## SOME NON-NEGATIVE TERMS :-

(i) $\sqrt{x}$, $\sqrt[4]{x}$, $\sqrt[6]{x}$, - - - - - $x^{1/2n}$ etc , $n \in I^+$

(ii) $x^2$, $x^4$, $x^6$, - - - - - - $x^{2n}$ etc. , $n \in I^+$

(iii) $|x|$, $|x-2|$, $|-x|$ etc.

(iv) $1-\sin x$ , $1-\cos x$

(v) $\{x\}$ , $0 \le \{x\} < 1$.

RESULT :- If sum of several non-negative terms is zero then each term is zero . i.e. if $u^2 + v^2 = 0$ then $u = 0 = v$.

(1.) Solve: $|x^2-1| + \sqrt{x^2-3x+2} + (x-1)^2 = 0$.

Sol$^n$:- Here, all the terms are non-negative, so

$|x^2-1| \geq 0 \Rightarrow x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \in (-\infty, -1] \cup [1, \infty)$. - - - (A)

But, $|x^2-1| = 0 \Rightarrow x = \pm 1$ , - - - (B)

$\sqrt{x^2-3x+2} = 0 \Rightarrow x^2-3x+2 = 0 \Rightarrow x = 1, 2$ - - - (C)

& $(x-1)^2 = 0 \Rightarrow x = 1$ - - - (D)

so, (A) $\cap$ (B) $\cap$ (C)

$\therefore \boxed{x = 1}$

(2.) Solve ! $|3^x-3| + (x^2+1) + \sqrt{x^2-x} = 2x$

Sol$^n$:- Here, $|3^x-3| + (x^2-2x+1) + \sqrt{x^2-x} = 0$

or, $|3^x-3| + (x-1)^2 + \sqrt{x^2-x} = 0$

$\therefore$ $3^x-3 = 0 \Rightarrow 3^x = 3 \Rightarrow x = 1$ - - - (A)

$\therefore$ $(x-1)^2 = 0 \Rightarrow x = 1$ - - - - (B)

& $\sqrt{x^2-x} = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$ - - - (C)

so, $A \cap B \cap C$

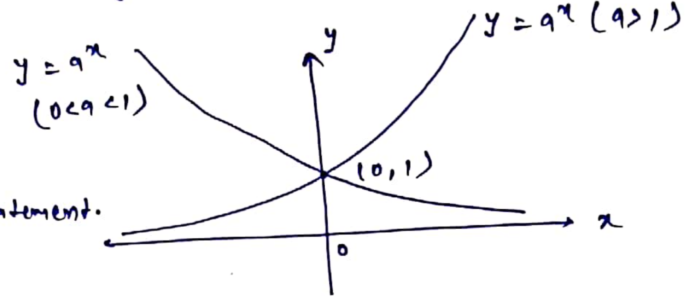
$\therefore \boxed{x = 1}$

**Exponential Concept :-**

Defined as, $y = a^x$ $(a>0, \neq 1)$

If $a=1$, it is constant funcⁿ

Graphically,




Also, $a^x < 0$ &

$a^x = 0$ are false statement.

**Note :-** $a^x > 0$ $\forall x \in R$.

(3.) Solve : $2^x = -1$

By defⁿ $2^x \neq$ -ve'. so, no solution.

(4.) Solve : $4^x - 3 \cdot 2^x + 2 = 0$

solⁿ :- Let $2^x = t$

$\therefore$ $t^2 - 3t + 2 = 0 \Rightarrow t = 1, 2$

Now, $2^x = 1 \Rightarrow 2^x = 2^0 \Rightarrow x = 0$

or, $2^x = 2 \Rightarrow x = 1$

$\therefore$ $x = 0, 1$.

(5.) Solve : $4^x - 2^x - 2 = 0$

→ students can try it.

**Important Results of Number property :-**

The sum of any number and its reciprocal doesn't lie in $(-2, 2)$

i.e. $\boxed{x + 1/x \geq 2}$ (if $x>0$, equality holds at $x=1$).

& $\boxed{x + 1/x \leq -2}$ (if $x<0$, equality holds at $x=-1$).

**Proof :-** let, $y = x + 1/x \Rightarrow xy = x^2 + 1 \Rightarrow x^2 - xy + 1 = 0$

$\therefore$ $x$ is real, so $D \geq 0$

Hence, $y^2 - 4 \geq 0 \Rightarrow y^2 \geq 4 \Rightarrow y \in (-\infty, -2] \cup [2, \infty)$

$\therefore$ $x + 1/x \leq -2$, for $x < 0$.

& $x + 1/x \geq 2$, for $x > 0$.

(6.) Solve: $\sin(e^x) = x + 1/x$

Sol$^n$:- as we know, $+2 \le x + 1/x$ or $x + 1/x \le -2$

But $-1 \le \sin e^x \le 1$.

$\therefore$ $\boxed{\text{No sol}^n.}$

(7.) $2^x + 2^{-x} = 2\cos^2\left(\frac{x^2+x}{6}\right)$

Sol$^n$:- Here, $2^x + \frac{1}{2^x} \geq 2$ $(2^x > 0)$

Also, $0 \le \cos^2\left(\frac{x^2+x}{6}\right) \le 1.$ $\Rightarrow 0 \le 2\cos^2\left(\frac{x^2+x}{6}\right) \le 2$

So, $2^x + 2^{-x} = 2\cos^2\left(\frac{x^2+x}{6}\right) = 2$

$\therefore$ $\boxed{x = 0}$

Exercise:-

(1.) JEE solve : $\sin e^x = 5^x + 5^{-x}$

(2.) solve : $x^2 + 1/x^2 = 2\cos^2\left(\frac{x^2-|x|}{6}\right)$

(3.) JEE solve : $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$

(4.) solve : $x + 4 < -\frac{2}{x+1}$

Fundamentals of eq$^n$ solving :-

(i) Avoid Cancelling on both sides, it may result in missing of roots.

ex: $\sqrt{x} = -x$

sq. on both sides,

$\therefore x = x^2$

or, $x = 0, 1$

But, $x = 0$ (not 1.)

