QUADRATIC EQUATION :-

Polynomial of degree 'n' :-

The eqn. of the form $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 = 0$, where $a_n, a_{n-1}, a_{n-2}, \cdots a_0$ are real constants & $a_n \neq 0$, $n \in W$ is known as polynomial of degree 'n'.

Quadratic eqn :-

The eqn of the form $ax^2 + bx + c = 0$, $a \neq 0$ is known as quad. eqn.

Here, $a^2x^2 + abx + ac = 0$

$$\Rightarrow \left(ax + b/2\right)^2 = b^2/4 - ac$$

$$\therefore x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a}, \quad x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a}$$

So, $\boxed{x_1 + x_2 = -b/a}$ & $\boxed{x_1 x_2 = c/a}$

Note :- If roots of a quad. eqn. are α & β then we can form eqn. as $\boxed{k\{x^2 - (\alpha+\beta)x + \alpha\beta\} = 0}$

Some important points :-

(i) If '$b^2 - 4ac$' is perfect square then the roots are rational only if $a, b, c \in Q$.

(ii) If $D \neq$ perfect square even then, roots may or may not be rational, but certainly irrational if $a, b, c \in Q$.

(iii) If roots are irrational then the roots are irrational conjugate of the form $p + \sqrt{q}$ & $p - \sqrt{q}$

(iv) If $D = 0$, roots are real & equal ($a, b, c \in R$).

(v) If $D < 0$, roots are complex conjugate of the form $p+iq$ & $p-iq$.

(v) For real coefficients if $D = b^2 - 4ac > 0$, roots are real & Distinct

*(vi) For roots to be integral,
(a) $a = 1$, (b) b & $c \in I$ (c) 'D' must be perfect sq.

So, its form is $\boxed{x^2 + bx + c = 0}$.

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Concept :-
'b' may be even or, odd

Reason :- If 'b' is even, then b^2-4ac i.e. 'b^2-4c' is even

$\therefore -b \pm \sqrt{b^2-4c}$ is even.

If 'b' is odd, then 'b^2-4c' is odd

$\therefore -b \pm \sqrt{b^2-4c}$ is even.

Q.1. Roots of eqn: $x^2+px+q=0$ & $x^2+qx+p=0$ differ by a same constants quantity then find $p+q$.

Solⁿ:- Here, $|\alpha_1-\beta_1| = |\alpha_2-\beta_2|$

or, $(\alpha_1-\beta_1)^2 = (\alpha_2-\beta_2)^2$

or $p^2-4q = q^2-4p$

or $p^2-q^2 = 4(q-p)$

$\therefore \boxed{p+q = -4}$

Q2. If roots of $ax^2+bx+c=0$ are α & β then find quadratic eqn. whose roots are (i) $1/\alpha$, $1/\beta$ (ii) $\frac{\alpha-1}{\alpha+1}$, $\frac{\beta-1}{\beta+1}$.

Solⁿ:- (i) taking $x = 1/\alpha \Rightarrow \alpha = 1/x$

putting in $ax^2+bx+c=0 \Rightarrow a/x^2 + b/x + c = 0$

or, $a+bx+cx^2=0$

i.e. $\boxed{cx^2+bx+a=0}$

(ii) taking $x = \frac{\alpha-1}{\alpha+1} \Rightarrow \alpha x + x = \alpha - 1 \Rightarrow \alpha x - \alpha = -1-x$

or $\alpha(x-1) = -(1+x) \Rightarrow \alpha = \frac{x+1}{1-x}$

Now, by putting $\alpha = \frac{x+1}{1-x}$ in $a\alpha^2+b\alpha+c=0$

so, $\boxed{a\left(\frac{x+1}{1-x}\right)^2 + b\left(\frac{x+1}{1-x}\right) + c = 0}$

By expanding it, you can get the final eqn.

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Q.3. $a_1, a_2, a_3, ---a_{50}$ be the sequence of real no. such that 'a_k' is 'k' less than sum of other 49 terms then find the eqn. whose roots are :

(i) a_{20}, a_{30} (ii) a_1, a_2, a_3.

Solⁿ:- let, $s = a_1 + a_2 + a_3 + ---- + a_{50}$

$$\therefore a_1 = s - a_1 - 1$$
$$a_2 = s - a_2 - 2$$
$$a_3 = s - a_3 - 3$$
$$-----------$$
$$-----------$$
$$a_{50} = s - a_{50} - 50$$

$$s = 50s - s - \frac{50\times51}{2}$$

so, $\boxed{s = \frac{50\times51}{96}}$

Now, by finding a_{20} & a_{30}, we can form the eqⁿ easily.

Q.4. p, a, q are in G.P. p, b, c are in A.P. then roots of $bx^2 - 2ax + c = 0$ are :

(a) complex (b) real, both positive (c) real, both negative (d) real, opp. in sign.

