Q.8. Roots of $x^2-4x+\alpha=0$ are a, b & roots of $x^2-36x+\beta=0$ are c, d such that a, b, c, d are in G.P., then find $\alpha+\sqrt{\beta}$.

Sol^n:- Here, let, $a=a$, $b=ak$, $c=ak^2$, $d=ak^3$

$$\therefore (a+b)=4 \Rightarrow a(1+k)=4$$

$$\text{Also, } ak^2(1+k)=36$$

$$\therefore k=\pm 3.$$

For $k=3$, $a=1$, $b=3$, $c=9$, $d=27$.

$$\text{i.e. } \boxed{\alpha+\sqrt{\beta}=3+9\sqrt{3}}$$

For $k=-3$, $\sqrt{\beta}$ is not defined.

Q.9. The curve $y=(\lambda+1)x^2+2$ intersects the curve $y=\lambda x+3$ in exactly one point, if $\lambda = \dots$

Sol^n:- Here, $(\lambda+1)x^2+2=\lambda x+3 \Rightarrow (\lambda+1)x^2-\lambda x-1=0$

$$\therefore D=0 \Rightarrow \lambda^2+4(\lambda+1)=0$$

$$\text{or } \lambda^2+4\lambda+4=0$$

$$\text{i.e. } \boxed{\lambda=-2}$$

Q.10. If the eq^n: $ax^2+2bx-3c=0$ has non-real roots & $\left(\frac{3c}{4}\right)<a+b$; then 'c' is always.

(a) <0 ✓ (b) >0 (c) ≥ 0 (d) zero.

Sol^n:- Here, $3c/4 < a+b \Rightarrow 4a+4b-3c>0$

$$\text{i.e. } f(2)>0 \Rightarrow f(0)>0$$

$$\therefore \boxed{c<0}$$

Q.11. If α, β be the roots of $375x^2-25x-2=0$ & $s_n=\alpha^n+\beta^n$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n} s_r = ?$

Sol^n:- Here, $\alpha+\beta=25/375$ & $\alpha\beta=-2/375$

Now, $$\lim_{n\to\infty}\sum_{r=1}^{n} s_r = (\alpha+\alpha^2+\alpha^3+\dots\alpha^n+\dots\infty)+(\beta+\beta^2+\beta^3+\dots+\beta^n+\dots\infty)$$

$$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}$$

$$\text{i.e. } \boxed{\lim_{n\to\infty}\sum_{r=1}^{n} s_r = \frac{1}{2}}$$

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Remainder Theorem :-

- If $p(x)$ is divided by $(x-\alpha)$ then the remainder is $p(\alpha)$ & we can write $p(x) = (x-\alpha)\,q + R$ i.e. $R = p(\alpha)$
- If $p(x)$ is divided by $(x-\alpha)(x-\beta)$ then, $p(x) = (x-\alpha)(x-\beta)\,q(x) + ax+b$. Here, a & b are unknowns which can be found by putting $x=\alpha$ & $x=\beta$.
- If $p(x)$ is divided by $(x-\alpha)(x-\beta)(x-\gamma)$ then, $p(x) = (x-\alpha)(x-\beta)(x-\gamma)\,q(x) + ax^2+bx+c$. Here, unknowns are a, b & c; can be found by putting $x=\alpha$, $x=\beta$ & $x=\gamma$.

Q.12. Find the value of: $1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots\infty}}}$

Solⁿ:- Let, $x = 1 + \cfrac{1}{1+\cfrac{1}{1+\cdots}}$ $\Rightarrow$ $x-1 = 1/x$ $\Rightarrow x = \frac{1+\sqrt{5}}{2}$ or $\frac{1-\sqrt{5}}{2}$

As 'x' is positive, $x = \frac{1+\sqrt{5}}{2}$

Note:-

Given $p(x)$ is divided by $(x-\alpha)^k$ & remainder is constant as 'c', then $p(x) = (x-\alpha)^k\,q(x) + c$.

$$\therefore\ c = p(\alpha).$$

Q.12. Find the remainder where $x^{2019} - 5.x^{201} + 3x^{21} - 7x^4 + 2x^2 + 3x - 9 = 0$ is divided by (x^2-x).

