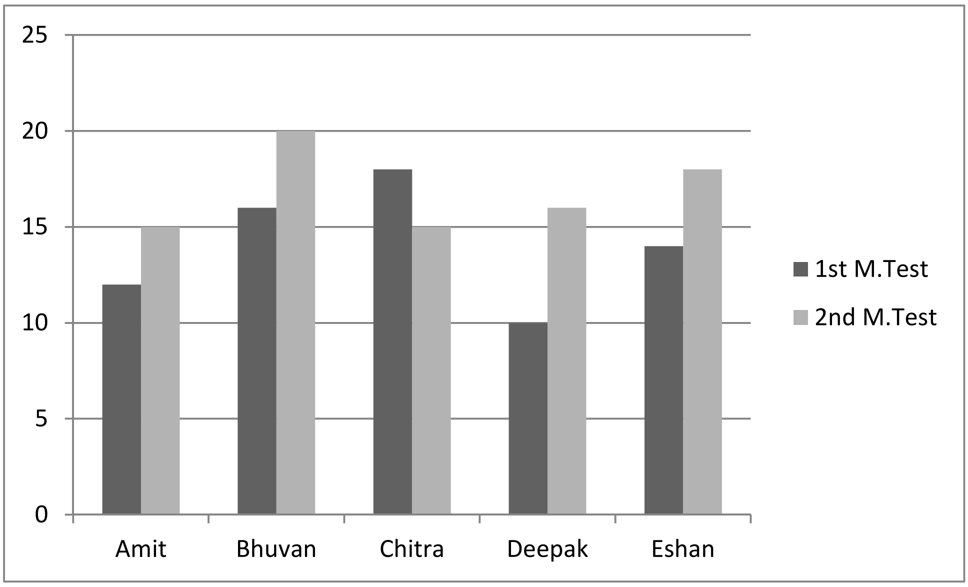
Q (1) Marks of two consecutive Monday Test of five children in a particular subject are given here:-

Students	Amit	Bhuvan	Chitra	Deepak	Eshan
1st M.Test	12	16	18	10	14
2nd M.Test	15	20	15	16	18

Choosing suitable scales draw a double bar graph to represent the data given above and answer the following –

(a) Who has improved most in that subject and by how many %?

(b) Whose performances are below than average (mean mark) in 1st M.test?



(a) Deepak has improved by 60% $\left[= \frac{6}{10} \times \frac{100}{100}\right]$ in that subject.

(b) Amit and Deepak's marks are below than average in 1st M. test.

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BOOKLET OF CLASSES X & XI

MATHEMATICS

1) NUMBER THEORY

Natural numbers : Counting numbers starting from 1 to infinite are known as Natural numbers. like N = {1,2,3,4,5,…………….}

Whole numbers : Numbers starting from 0 to infinite are known as Whole numbers.

W = {0,1,2,3,4,5,……………….}

Integers : Integers are the set of whole numbers & negative natural number.

I or Z = {………-4,-3,-2,-1,0,1,2,3,4,……..}

Rational number : A number which is in the form of a/b where a , b are an integer & b ≠ 0 , is known as Rational number.

Q = { ………,-3,-2,-3/2,-1,-1/2,0,1/2, 1,3/2,2,5/2,3,………}

Irrational number : A number whose decimal presentation is neither terminating nor recurring is known as Irrational number.

Q^- = { $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$,……….}

Real numbers : A set of all numbers is known as Real numbers.

2) BASIC MATHEMATICAL OPERATIONS

There are 4 basic mathematical operations . They are addition , subtraction , multiplication & division .

Point to remember : When we add two or more numbers , we arrange these numbers such that ones digits should be in ones column & tens digits should be in tens column.

Add : 234 , 1256 & add 1234 & 9876

$$\begin{array}{r} 234 \\ \underline{+1256} \\ \mathbf{1490} \end{array} \qquad \begin{array}{r} 1234 \\ \underline{9876} \\ \mathbf{11110} \end{array}$$

Subtract 1234 from 9876, Subtract 789 from 1000

$$\begin{array}{r} 9876 \\ \underline{-\ 1234} \\ 8642 \end{array} \qquad \begin{array}{r} 1000 \\ \underline{-\ \ 789} \\ 211 \end{array}$$

Sample Questions

1. Add 2345 & 98765
2. Add 5678 & 132456
3. Subtract 2341 from 5647
4. Subtract 8749 from 1000000

Multiplication :

```
    567 × 24                 4 5 6  ×  9 8 7
  ------------              -------------------
      2 2 6 8                   3 1 9 2
    1 1 3 4 ×                 3 6 4 8 ×
  ----------------          4 1 0 4 × ×
    1 3 6 0 8               ------------------
                            4 5 0 0 7  2
```

Sample Questions

1. 865×29
2. 5874×204

Division : Divide 510 from 5

```
5) 510  ( 102
   5
  ----
   0 1
     0
   -----
     1 0
     1 0
   -------
      00
```

Sample Questions

1) divide 612 by 6 2) divide 1248 by 12 3) H.C.F & L.C.M

HCF means Highest common factor : The H. C. F. of two or more numbers is the greatest number that divides each one of them exactly.

A) Factors of 6 = 1,2,3,6
Factors of 8 = 1,2 4,8
Common factors are 1,2
Highest common factor is 2

B) Find HCF of 24 , 30 & 36
Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30
F actors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36
Common factors are 1, 2, 3, 6
HCF = 6

Sample Questions

a) Find HCF of 18 , 32 , 40 .   b) Find HCF of 56, 72, 88.

L.C.M.

Means Lowest Common Multiple : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1) Find the L.C.M. of 16 , 20 , 24 by prime factorization method.
   Prime factorization of 16 is $2 \times 2 \times 2 \times 2$
   Prime factorization of 20 is $2 \times 2 \times 5$
   Prime factorization of 24 is $2 \times 2 \times 2 \times 3$
   L. C. M. = $2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$
2) Find L.C.M. of 16 , 24 , 36 , 54. By multiple method.
   Multiples of 16 =16,32,48,64,80,96,112,128,144,160,176,192,208,224,240, 256,272,288,304,320,336,352,368,384,400,416,**432**,448…………………..
   Multiples of 24 = 24,48,72,96,120,144,168,192,216,240,264,288,312,336,360,384,408,**432**,456…
   Multiples of 36 = 36,72,108,144,180,216,252,288,324,360,396,**432**,468
   Multiples of 54= 54,108,162,216,270,324,378,**432,**486……………………
   Therefore L.C.M. of 16, 24 , 36 & 54 = **432**

Sample Question:

i) Find L.C.M. of 18 , 20, 24.   ii) Find L.C.M. of 25 , 30 , 40.

4) BODMAS RULE

Means – Bracket of Division Multiplication Addition & Subtraction.

1) solve	2) (2 – 1/3) + 2/3 x 5/4 ÷ 3/7 of 21/8
10 + [5 – { 6 + (7 – 4)}]	(6 – 1)/3 + 2/3 x 5/4 ÷ 3/7 x 21/8
10 + [5 – { 6 + (3)}]	(5)/3 + 2/3 x 5/4 ÷ 9/8
10 + [5 – { 6 + 3}]	5/3 + 2/3 x 5/4 x 8/9
10 + [5 – 9]	5/3 + 20/27
10 + [-4]	(45 + 20)/27
10 – 4	65/27
6	

Sample Question:

i) 50 ÷ 5 + 15 of 6 – 4 x 7   iii) (2/5 + 1/3) ÷ 11/6 × 20/3

ii) $50 - [4 + 3\{6 - 2(34 - 28)\}]$ iv) $[10 - \{6 + 2(9 - 23)\}] = x$, find x.

5) All formulae according to syllabus of previous classes.

a) $(a + b)^2 = a^2 + 2ab + b^2$

b) $(a - b)^2 = a^2 - 2ab + b^2$

c) $a^2 - b^2 = (a-b)(a+b)$

d) $(x + a)(x + b) = x^2 + (a + b)x + ab$

Sample Question:

(a) $(2x + 3)^2$ (b) $(4x + 5/6)^2$ (c) $(3x - 7)^2$;

(d) $(2x/3 - 8/7)^2$ (e) $4x^2 - 9y^2$ (f) $32x^2 - 50$;

(g) $(x + 3)(x + 5)$ (h) $(y - 5)(y + 9)$.

FORMULA $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Sample Question:

Find (i) $(2x + 3y + 5z)^2$ (ii) $(5x - 7y + 4z)^2$

FORMULA $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Sample Question:

Find (i) $(2x + 3y)^3$ (ii) $(4x/3 + 6/5y)^3$

FORMULA $(x - y)^3 = x^3 - y^3 - 3xy(x-y)$

Sample Question:

Find (i) $(3x - 7)^3$ (ii) $(2x/3 - 6/11)^3$

FORMULA $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Sample Questions

Find (i) $8x^3 + y^3 + 27z^3 - 18xyz$ (ii) $27x^3 + y^3 + z^3 - 9xyz$

Area of triangle = ½ base x height

Hero's formula to find area of triangle = $[s(s-a)(s-b)(s-c)]^{1/2}$

Where $s = (a+b+c)/2$

Sample Question:

1) Find the area of triangle whose sides are 12cm ,14 cm & 16 cm.

2) Find area of an isosceles triangle whose equal sides are 16 cm & base is 20 cm.

Surface area of a cuboid = 2(lb + bh + hl)

1) Find the area of a cuboid whose l = 13cm , b = 9 cm & h = 7 cm.

Surface area of a cube = $6a^2$

1) Find the area of a cube whose edge = 18 cm.

Curved surface area of a cylinder = $2\pi rh$

Total surface area of a cylinder = $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$

Find curved surface area & total surface area of a cylinder whose diameter is 14 cm & height is 24 cm.

Curved surface area of a cone = πrl

Total surface area of a cone = $\pi r(r+l)$

Find curved surface area & total surface area of a cone whose diameter is 21 cm & slant height is 24 cm.

Surface area of a sphere = $4\pi r^2$

1) Find surface area of a sphere whose diameter is 21 cm.

Curved surface area of a hemisphere = $2\pi r^2$

1) Find curved surface area of a hemisphere whose diameter is 28 cm.

Total surface area of a hemisphere = $3\pi r^2$

1) Find total surface area of a hemisphere whose diameter is 35 cm.

Volume of a cuboid = lxbxh

1) Find the volume of a cuboid whose l = 18cm ,b = 15 cm & h = 9cm.

Volume of a cube = a^3

1) Find the volume of a cube whose edge = 13 m.

Volume of a cylinder = $\pi r^2 h$

1) Find the volume of a cylinder whose diameter is 14m & height is 18 m.

Volume of a cone = $1/3\ \pi r^2 h$

1) Find the volume of a cone whose diameter is 14m & height is 18 m.

Volume of a sphere = $4/3\ \pi r^3$

1) Find the volume of a sphere whose radius = 14 m.

Volume of a hemisphere = $2/3\ \pi r^3$

1) Find the volume of a hemisphere whose radius = 21 m.

6) LAWS OF INDICES

We know that $5 x 5 x 5 = 5^3$, read as 5 raised to the power 3.

Here 5 is knows **base** & 3 is knows as **exponent or index**.

Laws of Exponents or Laws of Indices.

For positive integer values of m & n , we have the following laws.