Solⁿ:- Here, $a^2 = pq$, $2b = p + c$, $2c = b + q$.

$$\therefore D = 4(a^2 - bc) = 4(pq - bc) = 4\{(2b-c)(2c-b) - bc\}$$
$$= 4\{4bc - 2b^2 - 2c^2\}$$
$$= -8(b-c)^2 < 0$$

$\therefore$ roots are complex.

i.e. $\boxed{\text{option 'a'.}}$

Note :- $ax^2 + bx + c = 0$ satisfied by only 2 values. Hence, $ax^2 + bx + c = 0$ is satisfied by more than 2 values then its identity

i.e. $\boxed{a = b = c = 0}$.

(This condⁿ is known as an identity).

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5.Q. $(a^2-4)x^2+(ab+a-2b-2)x+b^3+1=0$. Find all possible ordered pairs (a,b) such that given is an identity. If all possible (a,b) together with origin form a polygon, find area of the polygon.

soln:- The given eqn becomes an identity, iff

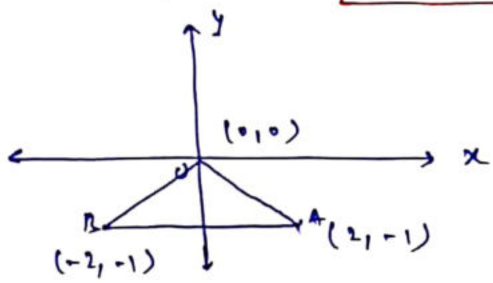
$$a^2-4=0 \;\&\; ab+a-2b-2=0 \;\&\; b^3+1=0$$

$$\therefore \quad a = \pm 2 \; --- \; (A)$$

$$a = -1, 2 \; --- \; (B)$$

$$b = -1 \; --- \; (C)$$

Hence, possible ordered pairs are $\boxed{(2,-1)\;,\;(-2,-1)}$.



so, area of $\Delta OAB = \frac{1}{2}\times 4\times 1 = \boxed{2 \text{ sq. units}}$.

Common Roots :-

(i) Exacty one root common :-

let, $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$ are two different quad. eqns. let 'α' be the common root

$$\therefore \quad a_1\alpha^2+b_1\alpha+c_1=0$$

$$\& \; a_2\alpha^2+b_2\alpha+c_2=0$$

By cross mul. method,

$$\therefore \quad \frac{\alpha^2}{b_1c_2-b_2c_1} = \frac{\alpha}{a_2c_1-a_1c_2} = \frac{1}{a_1b_2-a_2b_1}$$

i.e. $$\boxed{\frac{b_1c_2-b_2c_1}{a_2c_1-a_1c_2} = \frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}}$$

(ii) Both roots common :-

Let α & β be the common roots of the eqn. $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$.

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$\therefore \alpha+\beta = -b_1/a_1 = -b_2/a_2 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$ --- (i)

$\& \ \alpha \cdot \beta = c_1/a_1 = c_2/a_2 \Rightarrow \frac{a_1}{a_2} = c_1/c_2$ ---- (ii)

from (i) & (ii)

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

Q.6. If $x^2+px+qr=0$ & $x^2+qx+rp=0$ have exactly one root in common then prove that the eqn. formed by their other roots is $x^2+rx+pq=0$

Soln:- Here, $x^2+px+qr=0$

$x^2+qx+rp=0$

By cross multiplication,

$$\frac{\alpha^2}{p^2r-q^2r} = \frac{\alpha}{qr-rp} = \frac{1}{q-p}$$

$\therefore \ r=\alpha, \ \alpha = -(p+q)$

i.e. $r = -(p+q)$

Also, $\alpha+\beta_1 = -p \Rightarrow \alpha+\beta_1 = -p \Rightarrow r+\beta_1 = -p \Rightarrow \beta_1 = -(p+r)$

Similarly, $\beta_2 = -(q+r)$

$\therefore \beta_1 = q$ & $\beta_2 = p$.

so, eqn. having roots β_1 & β_2, $\Rightarrow x^2-(\beta_1+\beta_2)x+\beta_1\beta_2=0$

i.e. $x^2-(p+q)x+pq=0$

so, $\boxed{x^2+rx+pq=0}$

~~Q.~~ Note!- In $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$, if given that one root is common & any one eqn. has unreal or irrational roots then both eqns must have both roots common i.e. in case of conjugate roots! $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Q.7. eqns $ax^2+bx+c=0$ & $2x^2+4x+7=0$ have a common root then find $a:b:c$.

soln:- Here, roots of $2x^2+4x+7=0$ are imaginary as $D<0$. so both eqns has both roots common. so, $\boxed{a:b:c = 2:4:7}$.