Inequality :-

→ If $a = b$, known as equality Rel^n bet^n a & b.

→ If $a \neq b$, known as inequality Rel^n bet^n a & b.

so, $a \neq b$

$\Downarrow$

$a < b$ ('a' is less than 'b')

$a > b$ ('a' is greater than 'b').

Note :-

* $a \leq b$ means either $a < b$ or, $a = b$.
* $a \geq b$ means either $a > b$ or, $a = b$.
* $a < b < c$ means 'a' is less than 'b' and 'b' is less than 'c'.

Properties :-

(i.) If $a < b$, then $a \pm c < b \pm c$, $c \in R$.

(ii.) If $a < b$, then

↳ $a \cdot c < b \cdot c$, $c \in R^+$

↳ $a \cdot c > b \cdot c$, $c \in R^-$

(iii.) If $a < b$, then

↳ $a/c < b/c$, $c \in R^+$

↳ $a/c > b/c$, $c \in R^-$

(iv.) If $a < b$, then $1/a > 1/b$.

(v.) If $a < b$, then $a^2 < b^2$, $(a, b \in R^+)$.

(vi.) If $a < b$, then $a^2 > b^2$, $(a, b \in R^-)$.

(vii.) If $a \cdot b > 0$, then either a & b are positive or, a & b are negative

i.e. a & b are of same sign.

(viii.) If $a \cdot b < 0$, then ~~a~~ 'a' and 'b' are of different signs.

(ix.) If $a < b$ & $c < d$, then $a + c < b + d$

(Never subtract or, divide or, multiply two inequality).

Q.1. Solve: $2x + 1 < 3x + 2$

Sol^n: Sub. '1' on both sides,

$\therefore\ 2x + 1 - 1 < 3x + 2 - 1 \Rightarrow 2x < 3x + 1$

Sub. '2x' on both sides,

$\therefore\ 0 < x + 1 \Rightarrow -1 < x$ (sub. '1' on both sides) $\Rightarrow \boxed{x > -1}$

2.Q. Solve: $x \geq \sqrt{2-x}$

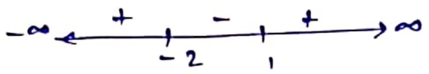
Sol^n:- Here, $\sqrt{2-x}$ is real iff $2-x \geq 0 \Rightarrow x \leq 2$ ---- (A)

Also, L.H.S. is positive because, $x \geq \sqrt{2-x}$.

So, Sq. on both sides,

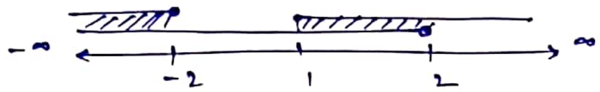
$\therefore x^2 \geq 2-x \Rightarrow x^2+x-2 \geq 0$

or, $x^2+2x-x-2 \geq 0 \Rightarrow (x+2)-1(x+2) \geq 0 \Rightarrow (x-1)(x+2) \geq 0$



So, $x \in (-\infty, -2] \cup [1, \infty)$. ---- (B)

Hence, finding, $A \cap B$,



So, $x \in (-\infty, -2) \cup (1, 2)$. ---- (C)

Also, we know, 'x' is positive.

So, $\boxed{x \in (1, 2)}$

3.Q Solve for 'a', $\frac{\sqrt{a+4}}{1-a} \leq 1$.

Sol^n:- Here, we have two cases:

Case I :- $a > 1$ i.e. $1-a < 0$.

So, LHS = '-ve'

RHS = '+ve'

Also, LHS $\leq$ RHS (True)

$\therefore a \in (1, \infty)$ ---- (A)

Case II: $1-a > 0$ i.e. $a < 1$.

$\therefore \frac{a+4}{(1-a)^2} \leq 1$

or $\frac{(a+4)-(1-a)^2}{(1-a)^2} \leq 0$

or, $\frac{a+4-(1-2a+a^2)}{(1-a)^2} \leq 0$

or $\frac{a+4-1+2a-a^2}{(1-a)^2} \leq 0$

or, $\frac{3+3a-a^2}{(1-a)^2} \leq 0$.

or $\frac{a^2-3a-3}{(a-1)^2} \geq 0$.