Law I. $a^m \times a^n = a^{m+n}$

$5^2 \times 5^4 = 5^{2+4} = 5^6$

Sample Question:

(i) $7^9 \times 7^8 = \ldots\ldots$ (ii) $3^{12} \times 3^{14} = \ldots\ldots.$

Law II . $a^m \div a^n = a^{m-n}$

(i) $4^8 \div 4^5 = 4^{8-5} = 4^3$

Sample Question:

(i) $9^{12} \div 9^7 = \ldots\ldots\ldots$ (ii) $12^{34} \div 12^{25} = \ldots\ldots\ldots$

Law III . $(a^m)^n = a^{mxn}$

$(2^4)^3 = 2^{4x3} = 2^{12}$

Sample Question:

(i) $(3^4)^5 = \ldots\ldots$ (ii) $(7^5)^7 = \ldots\ldots$

Law IV. $A^0 = 1$

Sample Question:

(i) $3^0 = \ldots$ (ii0 $(999999999999999)^0 = \ldots\ldots$

Law V. $a^{-1} = \frac{1}{a}$

Sample Question:

(i) 2^{-3} = …… (ii) b^{-8} = ……

7) LINEAR EQUATION

$Ax + B = 0$ is known as linear equation in one variable .

Its solution is $x = -B/A$

(i) $2x + 9 = 0$

$x = -9/2$ is the solution of this equation.

(ii) $4x – 12 = 0$

$x = 12/4$

$x = 3$

Sample Question:

Find the value of x in the following equations.

Type 1) $3x/4 – 5/6 = 0$

Type 2) $x + 5 = 9$

Type 3) $2x – 5 = x – 9$

Type 4) $2(x – 5) = 3x – 8$

Type 5) $3(x + 4) / 5 = 3/6$

Type 6) $(5x – 7)/(4x + 3) = 6/5$

Type 7) $9 – 3(x+ 6) = 30$

Type 8) $[2 – 4(x + 7)]/(9-x)=12$

Type 9) $(9x – 6)/7 = (5x +12)/8$

Type 10) $x[5 – 7\{ 4 – 3(9 – 13)\}] = 50$

8) FACTORISATION

Case1) When a monomial is the common factor of all the terms.

$6xy – 9x^2$

$3y(2x – 3y)$

Sample Question:

(i) $25x^2y^3 - 15xy$ (ii) $24xyz + 18x^2yz^2$

Case2) When the given algebraic expression has a common binomial or trinomial.

$X(X + 4) + 3(X+4)$

$(X+ 4)(X+3)$

Sample Question:

(i) $x(x-7) - 9(x-7)$ (ii) $xy(z-2) - 5(z-2)$

Case3) When grouping gives rise to common factors.

$AX + BX + AY + BY$

$X(A + B) + Y(A+B)$

$(A+B)(X+Y)$

Sample Question:

(i) $x^2 - ax - bx + ab$ (ii) $x^2 + y - xy - x$

Case 4) When the given expression is expressible as the difference of two square.

$x^2 - 4$

$x^2 - 2^2$

$(x-2)(x+2)$

Sample Question:

(i) $x^2 - 9$ (ii) $x^2 - 7$

Case5) When the given expression is a perfect square trinomial:

(i) $a^2 + 2ab + b^2 = (a + b)^2$

$X^2 + 10X + 25$

$X^2 + 2 X 5 + 5^2$

$(X + 5)^2$

Sample Question:

(i) $X^2 + 8X + 16$ (ii) $X^2 + 6AX + 9A^2$

(iii) $a^2 - 2ab + b^2 = (a - b)^2$ (iv) $X^2 - 20X + 100$ 2) $4X^2 - 20X + 25$

9) LOGARITHM & ITS LAWS

If a is a positive real number and $a \neq 1$ such that $a^x = m$, then x is called the logarithm of m to the base a, written as $\log_a m$.

Therefore $a^x = m \Leftrightarrow \log_a m = x$.

LAWS OF LOGARITHM

(i) $\log_a(mn) = \log_a m + \log_a n$.

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^k = k.\log_a m$

(iv) $\log_a 1 = 0$

(v) $\log_a a = 1$

(vi) $\log_a m = \frac{\log m}{\log a}$

(vii) $\log_a b = \frac{1}{\log_b a}$

Example:

Q 1. Find x , if $\log_x \frac{1}{4} = 2$

Solution: $x^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2 \Longrightarrow x = \frac{1}{2}$.

Q.2 Find x: $3\log 2 + \log 3 = \log x$.

Solution: $\Rightarrow \log 2^3 + \log 3 = \log x$

$\Rightarrow \log 8 \times 3 = \log x$

$\Longrightarrow \log 24 = \log x \Longrightarrow x = 24.$

Sample questions

i) Find x : $\log x = 4\log 2 - 2\log x$

ii) Simplify: $\log_2 16 + \log_2 8.$

ii) Find x : $\log x - \log(x-1) = 1.$

10) BASIC CONCEPT OF DRAWING GRAPH

The process of drawing rough sketch of a given function is called curve tracing/sketching.

Procedure of curve tracing

(1) **Symmetry**.

(i) **Symmetry about x-axis.** If the equation of the curve remains unaltered by replacing y by –y, then the curve is symmetric about x-axis.

(ii) **Symmetry about y-axis.** If the equation of the curve remains unaltered by replacing x by –x, then the curve is symmetric about y-axis.

(iii) **Symmetry in opposite quadrants.** If the equation of the curve remains unaltered when both x and y are replaced by–x and –y respectively, then the curve is symmetric opposite quadrants.

(iv) **Symmetry about line y=x.** If the equation of the curve remains unaltered by interchanging the places of x and y, then the curve is symmetric about line y=x..

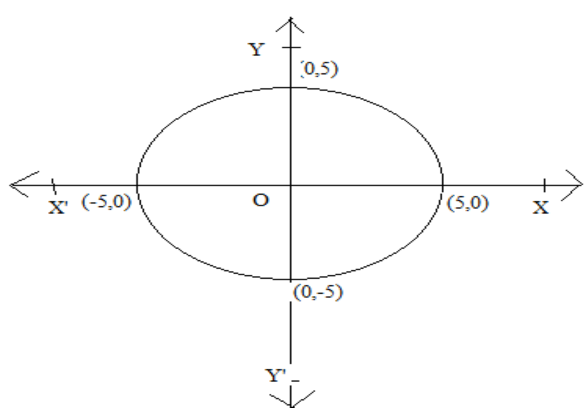
(2) **Origin.** Find whether the curve is passing through origin or not ,i.e. by satisfying the equation by the point (0,0).

(3) **Axes of intersection .** The points where the curve intersects the axes by putting x=0 and y=0 in the equation of the curve.

Example:

(i) Trace the curve: $x^2 + y^2 = 25$

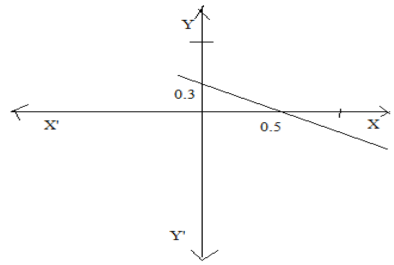
(i) Since by replacing x & y by –x & -y respectively the equation of the curve remains unaltered. So, the curve is symmetric about both the axes..
(ii) Also, interchanging x and y the equation remains unaltered. So, the curve is symmetric about opposite quadrant.
(iii) Also by putting x=0 and y=0 we get the point of intersection as (0,5), (0,-5), (5,0) & (-5,0).
Therefore the curve is as:



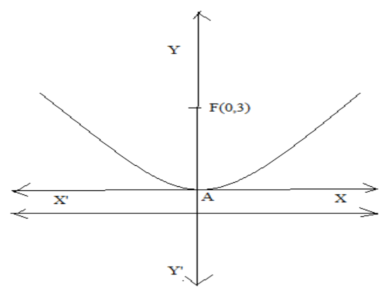
(ii) Draw the graph of

X	0	0.3
Y	0.5	0

3x+2y=1



(iii) Draw the graph of $x^2 = 12y$



Sample questions

(i) Trace the curve: $x^2 = 4y$

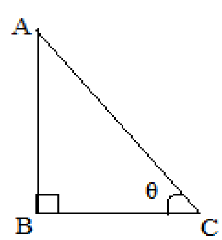
(ii) Trace the curve: $y = |x|$

(iii) Trace the curve: $\frac{x}{2} + \frac{y}{5} = 5$

11) BASICS OF TRIGONOMETRY

The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Trigonometric ratios:



(i) $\sin\theta = \frac{AB}{AC}$ (ii) $\cos\theta = \frac{BC}{AC}$ (iii) $\tan\theta = \frac{AB}{BC}$

(iv) $\operatorname{cosec}\theta = \frac{AC}{AB}$ (v) $\sec\theta = \frac{AC}{BC}$ (vi) $\cot\theta = \frac{BC}{AB}$

Example:

1) Find the other trigonometric ratios if $\tan\theta = \frac{4}{3}$.

Solution: $\tan\theta = \frac{AB}{BC} = \frac{4}{3}$

$\Longrightarrow AB = 4k \; and \; BC = 3k$

$\therefore AC = 5k$ (by Pythagoras theorem)

$\therefore \sin\theta = \frac{AB}{AC} = \frac{4}{5}$, $\cos\theta = \frac{BC}{AC} = \frac{3}{5}$,cosec $\theta = \frac{AC}{AB} = \frac{5}{4}$, $\sec\theta = \frac{AC}{BC} = \frac{5}{3}$, $\cot\theta = \frac{BC}{AB} = \frac{3}{4}$.

Sample Questions:

(i) If $\sin A = \frac{3}{4}$, calculate $\cos A \; and \; \tan A$.

(ii) If $\cot A = \frac{7}{8}$, evaluate : $\frac{(1+\sin A)(1-\sin A)}{(1+\cos A)(1-\cos A)}$.

Trigonometric Identities:

(i) $\sin^2 A + \cos^2 A = 1$ (ii) $1 + \tan^2 A = \sec^2 A$ (iii) $1 + \cot^2 A = \text{cosec}^2 A$

Examples: (i) If $\sin A = \frac{3}{4}$, find $cosA$ and $tan\ A$.

Solution: $cos\ A = \sqrt{1 - \sin^2 A}$ = $\sqrt{\mathbf{1} - \frac{\mathbf{9}}{\mathbf{16}}} = \sqrt{\frac{\mathbf{7}}{\mathbf{16}}} = \frac{\sqrt{7}}{\mathbf{4}}$.

$\tan A = \frac{\sin A}{\cos A} = \frac{3}{\sqrt{7}}$

Sample Questions:

(1) Using identity prove that: (i) $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

(ii) $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

Complementary angles : (i) $\sin(90-\theta) = cos\theta$ (ii) $\cos(90-\theta) = \sin\theta$

(iii) $\tan(90-\theta) = cot\theta$ (iv) $\cot(90-\theta) = tan\theta$

(v) $\text{cosec}(90-\theta) = sec\theta$ (vi) $\sec(90-\theta) = cosec\theta$

Example: Show that : $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

Proof: $\tan 48° \tan 23° \tan(90-48)° \tan(90-23°)$

$= \tan 48° \tan 23° \cot 48° \cot 23°$=1

Sample Questions: (i) Evaluate : $\frac{\sin 18°}{\cos 72°}$

(ii) If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle , find the value of A.

Trigonometric table for standard angles:

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example: (i) Evaluate $\frac{1-\tan^2 45}{1+\tan^2 45}$

Solution : $\because \tan 45° = 1$

$$\therefore \frac{1-\tan^2 45}{1+\tan^2 45} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Sample Questions:

Evaluate (i) $\frac{2\tan 30}{1-\tan^2 30}$

(ii) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(iii) $\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

*****

FORMULAE

[FORMULA - A special type of equation that shows the relationship between different variables.]

