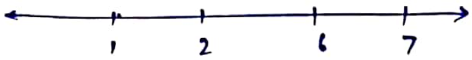
Q.10. Solve $|x-2|-|x-6|=10$ Graphically.

Sol^n:-

Step 1: find critical points

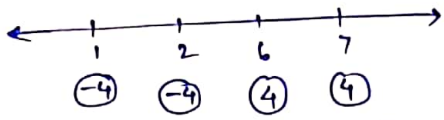
i.e. $x=2,6$

Step 2: plot the critical points on real number line, one more than the greatest value & '1' less than lowest value.

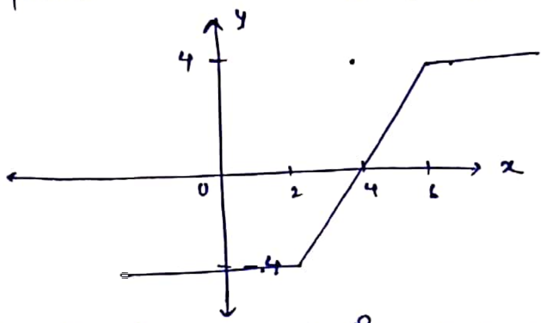


Step 3: find the values of y for respective values of x, where

$y=|x-2|-|x-6|$



Step 4: plot these points & draw the graph of $y=|x-2|-|x-6|$



Step 5: Draw the graph of $y=10$ in the same graph & see the intersection points. But in this case, $y=10$ & $y=|x-2|-|x-6|$ will not intersect even at single point. so, no solution.

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Q11. Solve : $x^2-|x+2|+x>0$

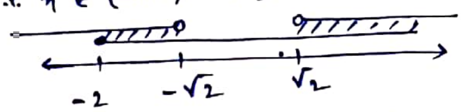
Sol^n:- Here, $|x+2|=\begin{cases} x+2, & x\geq -2 \\ -(x+2), & x<-2 \end{cases}$

Case (i) : $x+2\geq 0$ i.e. $x\geq -2$

$\therefore x^2-\cancel{x}-2+\cancel{x}>0$

or, $x^2>2$

i.e. $x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$ --- (A)



so, $x\in[-2,-\sqrt{2})\cup(\sqrt{2},\infty)$ ---(B)

Case (ii) : $x+2<0$ i.e. $x<-2$

$\therefore x^2+x+2+x>0$

or $x^2+2x+2>0$

$D=4-8<0$ & $1>0$

so, $x\in R$

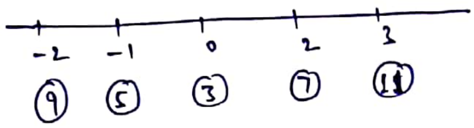
i.e. $x\in(-\infty,-2)$ ---- (C)

finding, Case (i) $\cup$ Case (ii)

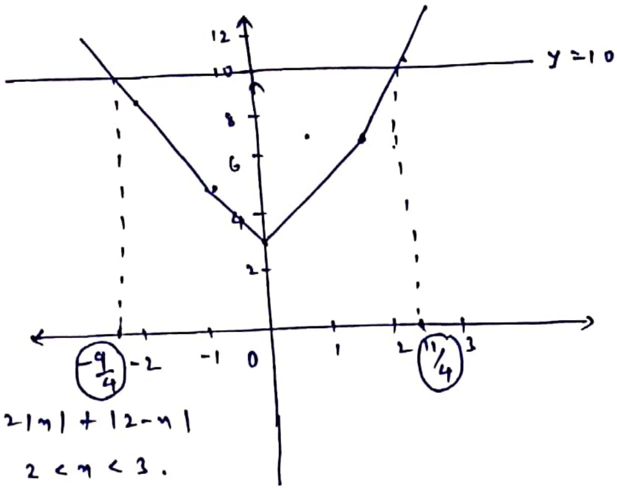
$x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

Q.12. $|x+1| + 2|x| + |2-x| \le 10$

Solⁿ:- $x = -1, 0, 2$



plotting points, & also drawing $y=10$.



we can see, the portion of $y = |x+1| + 2|x| + |2-x|$ intersected by $y=10$ is $x<-2$ or, $2<x<3$.

∴ In $x<-2$,

$x+1 < -1,\ 2-x>4$

~~so, $-(x+1) - 2x + (2-x) \le 10$~~

or, $y = -(x+1) - 2x + 2 - x$

or $y = -x - 1 - 2x + 2 - x$

i.e. $y = -4x+1$

or, In $2<x<3$,

$3 < x+1 < 4,\ -1 < 2-x < 0$

∴ $y = x+1+2x-(2-x)$

or $y = x+1+2x-2+x$

or, $y = 4x-1$

So, the point of intersections are, $(-9/4, 10)$ & $(\frac{11}{4}, 10)$

Hence, the solution is, $x \in [-9/4, 11/4]$

STUDENTS must try questions given below:-

Q.13. $|\sin x| + |\cos x| = |\sin x + \cos x|$, then find x in $-\pi \le x \le \pi$

Q.14. $x^2 - 5|x| + 4 = 0$.

