Positive logarithm & negative logarithm :-

* If $a>1$ & $x>1$ or, $0<a<1$ & $0<x<1$ then, $\log_a x$ will be positive.
* If $0<a<1$ & $x>1$ or, if $a>1$ & $0<x<1$ then, $\log_a x$ will be negative.

Q.10. Solve :

(a) $\log_2 (x^2-12x+29) = 2$

(b) $\log_2 (|x+2|) = 5$

Solⁿ:- (a) Here, $\log_2 (x^2-12x+29) = 2$

as $x^2-12x+29>0 \ \forall x \in R$.

So, $x^2-12x+29 = 2^2$ (Applying property).

i.e. $x^2-12x+25 = 0 \Rightarrow x^2-7x-5x+35=0$

i.e. $(x-7)(x-5) = 0 \Rightarrow x = 7$ or, 5

So, $x = 7$ or, 5

(b) $\log_2 |x+2| = 5$

$\therefore |x+2| > 0 \ \forall x \in R - \{-2\}$

So, $|x+2| = 2^5 \Rightarrow x+2 = \pm 32 \Rightarrow x = 30$ or, -34

So, $x = 30$ or, -34.

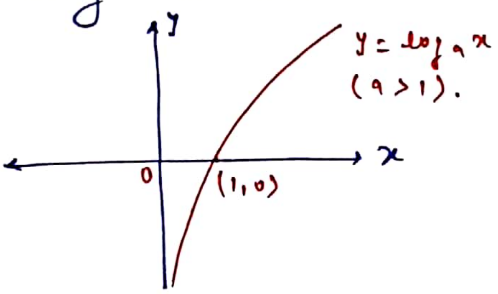
Q.11. Prove that : $\log_a N + \log_{1/a} N = 0$

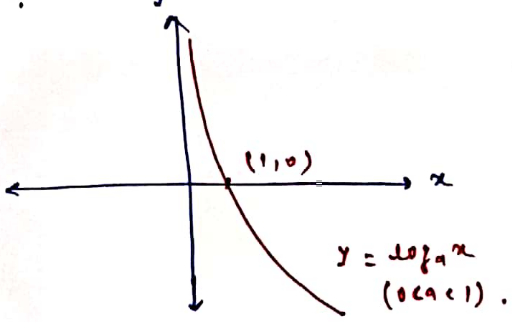
Solⁿ:- Here, $LHS = \log_a N + \log_{a^{-1}} N = \log_a N - \log_a N = 0$.

Logarithmic Inequality :-

Let $y = \log_a x$ $(a>0, x>0, a\neq 1)$

From the graph we can easily say that, '$\log_a x$' is increasing on $(1,\infty)$ & decreasing on $(0,1)$. for base $a>1$.





(i) If $a>1$,

$\log_a x > 0 \Rightarrow x > 1$

$\log_a x > p \Rightarrow x > a^p$

$\log_a x < 0 \Rightarrow 0 < x < 1$

(ii) If $0<a<1$,

$\log_a x > 0 \Rightarrow 0 < x < 1$

$\log_a x > p \Rightarrow 0 < x < a^p$

$\log_a x < 0 \Rightarrow x > 1$

$\log_a x < p \Rightarrow x > a^p$.

Q.12. Express $2\log 3 - \frac{1}{2}\log 16 + \log 12$ as a simple logarithm.

Solⁿ:-

$2\log 3 - \frac{1}{2}\log 2^4 + \log(3\times 4) = 2\log 3 - \frac{1}{2}\times 4\log 2 + \log 3 + \log 2^2$

$\Rightarrow 2\log 3 - 2\cancel{\log 2} + \log 3 + 2\cancel{\log 2}$

$= 3\log 3$

Q13. If $a^2+b^2 = 7ab$, prove that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$

Solⁿ:-

Here, $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$

or, $\log\left(\frac{a+b}{3}\right) = \log\sqrt{ab}$

or $\frac{a+b}{3} = \sqrt{ab} \Rightarrow (a+b)^2 = 9ab$

i.e. $a^2+b^2 = 7ab$

(14) Solve for x, y : $\frac{\log(x+y)}{\log 2} = \frac{\log(x-y)}{\log 5} = \frac{\log 4}{\log(0.5)}$

Solⁿ:- Here, $\frac{\log(x+y)}{\log 2} = \frac{\log 4}{-\log 2} = \frac{2\cancel{\log 2}}{-\cancel{\log 2}} = -2$

or, $\log(x+y) = -2\log 2 = \log(1/4)$

i.e. $x+y = 1/4$ ---- (1)