I. Number Theory

1 Rational Number (between two given rational numbers '$\boldsymbol{a}$' and '$\boldsymbol{b}$') = $a + \frac{n(b-a)}{N}$, where $\boldsymbol{n}$ is the sequence of the rational number and $\boldsymbol{N}$ is the total number of rational numbers which we need to find.

2 **Euclid's Division Lemma** $a = bq + r$, $0 \le r < b$, where a is dividend, b is divisor, q is quotient and r is remainder.

3 The Fundamental Theorem of Arithmetic $LCM \times HCF = Product\ of\ the\ numbers$

4 The Result of the Fundamental Theorem of Arithmetic

$$LCM\ (p, q, r) = \frac{p.q.r.HCF(p,q,r)}{HCF(p,q).HCF(q,r).HCF(p,r)}$$

$$HCF\ (p, q, r) = \frac{p.q.r.LCM(p,q,r)}{LCM(p,q).LCM(q,r).LCM(p,r)}$$

II. Algebra

1 $(a+b)^2 = a^2 + 2ab + b^2$

2 $(a-b)^2 = a^2 - 2ab + b^2$

3 $(a+b)(a-b) = a^2 - b^2$

4 $(a+b)^2 + (a-b)^2 = 2\ (a^2 + \ b^2)$

5 $(a+b)^2 - (a-b)^2 = 4ab$

6 $a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$

7 $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

8 $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

9 $(x+a)(x+b) = x^2 + (a+b)x + ab$

10 $(x-a)(x+b) = x^2 - (a-b)x - ab$

11 $(x+a)(x-b) = x^2 + (a-b)x - ab$

12 $(x-a)(x-b) = x^2 - (a+b)x + ab$

13 $(a+b)^3 = a^3 + 3ab(a+b) + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

14 $(a-b)^3 = a^3 - 3ab(a-b) - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$

15 $(a+b)^3 + \ (a-b)^3 = 2\ a^3 + \ 6\ ab^2$

16 $(a+b)^3 - \ (a-b)^3 = 2\ b^3 + \ 6\ a^2b$

17 $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3a^2b - 3ab^2$
$= \ (a+b)^3 - 3ab(a+b)$

18 $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3a^2b - 3ab^2$
$= \ (a-b)^3 - 3ab(a-b)$

19 $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

20 $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

21 $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - \ 3\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - \ 3x - 3 \times \frac{1}{x} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

22 $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x + \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3x - 3 \times \frac{1}{x} = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$

23 $(\pm a \pm b \pm c)^2 = a^2 + b^2 + c^2 \pm 2ab \pm 2bc \pm 2ca$

24 $(a + b + c)^3 = a^3 + b^3 + c^3 + 3ab(a + b) + 3bc(b + c) + 3ca(c + a) + 6abc$

25 $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

26 $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$

27 $a^m \times a^n = a^{m+n}$

28 $a^m \div a^n = a^{m-n}$

29 $(a^m)^n = a^{mn}$

30 $a^m \times b^m = (a \times b)^m$

31 $a^m \div b^m = (a \div b)^m$

32 $a^{-m} = \frac{1}{a^m}$

33 $\sqrt[m]{a} = a^{1/m}$

34 $a^0 = 1$

35 $log_a m + log_a n = log_a(m \times n)$

36 $log_a m - log_a n = log_a(m \div n)$

37 $log_a m^n = nlog_a m$

38 $log_a a = 1$

39 $log_b a = \frac{1}{log_a b} = \frac{log_e a}{log_e b}$

40 $log_b a = log_{c_1} a \times log_{c_2} c_1 \dots\dots\dots\dots \times log_{c_n} c_{n-1} \times log_b c_n$

41 *Remainder Theorem* - If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

42 *Factor Theorem* – $(x - a)$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$.

43 $Quadratic\ Formula\ \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,
where the values of x are the roots of the quadratic equation $ax^2 + bx + c = 0$ $and\ D(= b^2 - 4ac)\ is\ the\ Discriminant.$

(a) If D < 0, then the roots are non-real.
(b) If D = 0, then the roots are real and equal.
(c) If D > 0, then the roots are real and unequal.

If the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β, then

$sum\ of\ roots = \alpha + \beta = -\frac{b}{a} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and

$product\ of\ roots = \alpha . \beta = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

For any three sets A, B & C and Universal set ξ (xi), we have

44 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

45 $A \times (B \cap C) = (A \times B) \cap (A \times C)$

46 $A \times (B - C) = (A \times B) - (A \times C)$

47 $A \times B = B \times A \Leftrightarrow A = B$

48 $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

49 $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

50 $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

51 $A \times B = A \times C \Rightarrow B = C$

52 $Idempotent\ laws\ \ A \cup A = A\ and\ A \cap A = A$

53 $Identity\ laws\ \ A \cup \phi = A\ \ and\ \ A \cap \xi = A$

54 $Commutative\ laws\ \ A \cup B = B \cup A\ and\ A \cap B = B \cap A$

55 *Associative laws* $(A \cup B) \cup C = A \cup (B \cup C)$ *and* $(A \cap B) \cap C = A \cap (B \cap C)$

56 *Distributive laws* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

57 *De' Morgan's laws* $(A \cup B)' = A' \cap B'$ *and* $(A \cap B)' = A' \cup B'$

58 *Complementary laws* $A \cup A' = \xi$ *and* $A \cap A' = \phi$

59 Let S be the sample space associated with a random experiment. A set of events $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ is said to form a set of mutually exclusive and exhaustive system of events if

(i) $A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots, \cup A_n = S$

(ii) $A_i \cap A_j = \emptyset$ for $i \neq j$

60 Probability Function: - Let S = $\{w_1, w_2, \ldots\ldots\ldots, w_n\}$ be the sample space associated with a random experiment. Then a function P which assigns every event $A \subset S$to a unique non-negative real number P(A) is called the probability function if the following axioms hold:

A – 1 $0 \leq P(w_i) \leq 1$ for all $w_i \in S$

A – 2 P (S) = 1, i.e. $P(w_1) + P(w_2) + \ldots\ldots + P(w_n) = 1$.

A – 3 For any event $A \subset S$, P (A) = $\sum P(w_k)$, the number $P(w_k)$ is called the probability of elementary event w_k .

61 Probability of an event: - If there are n elementary events associated with a random experiment and m of them are favourable to an event A, then the probability of occurrence of A is defined as:

$$P(A) = \frac{m}{n} = \frac{Favourable\ number\ of\ elementary\ events}{Total\ number\ of\ elementary\ events}$$

The odds in favour of occurrence of the event A are defined by: $m : (n - m)$

The odds against the occurrence of A are defined by : $(n - m) : m$.

The probability of non-occurrence of A is given by $P(\bar{A}) = 1 - P(A)$

62 If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B).$$

63 If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

64 If A and B are two events associated with a random experiment, then

(i) $P\left(\bar{A} \cap B\right) = P(B) - P(A \cap B)$, i.e. probability of occurrence of B only

(ii) $P\left(A \cap \bar{B}\right) = P(A) - P(A \cap B)$, i.e. probability of occurrence of A only

(iii) Probability of occurrence of exactly one of A and B

$= P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$

If n is a natural number and r is a non-negative integer such that $0 \leq r \leq n$, then

65 $^nC_r = \frac{n!}{(n-r)!\,r!}$

66 $^nC_r \times r! = {}^nP_r$

67 $^nC_r = {}^nC_{n-r}$

68 $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

69 $^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1} = \frac{n}{r} \times \frac{(n-1)}{(r-1)} \cdot {}^{n-2}C_{r-2} = \ldots\ldots = \frac{n}{r} \times \frac{(n-1)}{(r-1)} \times \frac{(n-2)}{(r-2)} \times \ldots \times \frac{n-(r-1)}{1}$

70 $^nC_x = {}^nC_y \Rightarrow x = y$ **OR** $x + y = n$

71 If n is an even natural number, then the greatest among

$$^{n}C_0, ^{n}C_1, ^{n}C_2, \ldots\ldots, ^{n}C_n \text{ is } ^{n}C_{\frac{n}{2}}$$

72 If n is an odd natural number, then the greatest among

$$^{n}C_0, ^{n}C_1, ^{n}C_2, \ldots\ldots, ^{n}C_n \text{ is } ^{n}C_{\frac{n-1}{2}} \text{ or } ^{n}C_{\frac{n+1}{2}}$$

73 The number of ways of selecting **r** items or objects from a group of n distinct items or objects is

$$\frac{n!}{(n-r)!r!} = {}^{n}C_r$$

74 Binomial Theorem: - If x and a are real numbers, then for all n ∈ N, we have

$$(x+a)^n = {}^{n}C_0 x^n a^0 + {}^{n}C_1 x^{n-1} a^1 + {}^{n}C_2 x^{n-2} a^2 + \ldots\ldots + {}^{n}C_r x^{n-r} a^r + \ldots\ldots + {}^{n}C_{n-1} x^1 a^{n-1} + {}^{n}C_n x^0 a^n$$

i.e., $$(x+a)^n = \sum_{r=0}^{n} {}^{n}C_r x^{n-r} a^r$$

75 The sum S_n of n terms of an A.P. with first term a and common difference d is given by

$S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$, where l = last term = $a + (n-1)d$

76 If the sum S_n of n terms of a sequence is given, then n^{th} term a_n of the sequence can be determined by using the formula $a_n = S_n - S_{n-1}$

77 If n numbers $A_1, A_2, \ldots\ldots A_n$ are inserted between two given numbers a and **b** such that $a, A_1, A_2, \ldots\ldots A_n, b$ is an arithmetic progression, then $A_1, A_2, \ldots\ldots A_n$ are known as n arithmetic means between a and **b** and the common difference of the A. P. is $d = \frac{b-a}{n+1}$.

Also, $A_1, A_2, \ldots\ldots A_n = n\left(\frac{a+b}{2}\right)$.

78 Three numbers a, b, c are in A. P. iff (if and only if) $2b = a + c$.

79 The Arithmetic Mean of a and b is $\frac{a+b}{2}$.

80 The n^{th} term of a Geometric Progression with first term a and common ratio r is given by

$$a_n = ar^{n-1}$$

81 If the sum of n terms of a G. P. with first term a and common ratio is given by

$$S_n = a\left(\frac{r^n - 1}{r-1}\right) \quad for\ \boldsymbol{r} > 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \quad for\ \boldsymbol{r} < 1$$

$$S_n = n, \text{ if } r = 1$$

Also, $S_n = \frac{a - lr}{1-r}$ or $S_n = \frac{lr - a}{r-1}$, where l is the last term.

82 Let a and b be two given numbers. If n numbers $G_1, G_2, \ldots\ldots G_n$ are inserted between a and b such that the sequence $a, G_1, G_2, \ldots\ldots G_n, b$ is a G. P., then the numbers $G_1, G_2, \ldots\ldots G_n$ are known as n geometric means between a and b. The common ratio of the G.P. is given by $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

83 The Geometric Mean of a and b is given by $\sqrt{ab}$.

84 If A and G are respectively Arithmetic and Geometric Means between two positive numbers a and b, then

(i) A > G.

(ii) The quadratic equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$.

(iii) $a : b = \left(A + \sqrt{A^2 - G^2}\right) : \left(A - \sqrt{A^2 - G^2}\right)$.