Solⁿ:- Here, let $f(x) = x^{2019} - 5.x^{201} + 3x^{21} - 7x^4 + 2x^2 + 3x - 9$

$$\therefore\ f(x) = (x^3-x)\,g(x) + (ax^2+bx+c) \quad ---- (1)$$

putting, $x = 0$ in (1),

$c = -9$

putting $x=1$ in (1),

$-12 = a+b+c \Rightarrow a+b = -3$

putting $x=-1$ in (1),

$-7 = a-b$

Now, $a = -5$, $b = 2$ & $c = -9$.

So, $\boxed{R = -5x^2+2x-9}$.

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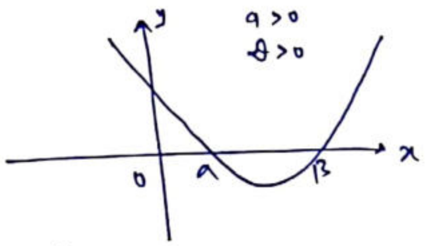
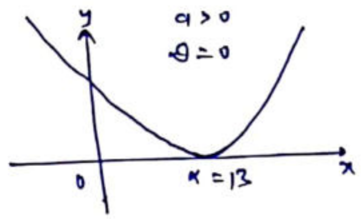
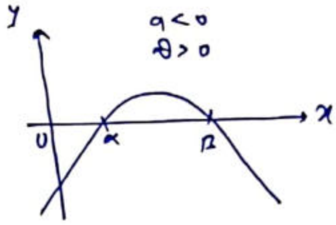
Location of Roots :-

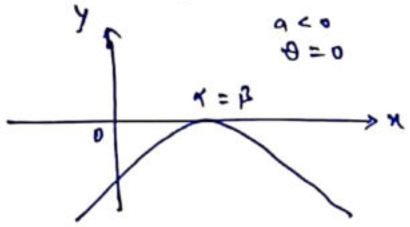
let $f(x) = ax^2 + bx + c = a(x-\alpha)(x-\beta)$.

(i) Both roots positive :









$\therefore$ $D \geq 0$ & sum of roots > 0 & prod. > 0.

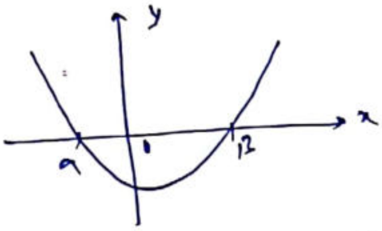
i.e. $\boxed{D \geq 0,\ -b/a > 0\ \&\ c/a > 0}$.

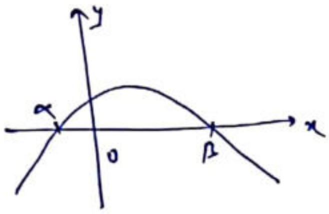
(ii) Both roots negative :

$\therefore$ $D \geq 0$, sum of roots < 0 & prod. > 0.

i.e. $\boxed{D \geq 0,\ -b/a < 0\ \&\ c/a > 0}$.

(iii) Roots of eq^n are opposite in sign :-

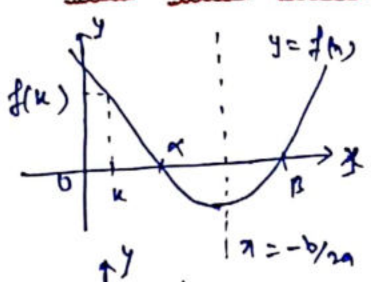


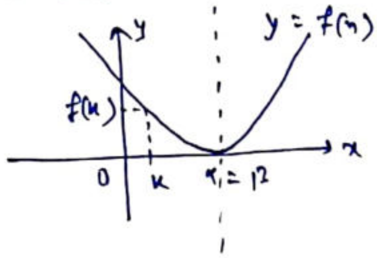


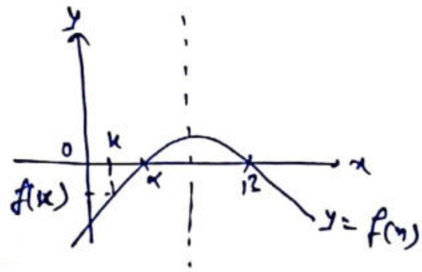
Here, we can't say anything about sum of roots. But definitely we can say prod. of roots is negative.