Q.15. $|x-5| + |4-x| = |2x-3|$

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Q.16. find the value of 'k' such that $y = 2|x| + |x+2| - \big||x+2| - 2|x|\big|$ have only two solutions.

Q.17. The product of all the solutions of the eqⁿ: $(x-2)^2 - 3|x-2| + 2 = 0$?

Q.18. The number of solutions of $|[x] - 2x| = 4$ is? (where, $[x]$ denotes G.I.F.).

Q.19. If $|x^2-9| + |x^2-4| = 5$, then the set of values of 'x' is ---

Q.20. Solution set of $|x^2-5x+7| + |x^2-5x-14| = 21$ is ---

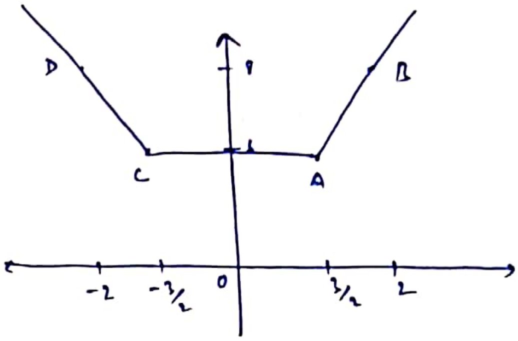
Q.21. The set of real values of 'p' for which the eqⁿ: $|2x+3| + |2x-3| = px+6$ has exactly two solⁿ:-

(A) $[0,4)$ (B) $(-4,4)-\{0\}$ (C) $R-\{-4,4,0\}$ (D) $\{0\}$

Solution for question (21)

Here, $|2x+3| + |2x-3| = px+6$

Drawing $y = |2x+3| + |2x-3|$



$-2 \quad -3/2 \quad 3/2 \quad 2$

$8 \quad 6 \quad 6 \quad 8$

So, To get two distinct solutions, $x \neq 0$, slope of AB & slope of CD.

$\therefore \; x \in R - \{-4, 4, 0\}$

So, option 'c'.

Q.22. Statement 1: Min^m value of $|x-2| + |x-5| + |x+3|$ is 8.

Statement 2: If $a < b < c$, then the min^m value of $|x-a| + |x-b| + |x-c|$ is $|b-a| + |b-c|$.

(A) Statement 1 is correct, statement 2 is also correct But statement 2 is the correct explanation of statement 1.

(B) Statement 1 is correct, statement 2 is correct But statement 2 is not the correct explanation of statement 1.

(C) Statement 1 is correct, statement 2 is false

(D) Statement 1 is false, statement 2 is correct

Sol^n:- Statement: 1 : By plotting critical points on real number line,

$-4 \quad -3 \quad 2 \quad 5 \quad 6$

$16 \quad 13 \quad 8 \quad 11 \quad 14$

So, clearly we can see, Min^m value $= 8$ at $x = 2$.

Statement: 2 : obviously true, and also it is the correct explanation of statement 1.

So, option 'A'.

Note:- Students can Remember the result given in statement 2.

Q.23. The complete sol^n set of the eq^n $|x^2-x| + |x+3| = |x^2-2x-3|$ is ...

Sol^n:- Apply, $|f(x)| + |g(x)| = |f(x) - g(x)|$ then $f(x) \cdot g(x) \leq 0$.

So, $(x^2-x)(x+3) \leq 0$

i.e. $x \in (-\infty, -3] \cup [0, 1]$.

Comprehension :-

Let $a_1 < a_2 < a_3 < \dots < a_n$, n is an odd natural number & $m, k \in N$.
consider the eqn. $|x-a_1| + |x-a_2| + |x-a_3| + \dots + |x-a_n| = kx + d$

Case I : when $2m - n = k$, for some $m < n$, then the equation has

(i) No soln. if $(a_{m+1} + a_{m+2} + \dots a_n) - (a_1 + a_2 + a_3 + \dots + a_m) > d$

(ii) Infinite solutions if $(a_{m+1} + a_{m+2} + \dots + a_n) - (a_1 + a_2 + \dots + a_m) = d$

(iii) Two solutions if $(a_{m+1} + a_{m+2} + \dots + a_n) - (a_1 + a_2 + \dots + a_m) < d$.

Case II : Let, when $2m - n \neq k$ for any $m < n$.

Two Cases arise

(A) If $|k| > n$, then there is one solution.