85 If AM and GM between two numbers are in the ratio m : n, then the numbers are in the ratio $\left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

86 Three numbers a, b, c are in G. P. iff (if and only if) $b^2 = ac$.

For any n ∈ N, we have

87 $$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

88 $$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

89 $$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

90 $$\sum_{k=1}^{n} k^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

In a series $a_1 + a_2 + a_3 + a_4 + \cdots + a_n + \cdots$, (for n ∈ N)

91 (i) If the differences $a_2 - a_1, a_3 - a_4, a_4 - a_3, \ldots \ldots$ are in A. P., then the n^th term is given by $a_n = an^2 + \ an + c$, where a, b and c are constants.

(ii) If the differences $a_2 - a_1, a_3 - a_4, a_4 - a_3, \ldots \ldots$ are in G. P., with common ratio r, then $a_n = ar^{n-1} + b_n + c$, where a, b and c are constants.

92 Let A = $[a_{ij}]$ be an $m \times n$ matrix. Then, the transpose of A, denoted by A^T, is an n x m

Matrix such that $(A^T)_{ij} = a_{ji}$ for all $i = 1, 2, \ldots . m; \ j = 1, 2, \ldots ., n$.

Following are the properties of transpose of a matrix:

(i) $(A^T)^T = A$ **(ii)** $(A+B)^T = A^T + B^T$ **(iii)** $(kA)^T = kA^T$ **(iv)** $(AB)^T = B^T A^T$
(v) $(ABC)^T = C^T B^T A^T$

93 If A is a non-singular matrix, then $A^{-1} = \frac{1}{|A|}(adj\ A)$

94 If A and B are two invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$

95 A system $AX = B$ of n linear equations in n equations has a unique solution given by $X = A^{-1}B$, if $|A| \neq 0$.
If $|A| = 0$ and $(adj\ A)B = 0$, then the system is consistent and has infinitely many solutions. If $|A| = 0$ and $(adj\ A)B \neq 0$, then the system is inconsistent.

96 A homogeneous system of n linear equations in n unknowns is expressible in the form AX =O. If $|A| \neq 0$, then AX =O has unique solution X = 0, i.e. $x_1 = x_2 = \cdots = x_n = 0$. This solution is called the Trivial solution. If $|A| = 0$, then AX =O has infinitely many solutions.

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors inclined at an angle θ, then

97 ***Scalar Product or Dot Product of two vectors*** – Let $\vec{a}$ and $\vec{b}$ be two vectors and let θ be the angle between them. Then, the scalar product or dot product of $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}.\vec{b}$, is defined as

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \text{ab} \cos\theta$.

Therefore $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)$

98 Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\,.\vec{b}}{|\vec{b}|} = \vec{a}.\ \hat{b}$

99 Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\,.\vec{b}}{|\vec{a}|} = \vec{b}.\ \hat{a}$

100 $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a}$ is perpendicular to $\vec{b}$

101 $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

102 $\vec{a}.\vec{a} = |\vec{a}|^2$

103 $m\vec{a}.\vec{b} = m(\vec{a}.\vec{b}) = \vec{a}.m\vec{b}$, for any scalar m

104 $m\vec{a}.n\vec{b} = mn(\vec{a}.\vec{b}) = \vec{a}.mn\vec{b}$, for scalars m ,n

105 $|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$

106 $|\vec{a}-\vec{b}| \geq |\vec{a}| - |\vec{b}|$

107 $|\vec{a}\pm\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2(\vec{a}.\vec{b})$

108 $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

109 $\vec{a}.\vec{b} > 0$, iff θ is acute.

110 $\vec{a}.\vec{b} < 0$, iff θ is obtuse.

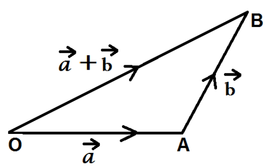
111 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors, then

$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

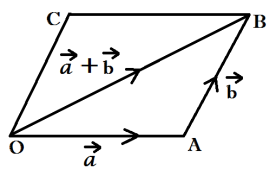
112 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

113 *Triangle law of Addition for Vectors* – In a ΔOAB, if $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\overrightarrow{OB}$ represents $(\vec{a}+\vec{b})$.



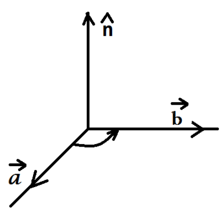
114 *Parallelogram Law of addition for vectors* – In a parallelogram OABC, if $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\overrightarrow{OB}$ represents $(\vec{a}+\vec{b})$.



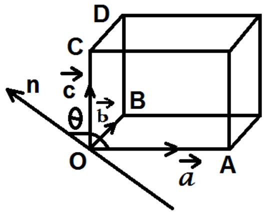
115 *Vector Product of two vectors* – Let $\vec{a}$ and $\vec{b}$ be two nonzero, nonparallel vectors and let θ be the angle between them such that $0 < \theta < \pi$. Then, the vector product of $\vec{a}$ and $\vec{b}$ is defined as

$\vec{a} \times \vec{b} = (|\vec{a}|\,|\vec{b}|\sin\theta)\,\hat{n} \Rightarrow \theta = \sin^{-1}\left\{\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}\right\}$,

Where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}$,$\vec{b}$, $\hat{n}$ form a right-handed system.



116 *Volume of Parallelopiped* – If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors then the scalar product of $\vec{a}$ with the vector product of $\vec{b}$ and $\vec{c}$ is called the scalar triple product of the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$. It is written as



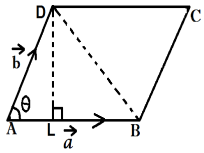
$$V = \vec{a}.(\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

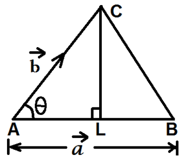
117 Vector Triple Product – If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors then the vector product of $\vec{a}$ with $(\vec{b} \times \vec{c})$, is called the vector triple product and is written as

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\vec{c})\,\vec{b} - (\vec{a}\,.\vec{b})\,\vec{c}$$

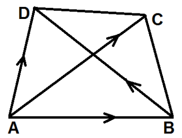
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \end{vmatrix}$$

118 (a) $ar\ (\parallel gm\ ABCD) = |\vec{a} \times \vec{b}|$, where $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$.





(b) $ar\ (\Delta\ ABC) = \frac{1}{2}|\vec{a} \times \vec{b}|$, where $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$.



(c) $ar(quad.ABCD) = \frac{1}{2}\,|\,\overrightarrow{AC} \times \overrightarrow{BD}\,|$, where AC and BD are its diagonals.

119 Co-linearity – The points A, B, C with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are collinear if and only if

$$(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$$

Area of Triangle – If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the points A, B, C, then *The area of triangle* $= \Delta = \frac{1}{2}\left[(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b})\right]$

120 Equation of a line passing through two given points –
Vector Form – The vector equation of a line L, passing through two given points A and B with position vectors $\vec{r_1}$ and $\vec{r_2}$, is given by

$$\vec{r} = \vec{r_1} + \lambda\,(\vec{r_2} - \vec{r_1})$$

Cartesian Form – (i) The equations of a line passing through two given points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) are given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

(ii) The Cartesian equations of a line with direction ratios *a*, b, c and passing through A (x_1, y_1, z_1) are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

121 (i) The vector equation of a line through a point with position vector $\vec{r_1}$ and parallel to $\vec{m}$ is

$$\vec{r} = \vec{r_1} + \lambda\, \vec{m}$$

(ii) The vector equation of a line through a point with position vectors $\vec{r_1}$ and $\vec{r_2}$ is

$$\vec{r} = \vec{r_1} + \lambda\, (\vec{r_2} - \vec{r_1})$$

122 *Cartesian Form* – Three given points A (x_1, y_1, z_1), B (x_2, y_2, z_2) and C (x_3, y_3, z_3) will be collinear if

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$$

Vector Form – Three given points A, B, C with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively are collinear if $\vec{c} = (1 - \lambda)\, \vec{a} + \lambda\, \vec{b}$

123 *Vector Form* – Let the vector equations of two given lines be $\vec{r} = \vec{r_1} + \lambda\, \vec{n_1}$ and $\vec{r} = \vec{r_2} + \mu\, \vec{n_2}$, where λ and μ are scalars. Let θ be the angle between these lines. Since the given lines are parallel to $\vec{n_1}$ and $\vec{n_2}$ respectively, the angle between the given lines must be equal to the angle between $\vec{n_1}$ and $\vec{n_2}$.

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}|\ |\vec{n_2}|}$$

These lines are parallel, if $\vec{n_1}$ is parallel to $\vec{n_2}$.
These lines are perpendicular, if $\vec{n_1} \cdot \vec{n_2} = 0$.
Cartesian Form – Let the Cartesian equations of two given lines be

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}.$$

Then, the direction ratios of these lines are a_1, b_1, c_1 and a_2, b_2, c_2 respectively. Let θ be angle between these lines. Then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

These lines are parallel, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

These lines are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

124 ***Vector Form*** – (i) The shortest distance between two skew lines $\vec{r} = \vec{a_1} + \lambda\, \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\, \vec{b_2}$ is given by $d = \left|\dfrac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$

(ii) Let $L_1\ (\vec{r} = \vec{a_1} + \lambda\, \vec{b_1})$ and $L_2\ (\vec{r} = \vec{a_2} + \mu\, \vec{b_2})$ be two parallel lines. Then these lines are coplanar if

$$\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$$

Distance between Parallel Lines = $|\overrightarrow{BM}| = \left|\dfrac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right|$

Condition for two given lines to intersect – Suppose that the lines $\vec{r} = \vec{a_1} + \lambda\, \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\, \vec{b_2}$ intersect. Then the shortest distance between them is zero, i.e.

$$[(\vec{a_2} - \vec{a_1})\, \vec{b_1}\ \vec{b_2}] = 0.$$

Cartesian Form - The shortest distance between two skew lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$$

where D = $\{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2\}$

125 Vector Form – If $\hat{n}$ is a unit vector normal to a given plane, directed from the origin to the plane and p is the length of the perpendicular drawn from the origin to the plane then the vector equation of the plane is $\vec{r} \cdot \hat{n}$ = p.

A vector normal to the plane $ax + by + cz + d = 0$ is $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$.

Cartesian Form – If a, b, c are the direction ratios of the normal to a plane, then the equation of the plane is $ax + by + cz + d = 0$

126 Vector Form – The vector equation of a plane passing through a point having position vector $\vec{a}$ and normal to $\vec{n}$ is

$$(\vec{r} - \vec{a}).\vec{n} = 0 \text{ or } \vec{r}.\vec{n} = \vec{a}.\vec{n}$$

Cartesian Form – The equation of a plane passing through a point P (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where a, b, c are constants.

127 The vector equation of a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$ is

$\vec{r} = \vec{a} + m\,\vec{b} + n\,\vec{c}$, where *m* and *n* are parameters.

or $\quad \vec{r}.(\vec{b} \times \vec{c}) = \vec{a}.(\vec{b} \times \vec{c})$

128 If a plane makes intercepts of lengths *a*, b, c with the *x*-axis, y-axis and z-axis respectively, the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

129 Vector Form – The equation of a plane through the intersection of two planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by

$$\vec{r}.(\overrightarrow{n_1} + \lambda\,\overrightarrow{n_2}) = d_1 + \lambda\, d_2$$

Cartesian Form – The equation of a plane through the intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by $(a_1x + b_1y + c_1z + d_1) + \lambda\,(a_2x + b_2y + c_2z + d_2) = 0$

130 The Cartesian equation of a plane passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

131 The equation of a plane parallel to the plane

(a) $\vec{r}.\vec{n} = d$ is $\vec{r}.\vec{n} = d_1$

(b) $ax + by + cz + d = 0$ is $ax + by + cz + \lambda = 0$

132 The length of perpendicular from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

133 The distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$\frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

134 The equation of the family of planes containing the lines $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0,$$

where λ is a parameter.