$\therefore a \in \left(-\infty, \frac{3-\sqrt{21}}{2}\right] \cup \left[\frac{3+\sqrt{21}}{2}, \infty\right)$ --- (B)

But, $a+4 \geq 0 \Rightarrow a \geq -4$. -- (C)

So, $(A \cup B) \cap C$.

i.e. $\boxed{a \in \left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)}$

4.Q. Solve : $\frac{\sqrt{21-4a-a^2}}{(a+1)} \le 1$.

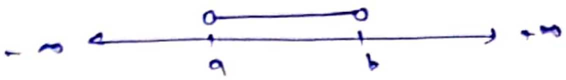
→ students must try this inequality.

Intervals :-

· showing some part of number system.

(i) closed interval :- If $a \le x \le b$, then $x \in [a,b]$.

(ii) open interval :- If $a < x < b$, then $x \in (a,b)$. or $]a,b[$

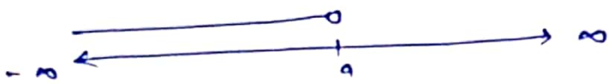


(iii) closed-open interval :- If $a \le x < b$, then $x \in [a,b)$ or $[a,b[$

(iv) open-closed interval :- If $a < x \le b$, then $x \in (a,b]$ or $]a,b]$.

(v) Infinite intervals :-

↳ If $x < a$, i.e. $x \in (-\infty, a)$.



↳ If $x \in R$ i.e. $x \in (-\infty, \infty)$

Note :- If 'm' lies betn 1 to 9 excluding 3 & 4. This statement can be represented mathematically as

$$m \in (1,9) - \{3,4\}.$$

5.Q. Here, $A = (-\infty,-3) \cup (5,17] \cup [26,29] \cup [70,2^{10}) - \{10,30\}$

& $B = [-17,2) \cup (15,21) \cup [20,25) \cup (250,\infty)$

then, find :-

(a) A^c (b) B^c (c) $A \cup B$ (d) $A - B$ (E) $A^c - B$.

Sol^n:-

(a) $A^c = [-3,5] \cup (17,26) \cup (39,70) \cup [2^{10},\infty) \cup \{10,30\}$.

(b) $B^c = [2,15] \cup [21,30] \cup [35,250]$

(c) $A \cup B = (-\infty,2) \cup (5,21) \cup [26,29] \cup [70,\infty) - \{10\}$

(d) $A - B = (-\infty,-17) \cup (5,15] \cup [26,30) \cup [35,29] \cup [70,250] - \{10\}$

(e) $A^c - B = [2,5] \cup [21,36) \cup (29,70) \cup \{10\}$.

WAVY-curve Method or Sign scheme :-

Sign-scheme of algebraic func^n tells that for which value of 'x' the expression is '-ve' or '+ve'.

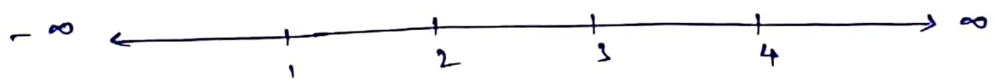
e.g. for which value of x, $f(x) = (x-2)^2 (1-x)(x-3)^3 (x-4)^2$ is '+ve' and '-ve'.

Sol^n:-

Step 1: Find roots of $f(x) = 0$.

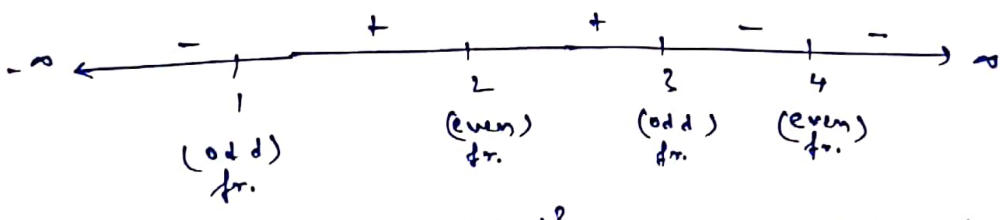
$\therefore x = 2, 2, 1, 3, 3, 3, 4, 4$ (Along with frequency).

Step 2: Represent roots on real no. line.

