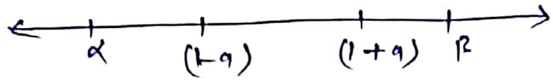
15. Q. For what real values of 'a' do the roots of eqn $x^2 - 2x - (a^2-1) = 0$ lie betn the roots of eqn $x^2 - 2(a+1)x + a(a-1) = 0$.

Solⁿ:- Now, solving $x^2 - 2x - (a^2-1) = 0 \Rightarrow x = 1 \pm a$.

Let, the roots of $x^2 - 2(a+1)x + a(a-1) = 0$ are, α & β.



so, $1 \cdot f(1-a) < 0$ & $1 \cdot f(1+a) < 0$

so, $(1-a)^2 - 2(a+1)(1-a) + a(a-1) < 0 \Rightarrow (a-1)\{(a-1) + 2(a+1) + a\} < 0$

or $(a-1)(4a+1) < 0 \Rightarrow \boxed{a \in (-1/4, 1)}$.

16. Q. If $(\alpha, \beta), (\beta, \gamma), (\gamma, \alpha)$ be the roots of $a_i x^2 + b_i x + c_i = 0$ $\forall\ i = 1, 2, 3$ resp. then find

$$\sqrt{\prod_{i=1}^{3} \left(\frac{a_i - b_i + c_i}{a_i}\right)} - \alpha - \beta - \gamma - \alpha\beta - \beta\gamma - \alpha\gamma - \alpha\beta\gamma.$$

Solⁿ:- Here, $\prod_{i=1}^{3} \left(\frac{a_i - b_i + c_i}{a_i}\right) = \prod_{i=1}^{3} \left(1 - \frac{b_i}{a_i} + c_i/a_i\right)$

$= \left(1 - b_1/a_1 + c_1/a_1\right)\left(1 - b_2/a_2 + c_2/a_2\right)\left(1 - b_3/a_3 + c_3/a_3\right)$

$= (1+\alpha)^2 (1+\beta)^2 (1+\gamma)^2$

so, $\sqrt{\prod_{i=1}^{3} \left(\frac{a_i - b_i + c_i}{a_i}\right)} = (1+\alpha)(1+\beta)(1+\gamma)$.

so, $\boxed{\sqrt{\prod_{i=1}^{3} \left(\frac{a_i - b_i + c_i}{a_i}\right)} - \alpha - \beta - \gamma - \alpha\beta - \beta\gamma - \alpha\gamma - \alpha\beta\gamma = 1}$

17. Q. Find no. of possible triplets (a, b, c) if $x^2 - (12+a)x + 12a + 2 = (x+b)(x+c)$. where b & c are integers.

Solⁿ:- Here, $-b$ & $-c$ must be their roots.

$\therefore$ 'D' must be perfect sq.

$= (a+12)^2 - 4(12a+2)$

$= (a-12)^2 - 8$

$\therefore$ $a-12 = \pm 3$ i.e. $a = 15$ or, 9.

for $a = 15 \Rightarrow x = 14, 13 \Rightarrow b = -14, c = -13$

for $a = 9 \Rightarrow b = -11$ or, -10

$\therefore$ possible triplets are $(15, -14, -13)$, $(15, -13, -14)$, $(9, -11, -10)$ & $(9, -10, -11)$.

18. The value of 'p' for which both roots of the eq^n: $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2, lies in

→ Students must try this.

19. Let 's' be the set of values of 'a' for which 2 lies betn the roots of the quad. eq^n: $x^2 + (a+2)x - (a+3) = 0$, then 's' lies in—

→ Students must try this.