135 The equations of the planes bisecting the angles between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are given by

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a^2 + b^2 + c^2}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a^2 + b^2 + c^2}}$$

136 The angle θ between a line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and a plane $ax + by + cz + d = 0$ is the complement of the angle between the line and normal to the plane and is given by

$$\sin\theta = \frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$

The angle θ between the line $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r}.\vec{n} = d$ is given by

$$\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$$

137 Two lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

and the equation of the plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

III. Commercial Mathematics

1 $Cost\ Price = Buying\ Price + Overhead\ Expenses$

2 $Profit = Sale\ Price - Cost\ Price$

3 $Loss = Cost\ Price - Sale\ Price$

4 $Profit\ \% = \dfrac{Profit}{Cost\ Price} \times 100\%$

5 $Loss\ \% = \dfrac{Loss}{Cost\ Price} \times 100\%$

6 $Sale\ Price = \left(\dfrac{100\ \% + Profit\ \%}{100\ \%}\right) \times Cost\ Price$

7 $Sale\ Price = \left(\frac{100\ \% - Loss\ \%}{100\ \%}\right) \times Cost\ Price$

8 $Discount = Marked\ Price - Sale\ Price = Discount\ \%\ of\ Marked\ Price$

9 $Sales\ Tax = Tax\ \%\ of\ Bill\ Amount$

10 $Simple\ Interest = \dfrac{PRT}{100}$

11 $Amount = Principal + Simple\ Interest$

12 $If\ the\ Interest\ is\ compounded\ annually, then$

$$A = P\left(1 + \frac{R}{100}\right)^n,$$

$where\ \boldsymbol{A}\ is\ Amount, \boldsymbol{P}\ is\ Principal, \boldsymbol{R}\ is\ Rate\ of\ Interest\ and\ \boldsymbol{n}\ is\ time\ period.$

13 $If\ the\ Interest\ is\ compounded\ half\ yearly, then$

$$A = P\left(1 + \frac{R}{200}\right)^{2n},$$

14 $If\ the\ Interest\ is\ compounded\ quarterly, then$

$$A = P\left(1+\frac{R}{400}\right)^{4n},$$

15 $$Compound\ Interest = P\left[\left(1+\frac{R}{100}\right)^{n} - 1\right]$$

16 Speed = $\frac{Distance}{Time}$

IV. Geometry

1 $Sum\ of\ all\ the\ interior\ angles\ of\ a\ polygon\ =\ (n-2)\times\ 180°,$
$where\ 'n'\ is\ the\ number\ of\ sides\ of\ the\ polygon$

2 $$Measure\ of\ each\ Exterior\ Angle\ = \frac{360°}{n}$$

3 $$Measure\ of\ each\ Interior\ Angle\ = 180° - \frac{360°}{n} = \frac{(n-2)\times 180°}{n}$$

4 The relation between three systems of measurement of an angle is

$$\frac{D}{90°} = \frac{G}{100} = \frac{2R}{\pi}$$

Where the three systems of measuring angles are:

(i) Sexagesimal system, in which we have
1 right angle = 90 degrees (= 90°)
$1° = 60\ minutes\ (= 60')$
$1' = 60\ seconds\ (= 60'')$

(ii) Centesimal system, in which we have
1 right angle = 100 grades
1grade = 100 minutes
1 minute = 100 seconds

(iii) Circular system, in which the unit of measurement is radian. One radian is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.
Π radians = 180°

V. Coordinate Geometry

1 $The\ Distance\ Formula$

$$-\ The\ Distance\ between\ the\ points\ P(x_1, y_1)\ \&\ Q(x_2, y_2)$$
$$= PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2 $The\ Section\ Formula - The\ coordinates\ of\ point\ P(x, y), which\ divides\ the\ line\ segment$
$joining\ the\ points\ A(x_1, y_1)\ and\ B(x_2, y_2), in\ the\ ratio\ m_1{:}\,m_2\ are$
$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right), internally\ and\ \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$, externally.

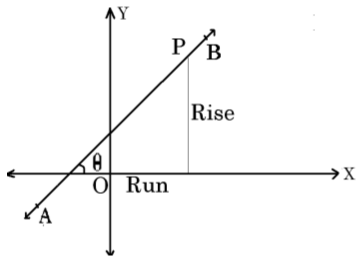
3 $The\ Mid\ point\ Formula$ – The coordinates of the midpoint P of the
$A(x_1, y_1)$ and $B(x_2, y_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

4 $$Area\ of\ a\ \Delta\ ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)],$$
$$where\ A(x_1, y_1), B(x_2, y_2)\ and\ C(x_3, y_3)\ are\ the\ vertices\ of\ the\ \Delta\ ABC.$$

5 $The\ Slope\ of\ a\ line\ AB\ = m = tan\theta = \dfrac{Rise}{Run}$

$$= \frac{Difference\ of\ Ordinates}{Difference\ of\ abscissae} = \frac{y_2 - y_1}{x_2 - x_1},$$

$Where\ (x_1, y_1)\ and\ (x_2, y_2)\ are\ the\ two\ on\ line\ AB.$



6 An acute angle θ between the lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \text{ where } 1 + m_1 m_2 \neq 0.$$

7 The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

i.e. $PQ = \sqrt{(Difference\ of\ abscissae)^2 + (Difference\ of\ ordinates)^2}$

8 The distance of a point ***P*** $(\boldsymbol{x}, \boldsymbol{y})$ from the origin **O (0, 0)** is given by $OP = \sqrt{x^2 + y^2}$.

9 The area of the triangle, the coordinates of whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is the absolute value of

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ or $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

10 If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

11 The coordinates of the mid-point of the line segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

12 The coordinates of the centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and(x_3, y_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

13 ***Slope-Intercept form*** – The equation of a line with slope m and making an intercept ***c*** on y-axis is $y = mx + c$.

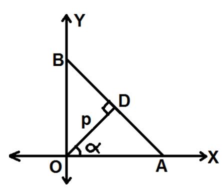
14 ***Point-Slope Form*** – The equation of the line which passes through the point (x_1, y_1) and has slope m is $y - y_1 = m(x - x_1)$

15 The equation of the line which passes through the point (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

16 ***Intercept Form*** – The equation of the line making intercepts a and b on x and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

17 Normal form of a straight line $x \cos\alpha + y \sin\alpha = p$, where p is the length of perpendicular from origin to the straight line and α is the angle made by this perpendicular with the positive direction of *x*-axis.



18 General Equation of Circle – $(x-h)^2+(y-k)^2=r^2$
Where *(h, k)* is the centre of the circle and *r* is radius of the circle.

19 Standard Equation of circle – $x^2+y^2+2gx+2fy+c=0$
Where $(-g,-f)$ is the centre of the circle and $r\left(=\sqrt{g^2+f^2-c}\right)$ is the radius of the circle.

20 If (α,β) is the focus and $ax+by+c=0$ is the equation of the directrix of a parabola, then its equation is $(x-\alpha)^2+(y-\beta)^2=\frac{(ax+by+c)^2}{a^2+b^2}$. This equation is of the form $ax^2+2hxy+by^2+2gx+2fy+c=0$ satisfying the conditions $abc+2fgh-af^2-bg^2-ch^2\neq 0$ and $h^2-ab=0$.

21 Following are four standard forms of parabola:

	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
Coordinates of vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Coordinates of focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Equation of the directrix	$x=-a$	$x=a$	$y=-a$	$y=a$
Equation of the axis	$y=0$	$y=0$	$x=0$	$x=0$
Length of the Latus – rectum	$4a$	$4a$	$4a$	$4a$
Focal distance of a point P (x,y)	$a+x$	$a-x$	$a+y$	$a-y$

22 (i) If S is the focus and ZZ' is the directrix and P is any point on the ellipse such that M is the foot of perpendicular from P on ZZ' then SP = e . PM
The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents an ellipse, if
$\Delta=abc+2fgh-af^2-bg^2-ch^2\neq 0$ and $h^2<ab$.

(ii) If the centre of the elipse is at the point (h,k) and the directions of the axes are parallel to the coordinate axes, then its equation is
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

(iii) The sum of the focal distances of any point on the elipse
= SP + S 'P = $a-ex+a+ex=2a$ = major axis (= constant),
Where P(*x*, *y*) is any point on the elipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

23 The equation of the ellipse whose axes are parallel to the coordinate axes and whose centre is at the origin, is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the following properties:

	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a<b$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(a,0)$ and $(-a,0)$	$(0,b)$ and $(0,-b)$
Coordinates of foci	$(ae,0)$ and $(-ae,0)$	$(0,be)$ and $(0,-be)$
Length of the major axis	$2a$	$2b$
Length of the minor axis	$2b$	$2a$
Equation of the major axis	$y=0$	$x=0$
Equation of the minor axis	$x=0$	$y=0$
Equations of the directrices	$x=\frac{a}{e}$ and $x=-\frac{a}{e}$	$y=\frac{b}{e}$ and $y=-\frac{b}{e}$
Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
Length of the latus-rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Focal distances of a point (x,y)	$a\pm ex$	$b\pm ey$

24 The equation of the hyperbola having its centre at the origin and axes along the coordinate axes is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with the following properties:

	Hyperbola $\frac{x^2}{y^2} - \frac{y^2}{b^2} = 1$	Conjugate hyperbola $-\frac{x^2}{y^2} + \frac{y^2}{b^2} = 1$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(a, 0)$ and $(-\ a, 0)$	$(0, b)$ and $(0, -b)$
Coordinates of foci	$(\pm\ ae, 0)$	$(0, \pm\ be)$
Length of the transverse axis	$2a$	$2b$
Length of the conjugate axis	$2b$	$2a$
Equation of the transverse axis	$y = 0$	$x = 0$
Equation of the conjugate axis	$x = 0$	$y = 0$
Equations of the direectices	$x = \ \pm\ \frac{a}{e}$	$y = \pm\ \frac{b}{e}$
Eccentricity	$e = \sqrt{1 + \frac{b^2}{a^2}}$ or $b^2 = a^2(e^2 - 1)$	$e = \sqrt{1 + \frac{a^2}{b^2}}$ or $a^2 \ = b^2(e^2 - 1)$
Length of the latus-rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

25 If the centre of the hyperbola is at the point (h, k) and the directions of the axes are parallel to the coordinate axes, then its equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

26 Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

27 Section Formula: If R divides PQ in the ratio $m{:}\,n$, then

$$R_x = \frac{mQ_x \pm nP_x}{m \pm n}$$

$$R_y = \frac{mQ_y \pm nP_y}{m \pm n}$$

$$R_z = \frac{mQ_z \pm nP_z}{m \pm n}$$

Note:- "+" sign for internal division & "-" sign for external division.