Also, $\frac{\log(x-y)}{\log 5} = \frac{\log 4}{\log(0.5)} = \frac{2\cancel{\log 2}}{-\cancel{\log 2}} = -2$

or, $\log(x-y) = -2\log 5 = \log(1/25)$

i.e. $x-y = 1/25$ --- (ii)

solving (i) & (ii),

$$\begin{aligned} x + \cancel{y} &= 1/4 \\ x - \cancel{y} &= 1/25 \\ \hline 2x &= 1/4 + \frac{1}{25} = \frac{29}{100} \end{aligned}$$

i.e. $x = \frac{29}{200}$ & $y = \frac{21}{200}$

(15.) show that : $\frac{\log_a(\log_b a)}{\log_b(\log_a b)} = \log_a(1/b)$

soln:- $\text{LHS} = \frac{\log_a(\log_b a)}{\log_b(\log_a b)} = \frac{\log(\log_b a)}{\log a} \times \frac{\log b}{\log(\log_a b)}$

$= \frac{\cancel{\log(\log_b a)}}{\log a} \times \frac{\log b}{-\cancel{\log(\log_b a)}}$

$= -\log_a b$

$= \log_a(1/b)$.

(16.) show that $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} = 1$.

soln:- $\text{LHS} = \frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)}$

$\Rightarrow \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc}(abc) = 1.$

Q.17. $\log(1+2+3) = \log 1 + \log 2 + \log 3$ is True?

Sol^n:- Here, $\log(1+2+3) = \log(1\cdot2\cdot3) = \log 1 + \log 2 + \log 3$ (True).

Q.18. Find $\log\tan 1^\circ + \log\tan 2^\circ + \log\tan 3^\circ + --- + \log\tan 89^\circ$.

Sol^n:- We know, $\tan 1^\circ \times \tan 89^\circ = 1 = \tan 2^\circ \tan 88^\circ = ---$

So, $\ln(\tan 1^\circ) + \log\tan 2^\circ + \log\tan 3^\circ + -- \log\tan 89^\circ = \log(\tan 1^\circ \cdot \tan 2^\circ \cdots \tan 89^\circ)$

$= \log 1$

$= 0.$

Q.19. Find: $\log\tan 1^\circ . \log\tan 2^\circ . \log\tan 3^\circ ---- \log\tan 89^\circ$.

Sol^n:- as we know $\log\tan 45^\circ = \log 1 = 0.$

So, '0'

Q.20. $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + --- + \frac{1}{\log_{43} n} = \frac{1}{\log_{43!} n}$.

Sol^n:-

Here, LHS $= \log_n 2 + \log_n 3 + \log_n 4 + ---- + \log_n 43$

$= \log_n(2\cdot3\cdot4 \cdots 43)$

$= \log_n(43!)$

$= \frac{1}{\log_{43!} n}$

Q.21. If $\log_4 10 = x$, $\log_2 20 = y$ & $\log_5 8 = z$, then find the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$.

→ students can try this.

Q.22. Solve for x, $\log_3(5 + 4\log_3(x-1)) = 2$

Sol^n:- $5 + 4\log_3(x-1) = 3^2 = 9$

or $\cancel{4}\log_3(x-1) = \cancel{4}$

or $x-1 = 3^1$

$\therefore \boxed{x = 4}$

Q.23. Solve $\log_2(3-x) + \log_2(1-x) = 3$.

Solⁿ:-

Here, $3-x>0$ & $1-x>0 \Rightarrow x<3$ & $x<1 \Rightarrow x<1$

Also, $\log_2\{(3-x)(1-x)\} = 3$

or $(3-x)(1-x) = 2^3 = 8 \Rightarrow x^2-4x+3 = 8$

or, $x^2-4x-5=0 \Rightarrow x^2-5x+x-5=0$

or, $(x-5)(x+1)=0$

$\therefore x = 5, -1$

So, $\boxed{x=-1}$

Q.24. Evaluate : $\dfrac{\left(81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}\right)}{409} \times \left[(\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right]$

Solⁿ:- $\dfrac{\left(81^{\log_9 5} + 3^{3\log_3 \sqrt{6}}\right)}{409} \times \left[7^{\frac{1}{\cancel{2}} \times \cancel{2}\log_7 25} - 5^{3\log_{25} 6}\right]$

$= \dfrac{25+(\sqrt{6})^3}{409} \times \left(25-(\sqrt{6})^3\right)$

$= \dfrac{(25+6\sqrt{6})(25-6\sqrt{6})}{409}$

$= \dfrac{(25)^2-(6\sqrt{6})^2}{409}$

$= \dfrac{625-216}{409}$

$= \dfrac{409}{409} = 1$

Q.25. Solve : $(\log_2 x)^2 - \log_2 x^3 = -2$

Solⁿ:- Let, $t = \log_2 x$ $(x>0)$

$\therefore t^2-3t+2=0 \Rightarrow t = 1, 2$

So, $\log_2 x = 1 \Rightarrow x = 2$ i.e. $\boxed{x = 2 \text{ or, } 4.}$

& $\log_2 x = 2 \Rightarrow x = 4$