20. If 'a' be a positive integer, the no. of values of a satisfying

$$\int_0^{\pi/2} \left[a^2\left(\frac{\cos 3x}{4} + \frac{3}{4}\cos x\right) + a\sin x - 20\cos x\right] dx \le -\frac{a^2}{3} \text{ is}$$

Soln: Here,

$$\left[a^2\left(\frac{\sin 3x}{12} + \frac{3\sin x}{20}\right) - a\cos x - 20\sin x\right]_0^{\pi/2} \le -\frac{a^2}{3}$$

or, $\left\{a^2\left(-\frac{1}{12} + \frac{3}{20}\right) - 0 - 20\right\} - \left\{0 - a - 0\right\} \le -\frac{a^2}{3}$

or $\frac{a^2}{15} - 20 + a \le -\frac{a^2}{3}$

i.e. $\boxed{a = 1, 2, 3, 4.}$

21. If t_n denotes the n^{th} terms of an A.P. & $t_p = 1/q$ & $t_q = 1/p$, then which of the following is necessarily a root of the eq^n

$(p + 2q - 3r)x^2 + (q + 2r - 3p)x + (r + 2p - 3q) = 0$ is ...

(a) t_p (b) t_q (c) t_{pq} (d) t_{p+q}.

Soln:. $t_p = A + (p-1)D = 1/q$, $t_q = A + (q-1)D = 1/p$

so, $t_{pq} = 1$

Also, $x = 1$ is the root of above eq^n so, $\boxed{\text{option 'c'}}$

Concept of polynomials :-

→ Degree = Number of roots.

→ $ax^2+bx+c=0$ has only 2 roots.

Note:-
If two polynomial expressions have same roots, it's not necessary that both expressions are same.
i.e. two polynomials are equal only if their corresponding co-efficients are equal.

ex:- $x^2-5x+6=0 \rightarrow 2,3$

$2x^2-15x+18=0 \rightarrow 2,3$

But $x^2-5x+6 \neq 2x^2-15x+18$

⇒ If $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 = 0$ has roots $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ then,

$$\boxed{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 = a_n(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)}$$

Relation betn roots and co-efficients :-

consider a polynomial,

$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 = 0$ has roots $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$. ------ (i)

so, $(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_n) = 0$ ---- (ii)

Also, $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 = a_n(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$

or $x^n + \frac{a_{n-1}}{a_n}x^{n-1} + \frac{a_{n-2}}{a_n}x^{n-2} + \cdots + \frac{a_0}{a_n} = (x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$

Now, comparing co-eff. of x^{n-1} on both sides,

i.e. $$\boxed{\alpha_1+\alpha_2+\alpha_3+\cdots+\alpha_n = -\frac{a_{n-1}}{a_n}}$$

comparing co-eff. of x^{n-2} on both sides,

i.e. $$\boxed{\underbrace{\alpha_1\alpha_2+\alpha_2\alpha_3+\cdots}_{{}^nC_2 \text{ elements.}} = \frac{a_{n-2}}{a_n}}$$

comparing co-efficient of x^{n-3},

$$\boxed{\underbrace{\alpha_1\alpha_2\alpha_3+\alpha_2\alpha_3\alpha_4+\cdots}_{{}^nC_3 \text{ elements.}} = -\frac{a_{n-3}}{a_n}}$$

and so, on - - -

comparing ~~coefficient of x~~ constant term,

i.e. $\boxed{\alpha_1\alpha_2\alpha_3 \cdots \alpha_n = \frac{a_0}{a_n}(-1)^n}$

Q.22. If roots of $x^6 - 12x^5 + ax^4 + 2bx^3 + 3cx^2 + 4dx + 64 = 0$ are all positive then find $a+b+c+d$.