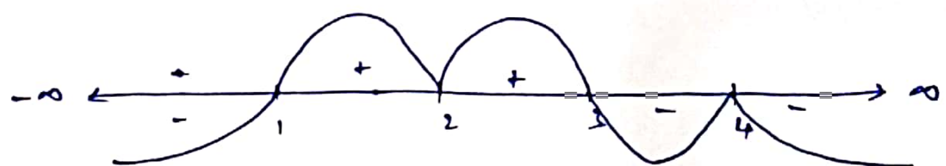


Step 3: keep the sign of different intervals i.e. for even freq. of roots, same sign in sub intervals.

For odd freq $\rightarrow$ different signs in sub intervals.



Step 4: Draw the curve, if there is a negative part, draw curve below the line and if curve is above the line, then sign of that interval is positive.



So, if $f(x) > 0 \Rightarrow x \in (1,2) \cup (2,3)$

if $f(x) < 0 \Rightarrow x \in (-\infty,1) \cup (3,4) \cup (4,\infty)$

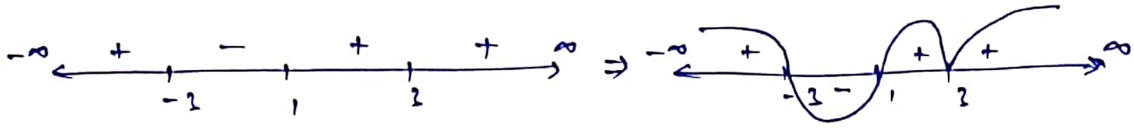
if $f(x) \geq 0 \Rightarrow x \in [1,3]$

if $f(x) \leq 0 \Rightarrow x \in (-\infty,1] \cup [3,\infty) \cup \{2\}$.

7.Q. If $f(x) = (x^2+x+1)(x+3)(x-1)(x-3)^2$ then find 'x' for

(a) $f(x) > 0$ (b) $f(x) < 0$ (c) $f(x) \geq 0$. (d) $f(x) \leq 0$.

Sol^n:- Here, $f(x) = 0 \Rightarrow x = -3, 1, 3, 3$ $(x^2+x+1 > 0 \ \forall x \in R)$.



(a) $f(x) > 0 \Rightarrow x \in (-\infty, -3) \cup (1, 3) \cup (3, \infty)$

(b) $f(x) < 0 \Rightarrow x \in (-3, -1)$.

(c) $f(x) \geq 0 \Rightarrow x \in (-\infty, -3] \cup [1, \infty)$

(d) $f(x) \leq 0 \Rightarrow x \in [-3, -1] \cup \{3\}$.

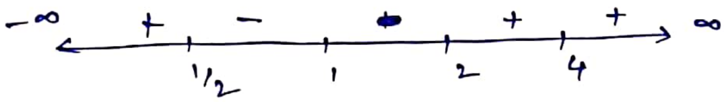
8.Q. If $f(x) = \dfrac{(2x-1)(x-1)^2(x-2)^2}{(x-2)(x-4)^2}$. then find '$x$' for

(a) $f(x) \geq 0$ (b) $f(x) \leq 0$.

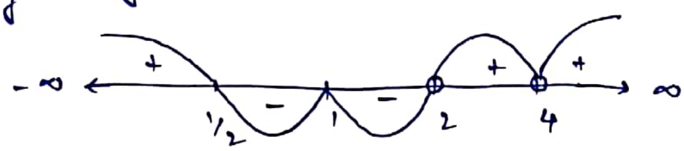
Sol^n:-
Here, for rational functions, find the roots of Numirator as well as Denominator,

$\therefore x = 1/2, 1, 1, 2, 2$ $(N^r = 0)$

or $x = 2, 4, 4$. $(D^r = 0)$



Drawing weavy curve,



so, (a) $f(x) \geq 0 \Rightarrow x \in (-\infty, 1/2] \cup (2, 4) \cup (4, \infty)$

(b) $f(x) \leq 0 \Rightarrow x \in [1/2, 1] \cup [1, 2)$ i.e. $[1/2, 2)$.

Here, we have to exclude 1 & 2. Because, $x = 1, 2$ are roots of $D^r = 0$.

Also, for $f(x) > 0 \Rightarrow x \in (-\infty, 1/2) \cup (2, 4) \cup (4, \infty)$

& for $f(x) < 0 \Rightarrow x \in (1/2, 1) \cup (1, 2)$.