VI. Mensuration

1 $Perimeter\ of\ a\ Polygon\ = \ Sum\ of\ all\ the\ sides\ of\ the\ polygon$

2 $Perimeter\ of\ Rectangle\ = \ 2(length + breadth) = 2(l + b)$

3 $Perimeter\ of\ Square\ = \ 4 \times side = 4a$

4 $Circumference\ of\ a\ Circle\ = \ C = \ 2\pi r, where\ \boldsymbol{r}\ is\ the\ radius\ of\ the\ circle$

5 $Perimeter\ of\ a\ Semicircle = \pi r + 2r = (\pi + 2)r$

6 $Area\ of\ Circle\ = \pi r^2$

7 $Area\ of\ a\ Semicircle = \frac{1}{2}\pi r^2$

8 $Length\ of\ an\ Arc = \frac{\theta}{360°} \times 2\pi r$

9 $Area\ of\ a\ Sector = \frac{\theta}{360°} \times \pi r^2$

10 $Heron's\ Formula\ -\ Area\ of\ a\ Scalene\ Triangle\ = \sqrt{s(s-a)(s-b)(s-c)},$

$where\ \boldsymbol{s}\ is\ the\ semi-perimeter\ of\ the\ Scalene\ Triangle, \quad \boldsymbol{s} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$

$and\ \boldsymbol{a}, \boldsymbol{b}\ \&\ \boldsymbol{c}\ are\ the\ sides\ of\ the\ Scalene\ Triangle.$

11 $Area\ of\ a\ Right\ Angled\ Triangle = \frac{1}{2} \times base \times height = \frac{1}{2} \times b \times h$

12 $Area\ of\ an\ Isosceles\ Triangle = \frac{1}{2} \times \mathrm{b} \times \sqrt{\mathrm{a}^2 - \left(\frac{\mathrm{b}}{2}\right)^2}$

Where ‘**b**’ is the base and ‘**a**’ is one of the equal sides of the Isosceles Triangle

13 $Area\ of\ an\ Equilateral\ Triangle = \frac{\sqrt{3}}{4} \times side \times side = \frac{\sqrt{3}}{4}a^2$

14 $Area\ of\ Rhombus$

$$= \frac{1}{2} \times d_1 \times d_2, where\ \boldsymbol{d_1}\ \&\ \boldsymbol{d_2}\ are\ the\ diagonals\ of\ the\ Rhombus$$

15 $Area\ of\ Parallelogram = base \times height = b \times h$

16 $Area\ of\ Rectangle = Length\ \times breadth = l \times b$

17 $Area\ of\ Square = side\ \times side = a^2$

18 $Area\ of\ Trapezium\ or\ Trapezoid = \frac{1}{2} \times sum\ of\ parallel\ sides \times height$

$$= \frac{1}{2} \times (a + b) \times h$$

19 $Euler's\ Formula \quad \boldsymbol{F} + \boldsymbol{V} - \boldsymbol{E} = \mathbf{2},\ \ where\ ‘\boldsymbol{F}’\ stands\ for\ number\ of\ faces,$
$‘\boldsymbol{V}’\ stands\ for\ number\ of\ vertices\ and\ '\boldsymbol{E}'stands\ for\ number\ of\ edges$

20 $Area\ of\ four\ walls\ of\ a\ rectangular\ box\ or\ Curved\ Surface\ Area\ of$
$Cuboid = 2(l + b \times h$

21 $Total\ Surface\ Area\ of\ a\ Cuboid = 2\ (lb + bh + hl)$

22 $Volume\ of\ a\ Cuboid = l \times b \times h$

23 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cube = 4\ l^2$

24 $Total\ Surface\ Area\ of\ a\ Cube = 6\ l^2$

25 $Volume\ of\ a\ Cube = l^3$

26 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cylinder = 2\pi rh$

27 $Total\ Surface\ Area\ of\ a\ Cylinder = 2\pi r(r + h)$

28 $Volume\ of\ a\ Cylinder =\ \pi r^2 h$

29 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cone = \pi rl,$

$$where\ \boldsymbol{l}\ \left(= \sqrt{r^2 + h^2}\right)\ is\ slant\ height\ of\ cone$$

30 $Total\ Surface\ Area\ of\ Cone = \pi r(r + l)$

31 $Volume\ of\ a\ Cone = \dfrac{1}{3}\pi r^2 h$

32 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Prism$
$= height \times Perimeter\ of\ the\ base = h \times P$

33 $Total\ Surface\ Area\ of\ a\ Prism$
$= 2 \times Area\ of\ the\ base + h \times Perimeter\ of\ the\ base = 2A + hP$

34 $Volume\ of\ a\ Prism = Area\ of\ base \times height = Ah$

35 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Pyramid$
$= Number\ of\ sides\ at\ the\ base \times Area\ of\ a\ triangle = n \times \frac{1}{2} \times b \times h$

36 $Total\ Surface\ Area\ of\ a\ Pyramid$
$= Area\ of\ base + Combined\ Area\ of\ the\ lateral\ faces = A + n \times \frac{1}{2} \times b \times h$

37 $Volume\ of\ a\ Pyramid = \dfrac{1}{3} \times Area\ of\ Base \times height = \dfrac{1}{3} \times A \times h$

38 $Surface\ Area\ of\ a\ Sphere = 4\pi r^2$

39 $Volume\ of\ a\ Sphere = \dfrac{4}{3}\pi r^3$

40 Total $Surface\ Area\ of\ a$ Solid $Hemisphere = 3\pi r^2$

41 $Volume\ of\ a\ Hemisphere = \dfrac{2}{3}\pi r^3$

42 $Total\ Surface\ Area\ of\ a\ Hollow\ Cylinder = 2\pi rh + 2\pi Rh + 2\pi(R^2 - r^2)$

43 $Volume\ of\ a\ Hollow\ Cylinder = \pi(R^2 - r^2)h$

44 $Curved\ Surface\ Area\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \pi l(r_1 + r_2),$
$where\ \boldsymbol{l}\left(= \sqrt{h^2 + (r_1 - r_2)^2}\right)\ is\ slant\ height\ of\ the\ frustum$

45 $Total\ Surface\ Area\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \pi l(r_1 + r_2) + \pi({r_1}^2 + {r_2}^2)$

46 $Volume\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2),\ \ where\ \boldsymbol{h}\ is\ vertical\ height\ of\ the\ frustum,$
$\boldsymbol{r_1}\ \&\ \boldsymbol{r_2}\ are\ radii\ of\ the\ two\ bases\ (ends) of\ the\ frustum$

VII. Statistics

1 $Mean\ (for\ ungrouped\ data) = \dfrac{Sum\ of\ all\ the\ observations}{Total\ number\ of\ observations}$

2 $Mean\ (for\ grouped\ data - The\ Direct\ Method) = \overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i},$
$where\ x_i\ are\ the\ class\ marks$

3 $Mean\ \left(for\ grouped\ data - The\ Assumed\ Mean\ Method\right) = \overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$
$where\ \boldsymbol{a}\ is\ the\ assumed\ mean, \boldsymbol{d_i}\ is\ the\ deviation\ from\ the\ class\ mark\ and$
$\boldsymbol{d_i = x_i - a}$

4 $Mean\ (for\ grouped\ data - The\ Step\ Deviation\ Method) = \overline{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h,$
$where\ \boldsymbol{h}\ is\ class\ size\ of\ each\ class\ interval\ and\ \boldsymbol{u_i} = \frac{x_i - a}{h}$

5 $Median\ (for\ ungrouped\ data\ \&\ Odd\ number\ of\ observations)$
$$= \left(\frac{N+1}{2}\right)^{th}\ observation, where\ \boldsymbol{N}\ is\ the\ total\ number\ of\ observations$$

6 $Median\ (for\ ungrouped\ data\ \&\ Even\ number\ of\ observations)$
$$= Mean\ of\ \left(\frac{N}{2}\right)^{th}\ \&\ \left(\frac{N}{2}+1\right)^{th}\ observations$$

7 $$Median (for\ grouped\ data) = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h,$$
$where\ \boldsymbol{l}\ is\ lower\ limit\ of\ median\ class, \boldsymbol{N}\ is\ total\ number\ of\ observations,$
$\boldsymbol{cf}\ is\ cumulative\ frequency\ of\ class\ preceding\ the\ median\ class, \boldsymbol{f}\ is$
$frequency\ of\ median\ class, \boldsymbol{h}\ is\ class\ size\ (assuming\ class\ size\ to\ be\ equal)$

8 $Mode (for\ ungrouped\ data)$
$$= The\ observation\ which\ occurs\ most\ number\ of\ times$$

9 $$Mode (for\ grouped\ data) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$
$where\ \boldsymbol{l}\ is\ lower\ limit\ of\ the\ modal\ class, \boldsymbol{h}\ is\ size\ of\ the\ class\ interval$
$(assuming\ all\ class\ sizes\ to\ be\ equal), \boldsymbol{f_1}\ is\ frequency\ of\ the\ modal\ class,$
$\boldsymbol{f_0}\ is\ frequency\ of\ the\ class\ preceding\ the\ modal\ class,$
$\boldsymbol{f_2}\ is\ frequency\ of\ the\ class\ succeeding\ the\ modal\ class$

10 Theoretical Probability = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

11 Experimental Probability = $\frac{Number\ of\ favourable\ trials}{Total\ number\ of\ trials}$

12 Probability (Multiplication Rule) – For two independent events E_1 and E_2

(i) $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$
(ii) $P(E_1 \cap \overline{E_2}) = P(E_1) \times P(\overline{E_2})$
(iii) $P(\overline{E_1} \cap E_2) = P(\overline{E_1}) \times P(E_2)$
(iv) $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$

13 Conditional Probability – Probability of A when B has already occurred

(i) $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{n\,(A\cap B)}{n(B)}, P(B) \neq 0$
(ii) $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{n\,(A\cap B)}{n(A)}, P(A) \neq 0$

14 Theorem of Total Probability: - Let $E_1, E_2, \ldots\ldots\ldots, E_n$ be mutually exclusive and exhaustive events associated with a random experiment and let E be an event occurs with some E_i, then

$$P(E) = \sum_{i=1}^{n} P\left(\frac{E}{E_i}\right).P(E_i)$$

15 Bayes' Theorem – Let $E_1, E_2, \ldots\ldots, E_n$ be mutually exclusive & exhaustive events, associated with a random experiment and let E be any event that occurs with some E_i then

$$P\left(\frac{E_i}{E}\right) = \frac{P\left(\frac{E}{E_i}\right).P(E_i)}{\sum_{i=1}^{n} P\left(\frac{E}{E_i}\right)P(E_i)}$$

16 Probability distribution of X is given by

X	x_1	x_2	……………………..	x_n
P(X)	P_1	P_2	……………………..	P_n

$$Where\ each\ P_i \geq 0\ \ \&\ \ \sum_{i=1}^{n} P_i = 1$$

$$Mean\ \ \mu\ = E(X) = \sum_{i=1}^{n} x_i P_i$$

$$Variance\ \sigma^2\ = \left(\sum x_i{}^2 P_i - \mu^2\right)$$

$$Standard\ Deviation\ \ \sigma = \sqrt{Variance}$$

17 Binomial Distribution: $P(X = r) = {}^n c_r\, p^r q^r$, where p & q are probability of success & failure and $p + q = 1$.

$$Mean\ \ \mu = \sum_{i=1}^{n} x_i P_i = np$$

$$Variance\ \sigma^2\ = \left(\sum x_i{}^2 P_i - \mu^2\right) = npq$$

$$Standard\ Deviation\ \ \sigma = \sqrt{Variance} = \sqrt{npq}$$

18 Mean deviation is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

(i) For individual observation, we have

$$M.D. = \frac{1}{n}\sum_{i=1}^{n} \left|x_i - a\right|,$$

where a = mean, median, mode

Also, $M.D. = a + h\left\{\frac{1}{N}\sum_{i=1}^{n}\left|u_i\right|\right\}$, where $u_i = \dfrac{x_i - a}{h}$.

(ii) For a discrete frequency distribution, we have

$M.D. = \frac{1}{N}\sum_{i=1}^{n} f_i\left|x_i - a\right|$, a = mean, median, mode

$M.D. = a + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$, where $u_i = \frac{x_i - a}{h}$.