Another way to solve it,

$\sqrt{x} + x = 0 \Rightarrow \sqrt{x}(1 + \sqrt{x}) = 0$

$\therefore$ $\boxed{x = 0}$

or, $1 + \sqrt{x} = 0$ (false statement).

Note:- Always check by putting the values of '$x$'. in question.

(ii) Avoid that root for which Denominator is zero.

ex: $\frac{x^2-3x+2}{x-2} = 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

$\therefore$ $\boxed{x = 1}$ (not 2).

(iii) If $a^{f(x)} = a^{g(x)}$ $(a>0)$ $\Rightarrow$ $f(x) = g(x)$ or, $a = 1$.

ex:- solve $|x|^{x^2-2x} = 1$ $\Rightarrow$ $|x|^{x^2-2x} = |x|^0$

$\therefore$ $x^2 - 2x = 0 \Rightarrow x = 0, 2$

Also, $|x| = 1 \Rightarrow x = \pm 1$

But, at $x = 0$, '$0^0$' $\rightarrow$ which is meaningless.

$\therefore$ $\boxed{x = \pm 1, 2}$

Students can try questions given below:-

(5.) solve: $\dfrac{x^2+x-2}{x-1} = 0$

(6.) solve: $x + \sqrt{x-1} = 0$

(7.) solve: $|x-3|^{\frac{x^2-4x+3}{x-2}} = 1$

(8.) solve: $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

(8.) The no. of sol^n of the eq^n: $\log(-2x) = 2\log(x+1)$ is ---

(A) 0 (B) 1 (C) 2 (D) NOT.

sol^n:- Here, $\log(-2x) = 2\log(x+1)$

so, $\left.\begin{array}{l} -2x > 0 \Rightarrow x < 0 \\ \& \;\; x+1 > 0 \Rightarrow x > -1 \end{array}\right\} \Rightarrow x \in (-1, 0)$

Again, $-2x = (x+1)^2 \Rightarrow x^2 + 4x + 1 = 0$

$\therefore$ $x = -2 \pm \sqrt{3}$

Hence, $x = -2 + \sqrt{3} \in (-1, 0)$

$\therefore$ $\boxed{\text{option 'B'.}}$

(9.) Greatest int. less than or, equal to the number $\log_2 15 \cdot \log_{1/6} 2 \cdot \log_3 1/6$ is ---

(A) 4 (B) 3 (C) 2 (D) 1

sol^n:- Let $x = \log_2 15 \cdot \log_{1/6} 2 \cdot \log_3 1/6 = \dfrac{\log 15}{\cancel{\log 2}} \times \dfrac{\cancel{\log 2}}{\cancel{\log 1/6}} \times \dfrac{\cancel{\log 1/6}}{\log 3} = \log_3(5\cdot 3)$

$= \log_3 3 + \log_3 5$

$= 1 + \log_3 5 > 2$ but $< 3$

so, $[x] = 2$. $\therefore$ $\boxed{\text{option 'C'}}$

(10.) If $\frac{|x+2|-x}{x} < 2$, then the set of values of $x$ is ---

(A) $(-\infty,1)\cup(2,\infty)$ (B) $(-\infty,0)\cup(1,\infty)$ (C) $(-\infty,-1)\cup(0,\infty)$ (D) NOT.

Soln:- Here, $\frac{|x+2|-x}{x} < 2$

for, $x \geq -2$,

$\frac{\cancel{x}+2-\cancel{x}}{x} < 2$

or $\frac{1-x}{x} < 0$

So, $x \in (-\infty,0)\cup(1,\infty)$

i.e. $x \in [-2,0)\cup(1,\infty)$ ---(A)

for $x < -2$,

$\frac{-x-2-x}{x} < 2 \Rightarrow \frac{-2x-2}{x} < 2$

$\therefore \frac{-2x-1}{x} < 0 \Rightarrow \frac{2x+1}{x} > 0$

i.e. $x = (-\infty,-1/2)\cup(0,\infty)$

i.e. $x \in (-\infty,-2)$ --- (B)

$\therefore$ A $\cap$ B,

$x \in (-\infty,0)\cup(1,\infty)$

So, $\boxed{\text{option 'B'.}}$

(11.) $e^{e^{\ln(\ln 3)}}$ is simplified to ---

(A) $e^3$ (B) $\ln 3$ (C) $3$ (D) $\ln\ln(3)$.

Soln:. $e^{e^{\ln_e(\ln 3)}} = e^{\ln 3} = 3.$

So, $\boxed{\text{option 'C'}}$

(12.) Soln. set of the inequality $\log_e^2[2x] - \log_e[2x] \leq 0$

Soln:. Let, $t = \log[2x]$

So, $t^2 - t \leq 0 \Rightarrow t(t-1) \leq 0$

i.e. $0 \leq t \leq 1$

Also, $0 \leq \ln[x] \leq 1 \Rightarrow 1 \leq [2x] \leq e \Rightarrow [2x] = \{1,2\}$

if $[2x] = 1 \Rightarrow 1 \leq 2x < 2 \Rightarrow 1/2 \leq x < 1$ --- (A)

or, $[2x] = 2 \Rightarrow 2 \leq 2x < 3 \Rightarrow 1 \leq x < 3/2$ --- (B)

$\therefore x \in [1/2, 3/2)$

So, $\boxed{\text{option 'D'.}}$