(B) If $|k| < n$, then there is m such that $2m - (n-1) = k$.

(i) If $(a_{m+2} + a_{m+3} + \dots + a_n) - (a_1 + a_2 + \dots + a_m) > d$ (no soln.)

(ii) If $(a_{m+2} + a_{m+3} + \dots + a_n) - (a_1 + a_2 + \dots + a_m) = d$ (one soln.)

(iii) If $(a_{m+2} + a_{m+3} + \dots + a_n) - (a_1 + a_2 + \dots + a_m) < d$ (Two solns.).

(24.) Number of solutions of $|x-1| + |x-2| + |x-7| + |x-9| = 2x+5$ is ...

(a) 0 (b) 1 (c) 2 (d) Infinite.

(25.) Number of solutions of $|x-1| + |x-3| + |x-4| + |x-7| + |x-10| = 2x+1$ is ...

(a) 0 (b) 1 (c) 2 (d) NOT.

(26.) Number of solutions of $|x-1| + |x-4| + |x-6| + |x-7| = 2x+10$ is ...

(a) 0 (b) 1 (c) 2 (d) NOT.

soln.:-

(24.) Here, $n = 5$

so, $2m - n = 3 \Rightarrow 2m = 8$ i.e. $m = 4$

$\therefore a_5 - (a_1 + a_2 + a_3 + a_4) = 9 - (1+3+4+7) = 9 - 15 = -6 < 5$

i.e. Two solutions.

(25.) soln.:- $n = 5$

so, $2m - 5 = 2 \Rightarrow m = 7/2 \notin N$ (not possible)

so, by second case, $|k| = 2 < n \; (5)$

then, $2m - (5-1) = 2$

or $2m = 6 \Rightarrow m = 3$

$\therefore a_5 - (a_1 + a_2 + a_3) = 10 - (1+3+4) = 10 - 8 = 2 > d$ (No solⁿ ~~Two solⁿ~~)

(26.) Solⁿ :- $n = 5,\ 2m - n \neq k$

$|k| = 2 < n \Rightarrow m = 3$

$\therefore a_5 - (a_1 + a_2 + a_3) = 7 - (1+2+4) = 7 - 7 = 0 < d$ (Two solⁿs)

Q.27. If set of all real values of 'x' satisfying $|x^2 - 3x - 1| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|$, $x^2 - 3x - 1 \neq 0$ is $(-\infty, -a) \cup (-b, \infty)$, then find the value of $a + \log(ab)$.

Solⁿ :- Here, $|x^2 - 3x - 1| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|$

or, $|(3x^2 + 2x + 1) - (2x^2 + 5x + 2)| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|$

$\therefore (3x^2 + 2x + 1)(2x^2 + 5x + 2) > 0$

or, $2x^2 + 4x + x + 2 > 0$

or $(2x+1)(x+2) > 0$

i.e. $x \in (-\infty, -2) \cup (-1/2, \infty)$

So, $a + \log ab = 2 + \log(2 \times 1/2) = 2$

Q.28. Match the following :

Column 'I'

(A) $x|x| =$

(B) $|x-1| + |x+1| =$

(C) If $-1 \le x < 2$, then $2x - \{x\} =$
where $\{x\}$ denotes fractional part of x.

(D) If $-1 \le x < 2$, then $x[x] = \ldots$
where $[x]$ denotes G.I.F.

Column 'II'

(P.) $\begin{cases} -2x ; & x < -1 \\ 2 ; & -1 \le x \le 1 \\ 2x ; & x \geq 1 \end{cases}$

(q.) $\begin{cases} -x^2 ; & x \le 0 \\ x^2 ; & x > 0 \end{cases}$

(r.) $\begin{cases} -x ; & -1 \le x < 0 \\ 0 ; & 0 \le x < 1 \\ x ; & 1 \le x < 2 \end{cases}$

(s.) $\begin{cases} x-1 ; & -1 \le x < 0 \\ x+1 ; & 0 \le x < 2 \end{cases}$

(t.) $\begin{cases} x-1 ; & -1 \le x < 0 \\ x ; & 0 \le x < 1 \\ x+1 ; & 1 \le x < 2. \end{cases}$

Solⁿ :- Clearly, just expanding we will get result,

(A) → q , (B) → P.

(C) for $-1 \le x < 2 \Rightarrow \begin{cases} x-1 , & -1 \le x < 0 \\ x , & 0 \le x < 1 \\ x+1 , & 1 \le x < 2 \end{cases}$

So, (C) → t

& (D) → r

& (D) for $-1 \le x < 2 \Rightarrow \begin{cases} -x , & -1 \le x < 0 \\ 0 , & 0 \le x < 1 \\ x , & 1 \le x < 2 \end{cases}$