19 Variance is the arithmetic mean of the squares of deviations about mean $\overline{x}$

(i) For individual observations, we have

$$Variance = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2$$

Also, $Var\ (X) = \frac{1}{n}\sum_{i=1}^{n}{x_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$$Var\ (X) = h^2\left[\frac{1}{n}\sum_{i=1}^{n}{u_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n}u_i\right)^2\right], \text{ where } u_i = \frac{x_i - a}{h}.$$

(ii) For a discrete frequency distribution, we have

$$Var\ (X) = \frac{1}{N}\sum_{i=1}^{n}f_i\left(x_i - \overline{X}\right)^2$$

$$\Rightarrow Var\ (X) = \frac{1}{N}\sum_{i=1}^{n}f_i{x_i}^2 - \left(\frac{1}{N}\sum_{i=1}^{n}f_i x_i\right)^2$$

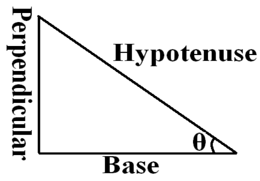
$$\Rightarrow Var\ (X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n}f_i{u_i}^2 - \left(\frac{1}{N}\sum_{i=1}^{n}f_i u_i\right)^2\right]$$

20 The coefficient of variation = C. V. = $\frac{\sigma}{\overline{X}} \times 100$

VIII. Trigonometry and Inverse Trigonometry

Standard Angles → / T. Ratios ↓	**0°**	**30°**	**45°**	**60°**	**90°**
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

1 $\sin\theta = \dfrac{Perpendicular}{Hypotenuse}$ 2 $\mathrm{cosec}\theta = \dfrac{Hypotenuse}{Perpendicular}$



3 $\cos\theta = \dfrac{Base}{Hypotenuse}$ 4 $\sec\theta = \dfrac{Hypotenuse}{Base}$

5 $\tan\theta = \dfrac{Perpendicular}{Base}$ 6 $\cot\theta = \dfrac{Base}{Perpendicular}$

7 $\sin\theta = \dfrac{1}{cosec\theta}$ 8 $\mathrm{cosec}\theta = \dfrac{1}{sin\theta}$

9 $\cos\theta = \dfrac{1}{sec\theta}$ 10 $\sec\theta = \dfrac{1}{cos\theta}$ 11 $\tan\theta = \dfrac{1}{cot\theta}$

12 $\cot\theta = \dfrac{1}{tan\theta}$ 13 $\tan\theta = \dfrac{sin\theta}{cos\theta}$ 14 $\cot\theta = \dfrac{cos\theta}{sin\theta}$

15 $\sin^2\theta + \cos^2\theta = 1$ **Or** $\sin^2\theta = 1 - \cos^2\theta$ **Or** $\cos^2\theta = 1 - \sin^2\theta$

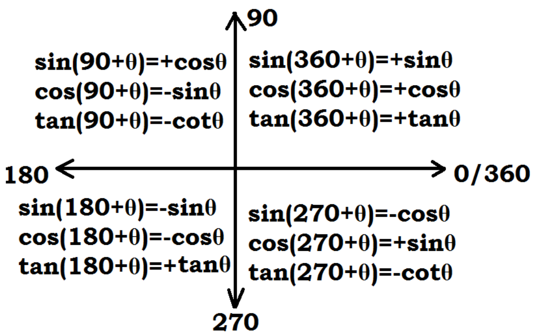
16 $1 + \tan^2\theta = \sec^2\theta$ **Or** $\tan^2\theta = \sec^2\theta - 1$ **Or** $\sec^2\theta - \tan^2\theta = 1$

Or $\sec\theta - \tan\theta = \dfrac{1}{sec\theta + \tan\theta}$

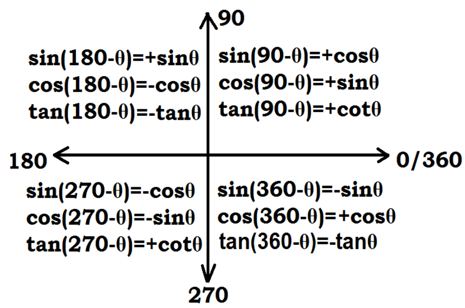
17 $1 + \cot^2\theta = \mathrm{cosec}^2\theta$ **Or** $\cot^2\theta = \mathrm{cosec}^2\theta - 1$ **Or** $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

Or $\mathrm{cosec}\theta - \cot\theta = \dfrac{1}{cosec\theta + \cot\theta}$

18



19



20 $\sin(-\theta) = -\sin\theta$ 21 $\cos(-\theta) = \cos\theta$

22 $\tan(-\theta) = -\tan\theta$ 23 $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\theta$

24 $\sec(-\theta) = \sec\theta$ 25 $\cot(-\theta) = -\cot\theta$

26 $\mathrm{Sin}(A + B) = \sin A\cos B + \cos A\sin B$ 27 $\mathrm{Sin}(A - B) = \sin A\cos B - \cos A\sin B$

28 $\cos(A + B) = \cos A\cos B - \sin A\sin B$ 29 $\cos(A - B) = \cos A\cos B + \sin A\sin B$

30 $\tan(A + B) = \dfrac{tanA + tanB}{1 - tanAtanB}$ 31 $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

32 $Sin(A + B) + sin(A - B) = 2sinAcosB$

33 $Sin(A + B) - sin(A - B) = 2cosAsinB$

34 $cos(A + B) + cos(A - B) = 2cosAcosB$

35 $cos(A - B) - cos(A + B) = 2\ sinAsinB$

36 $sin(A + B)sin(A - B) = sin2A - sin2B$

37 $cos(A + B)cos(A - B) = cos2A - sin2B$

38 $sinC + sinD = 2sin\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$

39 $sinC - sinD = 2cos\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$

40 $cosC + cosD = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$

41 $cosC - cosD = 2sin\left(\frac{C+D}{2}\right)sin\left(\frac{D-C}{2}\right)$

42 $sin2\theta = 2sin\theta cos\theta = \frac{2tan\theta}{1+tan^2\theta}$

43 $cos2\theta = cos2\theta - sin2\theta = 2cos2\theta - 1 = 1 - 2sin2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$

44 $\tan 2\theta = \frac{2tan\theta}{1-tan^2\theta}$

45 $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

46 $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

47 $\tan 3\theta = \frac{3tan\theta - \tan^3\theta}{1-3\,tan^2\theta}$

48 $1 + \cos 2\theta = 2\cos^2\theta$

OR

$\cos\theta = \sqrt{\frac{1+cos2\theta}{2}}$

49 $1 - \cos 2\theta = 2\sin^2\theta$

OR

$\sin\theta = \sqrt{\frac{1-cos2\theta}{2}}$

50 $\sin 18° = \frac{\sqrt{5}-1}{4}$

51 $\cos 36° = \frac{\sqrt{5}+1}{4}$

52 sin(A + B + C) = sinAcosBcosC + cosAsinBcosC + cosAcosBsinC - sinAsinBsinC

53 cos(A + B + C) = cosAcosBcosC − cosAsinBsinC − sinAcosBsinC − sinAsinBcosC

54 $tan(A + B + C) = \frac{\tan A + \tan B + \tan C - tanAtanBtanC}{1 - tanAtanB - tanBtanC - tanCtanA}$

55 $\sin^{-1}(sin\,\theta) = \theta$, for all $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

56 $\cos^{-1}(\cos\theta) = \theta$, for all $\theta \in [0, \pi]$

57 $\tan^{-1}(\tan\theta) = \theta$, for all $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

58 $cosec^{-1}(cosec\,\theta) = \theta$, for all $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\theta \neq 0$

59 $sec^{-1}(\sec\theta) = \theta$, for all $\theta \in [0, \pi]$, $\theta \neq \frac{\pi}{2}$

60 $cot^{-1}(\cot\theta) = \theta$, for all $\theta \in (0, \pi)$

61 $sin(sin^{-1}x) = x$, for all $x \in [-1, 1]$

62 $\cos(\cos^{-1}x) = x$, for all $x \in [-1, 1]$

63 $\tan(\tan^{-1}x) = x$, for all $x \in$ R

64 cosec $(cosec^{-1}x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

65 $\sec(sec^{-1}x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

66 $\cot(\cot^{-1}x) = x$, for all $x \in$ R

67 $\sin^{-1}(-x) = -\sin^{-1}x$, for all $x \in [-1, 1]$

68 $\cos^{-1}(-x) = \pi - \cos^{-1}x$, for all $x \in [-1, 1]$

69 $\tan^{-1}(-x) = -\tan^{-1}x$, for all $x \in$ R

70 $cosec^{-1}(-x) = -cosec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

71 $\sec^{-1}(-x) = \pi - \sec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

72 $\cot^{-1}(-x) = \pi - \cot^{-1}x$, for all $x \in$ R

73 $\sin^{-1}\left(\frac{1}{x}\right) = cosec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

74 $\cos^{-1}\left(\frac{1}{x}\right) = sec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

75 $$tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} cot^{-1}x, & for\ x > 0 \\ -\pi + \cot^{-1}x, & for\ x < 0 \end{cases}$$

76 $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, for all $x \in [-1, 1]$

77 $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, for all $x \in$ R

78 $\sec^{-1}x + \mathrm{cosec}^{-1}x = \frac{\pi}{2}$, for all $x \in (-\infty, -1] \cup [1, \infty)$

79
$$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ x > 0,\ y > 0\ and\ xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ x < 0,\ y < 0\ and\ xy > 1 \end{cases}$$

80
$$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ x > 0,\ y < 0\ and\ xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ x < 0,\ y > 0\ and\ xy < -1 \end{cases}$$

81 If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n \in$ R, then

$$\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots\ldots + \tan^{-1}x_n = \left(\frac{S_1 - S_3 + S_5 - S_7 + \cdots\ldots}{1 - S_2 + S_4 - S_6 + S_8 + \cdots\ldots}\right),$$

where S_k denotes the sum of the products of $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ taken k at a time.

82 $\sin^{-1}x + \sin^{-1}y$

$$= \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), & if\ -1 \le x, y \le 1\ and\ x^2 + y^2 \le 1 \\ & or \\ & if\ xy < 0\ and\ x^2 + y^2 > 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if\ 0 < x, y \le 1 and\ x^2 + y^2 > 1 \\ -\pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if\ -1 \le x, y < 0\ and\ x^2 + y^2 > 1 \end{cases}$$

83 $\sin^{-1} x - \sin^{-1} y$

$$= \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), & if -1 \le x, y \le 1 \; and \; x^2+y^2 \le 1 \\ & or \\ & if \; xy > 0 \; and \; x^2+y^2 > 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & if \; 0 < x \le 1, -1 \le y \le 0 \; and \; x^2+y^2 > 1 \\ -\pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if -1 \le x < 0, 0 < y \le 1 \; and \; x^2+y^2 > 1 \end{cases}$$

84 $\cos^{-1} x + \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x+y \ge 0 \\ 2\pi - \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x+y \le 0 \end{cases}$$

85 $\cos^{-1} x - \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x \le y \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right), & if \;\; 0 \le x \le 1, -1 \le y \le 0 \; and \; x \ge y \end{cases}$$

86

$$2\sin^{-1} x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if \; -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if \; \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$$

87

$$3\sin^{-1} x = \begin{cases} \sin^{-1}(3x - 4x^3), & if \; -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3), & if \; \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x - 4x^3), & if -1 \le x < -\frac{1}{2} \end{cases}$$

88

$$2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2 - 1), & if \; 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), & if -1 \le x \le 0 \end{cases}$$

89

$$3\cos^{-1} x = \begin{cases} \cos^{-1}(4x^3 - 3x), & if \; \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x), & if \; -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x), & if \; -1 \le x \le -\frac{1}{2} \end{cases}$$

90

$$2\tan^{-1} x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; x < -1 \end{cases}$$

91

$$3\tan^{-1} x = \begin{cases} \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; x < -\frac{1}{\sqrt{3}} \end{cases}$$

92

$$2\tan^{-1} x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; x < -1 \end{cases}$$

93 $2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if\ 0 \le x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if\ -\infty < x \le 0 \end{cases}$

94 $\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$= \text{cosec}^{-1}\left(\frac{1}{x}\right)$

95 $\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right)$

$= \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

96 $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right)$

$= \text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

IX. Calculus

1 $\lim_{x\to a} f(x)$ exists $\Leftrightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

2 Let $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$. If l and m both exist, then

(i) $\lim_{x\to a} kf(x) = k\ \lim_{x\to a} f(x)$

(ii) $\lim_{x\to a}(f \pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = l \pm m$

(iii) $\lim_{x\to a}(fg)(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x) = lm$

(iv) $\lim_{x\to a}\left(\frac{f}{g}\right)(x) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{l}{m}$, $m \neq 0$.