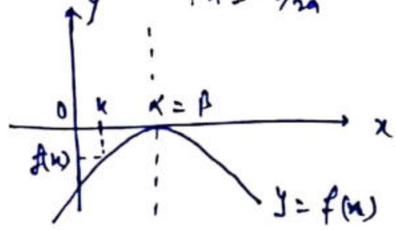
i.e. $c/a < 0$

(iv) Both roots greater than 'k' :



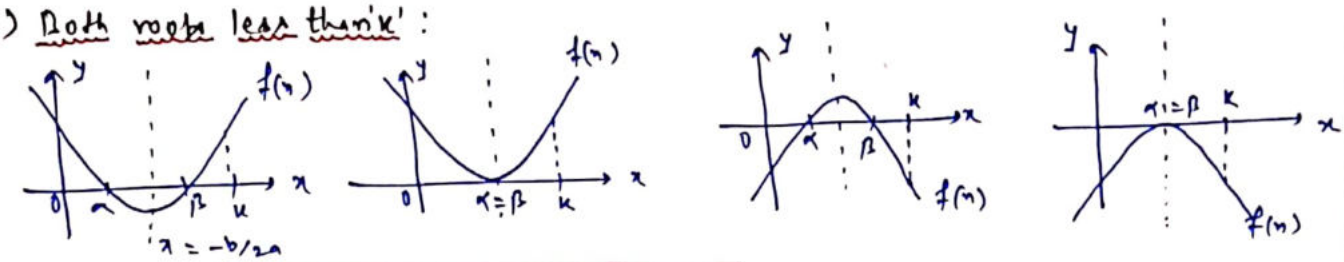






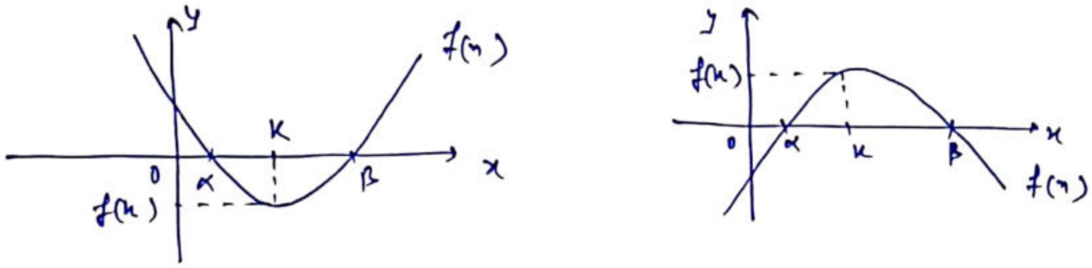
i.e. $\boxed{D \geq 0,\ -\frac{b}{2a} > k,\ a f(k) > 0}$.

(v) Both roots less than 'k':



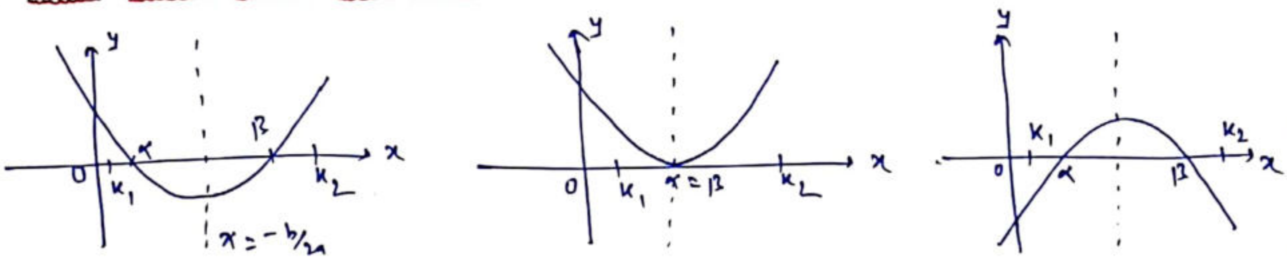
i.e. $\boxed{D \geq 0,\ a f(k) > 0,\ -b/2a < k}$