Sol^n:- let $\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_6$ are the roots of given eq^n

$$\therefore \alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_6 = 12 \quad \& \quad \alpha_1\alpha_2\alpha_3 \cdots \alpha_6 = 64$$

$$\therefore \frac{\alpha_1 + \alpha_2 + \cdots \alpha_6}{6} = 2 = (\alpha_1\alpha_2\alpha_3 \cdots \alpha_6)^{1/6}$$

i.e. AM = GM

which is possible, iff $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_6 = 2$

Then $a = \underbrace{\alpha_1\alpha_2 + \alpha_2\alpha_3 + \cdots}_{^6C_2 \text{ times}} = 4 \times 15 = 60$

$$b = \frac{2^3 \times 20}{-2} = -80$$

III^{ly}, $c = 80$

$d = -48$

so, $a+b+c+d = 60 - \cancel{80} + \cancel{80} - 48 = \boxed{12}$

Q.23. If a, b, c, d are the roots of $x^4 + P_1x^3 + P_2x^2 + P_3x + P_4 = 0$ then (i) $(1-a^2)(1-b^2)(1-c^2)(1-d^2) = ?$

(ii) $(1+a^2)(1+b^2)(1+c^2)(1+d^2) = ?$

Sol^n:- $\therefore x^4 + P_1x^3 + P_2x^2 + P_3x + P_4 = (x-a)(x-b)(x-c)(x-d)$.

putting $x = 1$,

$$1 + P_1 + P_2 + P_3 + P_4 = (1-a)(1-b)(1-c)(1-d) \quad \text{--- (i)}$$

putting $x = -1$,

$$1 - P_1 + P_2 - P_3 + P_4 = (1+a)(1+b)(1+c)(1+d) \quad \text{--- (ii)}$$

(i) × (ii)

i.e. $(1 + P_2 + P_4)^2 - (P_1 + P_3)^2 = (1-a^2)(1-b^2)(1-c^2)(1-d^2)$

(i) $\boxed{(1-a^2)(1-b^2)(1-c^2)(1-d^2) = (1+P_2+P_4)^2 - (P_1+P_3)^2}$

(ii) putting $x = i$

$\therefore \quad 1 - iP_1 - P_2 + P_3 i + P_4 = (i-a)(i-b)(i-c)(i-d)$ --- (i')

putting $x = -i$

$1 + P_1 i - P_2 - P_3 i + P_4 = (i+a)(i+b)(i+c)(i+d)$ --- (ii')

(i')×(ii')

$$\boxed{(1 - P_2 + P_4)^2 + (P_1 + P_3)^2 = (1+a^2)(1+b^2)(1+c^2)(1+d^2)}$$

Note:-

let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0, \; a_n \neq 0$.

value of polynomial at $x = \infty$ or, $-\infty$ depends upon a_n & n only.

$$\therefore \quad P(x) = x^n \left\{ a_n + \frac{a_{n-1}}{x} + \cdots + \frac{a_0}{x^n} \right\}$$

$$\lim_{x \to \infty} f(x) = \begin{cases} \infty, & a_n > 0 \\ -\infty, & a_n < 0 \end{cases}$$

$$\text{Illy} \quad \lim_{x \to -\infty} f(x) = \begin{cases} \infty, & n \to \text{even} \;\&\; a_n > 0 \text{ or, } n \to \text{odd} \;\&\; a_n < 0 \\ -\infty, & n \to \text{odd} \;\&\; a_n > 0 \text{ or } n \to \text{even} \;\&\; a_n < 0. \end{cases}$$

Students should remember above result:-

Descarte's Rule of Signs:-

"The no. of positive real roots of a polynomial is bounded by the number of changes of sign in its coefficients."

"The no. of negative real roots of a polynomial of $f(x)$ is bounded by the no. of changes of sign in the coefficients of $f(-x)$.

for ex:- $f(x) = x^4 + 7x^3 - 4x^2 - x - 7$.

possible no. of '+ve' real roots $= 1$ as only one sign change occurs i.e

$+ \; + \; - \; - \; -$

Now, $f(-x) = x^4 - 7x^3 - 4x^2 + x - 7$.

$\therefore \quad + \; - \; - \; + \; - \Rightarrow 3$ changes in sign.

So, no. of negative real roots of $f(x) = 2$.

Note!- +ve & -ve roots are may not be exact, it shows only max^m. possible no. of roots