(v) $\lim_{x\to a}\{f(x)\}^{g(x)} = l^m$

3. $\lim_{x\to a}\frac{x^n - a^n}{x - a} = n\,a^{n-1}$

4. $\lim_{x\to 0}\frac{sinx}{x} = 1$

5. $\lim_{x\to 0}\frac{tanx}{x} = 1$

6. $\lim_{x\to a}\frac{\sin(x-a)}{x-a} = 1$

7. $\lim_{x\to a}\frac{\tan(x-a)}{x-a} = 1$

8. $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$

9. $\lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a, \qquad a > 0$

10. $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$

11. $\frac{d}{dx}(x^n) = nx^{n-1}$

12. $\frac{d}{dx}(x) = 1$

13. $\frac{d}{dx}(e^x) = e^x$

14. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

15. $\frac{d}{dx}(sin\ x) = \cos x$

16. $\frac{d}{dx}(cos\ x) = -\sin x$

17. $\frac{d}{dx}(tan\ x) = \sec^2 x$

18. $\frac{d}{dx}(cot\ x) = -cosec^2\, x$

19. $\frac{d}{dx}(sec\ x) = \sec x \tan x$

20. $\frac{d}{dx}(cosec\ x) = -\,\text{cosec}\, x \cot x$

21. $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0\ a \neq 1$

22. $\frac{d}{dx}(c) = 0, where\ c\ is\ a\ constant\ function$

23. $\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

24 $\frac{d}{dx}\{f(x) \times g(x)\} = g(x)\frac{d}{dx} f(x) + f(x)\frac{d}{dx} g(x)$

25 $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{\{g(x)\}^2}$

26. A real valued function $f(x)$ defined on (a, b) is said to be differentiable at $x = c \in (a, b)$, iff $\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ $exists\ finitely$

$$\Leftrightarrow \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

$$\Leftrightarrow \lim_{h \to 0^-} \frac{f(c - h) - f(c)}{-h} = \lim_{h \to 0^+} \frac{f(c + h) - f(c)}{h}$$

$$\Leftrightarrow (LHD\ at\ x = c) = (RHD\ at\ x = c)$$

27 *Rolle's Theorem* – Let *f* be a real valued function defined on the closed interval $[a, b]$ such that

(i) It is continuous on the closed interval $[a, b]$,
(ii) It is differentiable on the open interval (a, b).
(iii) $f(a) = f(b)$.

Then, there exists at least one real number $c \in (a, b)$ such that $f'(c) = 0$.

28 *Lagrange's Mean Value Theorem* – Let *f* be a function defined on $[a, b]$ such that

(i) It is continuous on $[a, b]$,
(ii) It is differentiable on (a, b).

Then, there exists at least one real number $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

29. If $P(x_1, y_1)$ is a point on the curve $y = f(x)$, then

$y - y_1 = \left(\frac{dy}{dx}\right)_P (x - x_1)$ is the equation of tangent at P.

$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_P}(x - x_1)$ is the equation of the normal at P.

30. The angle between the tangents to two given curves at their point of intersection is defined as the angle of intersection of two curves. If C_1 and C_2 are two curves having equations $y = f(x)$ and $y = g(x)$ respectively such that they intersect at point P. The angle θ of intersection of these two curves is given by

$$\tan\theta = \frac{\left(\frac{dy}{dx}\right)_{C_1} \sim \left(\frac{dy}{dx}\right)_{C_2}}{1+\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

If the angle of intersection of two curves is a right angle, then the curves are said to intersect orthogonally. The condition for orthogonality of two curves C_1 and C_2 is

$$\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$$

31. Two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ will intersect orthogonally, if

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$$

32. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

33. $\int \frac{1}{x} \, dx = log_e x + C$

34. $\int a^x \, dx = \frac{a^x}{log_e a} + C, \quad a \neq -1, a > 0$

35. $\int e^x \, dx = e^x + C$

36. $\int \sin x \, dx = -\cos x + C$

37. $\int \cos x \, dx = \sin x + C$

38. $\int \sec^2 x \, dx = \tan x + C$

39. $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

40. $\int \sec x \tan x \, dx = \sec x + C$

41. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$

42. $\int tan\, x \, dx = log\, |sec\, x| + C$

$= -\log \cos x + C$

43. $\int cot\, x \, dx = log\, |sin\, x| + C$

44. $\int sec\, x \, dx = log\, |sec\, x + tan\, x| + C = log \left|tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

45. $\int cosec\, x \, dx = log\, |cosec\, x - cot\, x| + C = log \left|tan \frac{x}{2}\right| + C$

46. $\int (ax+b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$

47. $\int \frac{1}{ax+b} \, dx = \frac{1}{a} log_e\, |ax+b| + C$

48. $\int a^{bx+c} \, dx = \frac{1}{b} \frac{a^{bx+c}}{log_e a} + C, \quad b > 1$

49. $\int e^{bx+c} \, dx = \frac{1}{b} e^{bx+c} + C$

50. $\int \sin(ax+b) \, dx = -\frac{1}{a} \cos(ax+b) + C$

51. $\int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + C$

52. $\int \tan(ax+b) \, dx = \frac{1}{a} log\, |sec\,(ax+b)| + C$

53. $\int \cot(ax+b) \, dx = \frac{1}{a} log\, |sin\,(ax+b)| + C$

54. $\int \sec^2(ax+b) \, dx = \frac{1}{a} \tan(ax+b) + C$

55. $\int \operatorname{cosec}^2(ax+b) \, dx = -\frac{1}{a} \cot(ax+b) + C$

56. $\int \sec(ax+b) \tan(ax+b) \, dx = \frac{1}{a} \sec(ax+b) + C$

57. $\int \text{cosec } (ax+b) \cot (ax+b) \; dx = -\frac{1}{a} \text{cosec } (ax+b) + C$

58. $\int \sec (ax+b) \; dx = \frac{1}{a} \; log \; |sec(ax+b) + tan \; (ax+b)| + C$

59. $\int \text{cosec } (ax+b) \; dx = \frac{1}{a} \log |\text{cosec}(ax+b) - \cot (ax+b)| + C$

60. $\int \frac{1}{a^2 + x^2} \; dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

61. $\int \frac{1}{x^2 - a^2} \; dx = \frac{1}{2a} \; log \left|\frac{x - a}{x + a}\right| + C$

62. $\int \frac{1}{a^2 - x^2} \; dx = \frac{1}{2a} \; log \left|\frac{a + x}{a - x}\right| + C$

63. $\int \frac{1}{\sqrt{a^2 - x^2}} \; dx = \sin^{-1} \frac{\mathrm{x}}{a} + C$

64. $\int \frac{1}{\sqrt{a^2 + x^2}} \; dx = log \; \left|x + \sqrt{a^2 + x^2}\right| + C$

65. $\int \frac{1}{\sqrt{x^2 - a^2}} \; dx = log \; \left|x + \sqrt{x^2 - a^2}\right| + C$

66. $\int \sqrt{a^2 - x^2} \; dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{\mathrm{x}}{a} + C$

67. $\int \sqrt{x^2 - a^2} \; dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \; log \; \left|x + \sqrt{x^2 - a^2}\right| + C$

68. $\int \sqrt{x^2 + a^2} \; dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \; log \; \left|x + \sqrt{x^2 + a^2}\right| + C$

69. If u and v are two functions of x, then

$$\int uv \; dx = u(\int v \; dx) - \int \left\{\tfrac{du}{dx} \int v \; dx\right\} dx$$

70. $\int e^x\{f(x) + f'(x)\} \, dx = e^x f(x) + C$

71. $\int e^{kx}\{k f(x) + f'(x)\} \, dx = e^{kx} f(x) + C$

72. $\int e^{ax} \sin(bx+c) \; dx = \frac{e^{ax}}{a^2 + b^2}\{a \sin(bx+c) - b \cos (bx+c)\} + \lambda$

73. $\int e^{ax} \cos (bx+c) \; dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos (bx+c) + b \sin(bx+c)\} + \lambda$

74 $\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} \; dx = M.\log(c \cos x + d \sin x) + N.x + C$

where $N^r = M \frac{d}{dx}(D^r) + N.D^r$

75 $\int \frac{a \cos x + b \sin x + c}{d \cos x + e \sin x + f} \; dx$

$$= M.\log(d \cos x + e \sin x) + N.x + \int \frac{dx}{d \cos x + e \sin x + f}$$

where $N^r = M \frac{d}{dx}(D^r) + N.D^r + P$

76 $\int \frac{x^2+1}{x^4+1}\,dx$ OR $\int \frac{1}{x^4+x^2+1}\,dx$ OR $\int \frac{x^2}{x^4+x^2+1}\,dx$

$\Leftrightarrow k\int \frac{1 \mp \frac{1}{x^2}}{\left(x \pm \frac{1}{x}\right)^2 \mp 2}\,dx \;\Leftrightarrow\; k\int \frac{dt}{t^2 \mp (\sqrt{2})^2}$

77 $\int \frac{dx}{P\sqrt{Q}}$ (i) If P & Q both are linear, put Q = t^2

(ii) If P is quadratic & Q is linear, put Q = t^2

(iii) If Q is quadratic, P is linear, put P = $\frac{1}{t}$

(iv) If P & Q are both pure quadratic put $x = \frac{1}{t}$.

78. $\int_a^b f(x)dx = \int_a^b f(t)dt$, i.e. integration is independent of the change of variable.

79. $\int_a^b f(x)dx = -\int_b^a f(x)dx$, i.e. if the limits of a definite integral are interchanged, then its value changes by minus sign only.

80. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.

The above property can be generalized into the following form

$$\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \ldots\ldots + \int_{c_n}^b f(x)dx$$

Where $a < c_1 < c_2 < c_3 \ldots\ldots < c_{n-1} < c_n < b$.

81. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

82. $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(x)\ is\ an\ even\ function \\ 0, & if\ f(x)\ is\ an\ odd\ function \end{cases}$

83. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0, & if\ f(2a-x) = -f(x) \end{cases}$

84. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

85. (a) If a differential equation is expressible in the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of *x*, then it is called a linear differential equation.

The solution of this equation is given by

$$y\left(e^{\int P\,dx}\right) = \int\left(Q\, e^{\int P\,dx}\right)dx + C$$

(b) If a differential equation is in the form $\frac{dx}{dy} + Rx = S$, where R and S are functions of *y,* the solution of this equation is given by

$$x\left(e^{\int R\,dy}\right) = \int\left(S\, e^{\int R\,dy}\right)dy + C$$