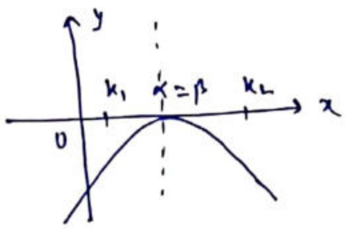
(vi) one is less than k & other is greater than k:



i.e. $\boxed{a f(k) < 0}$

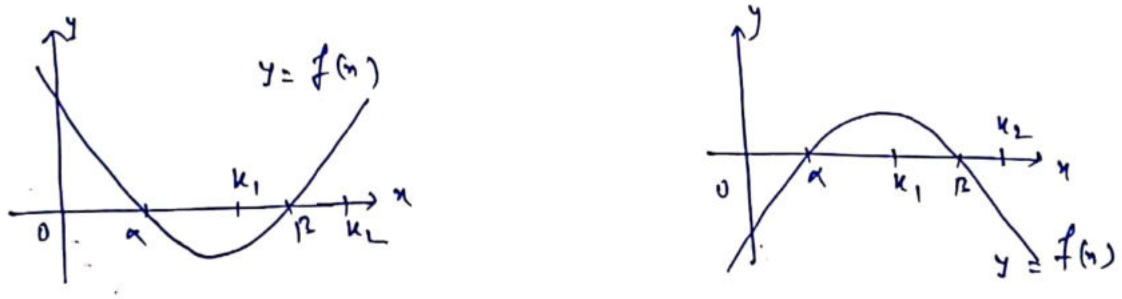
(vii) Both roots betn k_1 & k_2 :-





i.e. $\boxed{D \geq 0,\ k_1 < -\frac{b}{2a} < k_2,\ a f(k_1) > 0,\ a f(k_2) > 0}$

(viii) Exactly one root lies betn k_1 & k_2 :



$\therefore$ $\boxed{f(k_1) \cdot f k_2 < 0}$

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Q.14. Find the values of parameter 'a' for which the roots of quad eqⁿ. $x^2+2(a-1)x + (a+5)=0$ are

(i) Real & distinct

(ii) Equal (iii) Not equal (iv) opposite in sign

(v) Equal in magnitude but opp. in sign. (vi) positive (vii) negative

(viii) Such that one root is greater than 3 other is smaller than 3.

(ix) Greater than 3 (x) Smaller than 3.

(xi) Such that exactly one root lie in the interval (1, 3).

(xii) " " one root is greater than 3 & other smaller than 1.

Solⁿ :- (i) Here, given eqⁿ. is $x^2+2(a-1)x+(a+5)=0$

for real & distinct. $D>0 \Rightarrow 4(a-1)^2-4(a+5)>0$

or $4(a^2-2a+1)-4a-20>0$

or $4a^2-8a+4-4a-20>0$

or $4a^2-12a-16>0 \Rightarrow a^2-3a-4>0 \Rightarrow (a+1)(a-4)>0$

so, $\boxed{a \in (-\infty,-1) \cup (4,\infty)}$

(v) prod. of roots < 0 & sum of roots = 0.

$\therefore \frac{(a+5)}{1}<0 \Rightarrow a<-5$ ---- (A)

$-\frac{2(a-1)}{1}=0 \Rightarrow a=1$ --- (B)

$A \cap B \Rightarrow \phi$

so, $\boxed{a \in \phi}$

(vi) for this $D \geq 0$, $-b/a>0$ & $c/a>0$

$D \geq 0 \Rightarrow a \in (-\infty,-1] \cup [4,\infty)$. ---- (A)

$-b/a>0 \Rightarrow -\frac{2(a-1)}{1}>0 \Rightarrow a-1<0 \Rightarrow a \in (-\infty,1)$. --- (B)

& $c/a>0 \Rightarrow a+5>0 \Rightarrow a>-5$ --- (C)

so, $A \cap B \cap C \Rightarrow \boxed{a \in (-5,-1]}$

* Students must try rest cases.

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