(2.) GEOMETRIC PROGRESSION :- (G.P.)

Defⁿ :- A sequence, in which the ratio of any term & the preceeding term is constant throughout is known as G.P.

for ex :- 2, 4, 8, 16, 32, 64, ---- are in G.P. with common ratio '2'.

Also, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, ----- are in G.P. with common ratio '$\frac{1}{2}$'.

n^{th} term of a G.P. (General term) :-

Let 'a' = first term & r = common difference

then sequence of G.P. is,

$a, ar, ar^2, ar^3, ar^4, ---- ar^{n-1}$

$\therefore \boxed{t_n = ar^{n-1}}$

Note :-

→ n^{th} term from the end

let, $a, ar, ar^2, --- \underset{\to l}{ar^{n-1}}$

$\therefore \boxed{T_{n(end)} = l\left(\frac{1}{r}\right)^{n-1}}$

→ n^{th} term from the end of a G.P. containing 'm' terms (i.e. finite G.P.) $= ar^{m-n}$

→ If a = first term, b = last term, n = no. of terms

$\therefore b = ar^{n-1} \Rightarrow \frac{b}{a} = r^{n-1}$

$\therefore \boxed{r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}}$

Q 2nd, 3rd, 6th terms of A.P. form a G.P. then C.d. is --
(where a = first term & d = c.d. of A.P.).

Solⁿ:- A/c to question, $(a+d), (a+2d), (a+5d)$

so, $(a+2d)^2 = (a+d)(a+5d)$

or, $\cancel{a^2} + 4d^2 + 4ad = \cancel{a^2} + 5ad + ad + 5d^2$

or $d^2 + 2ad = 0 \Rightarrow \boxed{d = -2a}$

Q Prove that sequence given by, $a_n = 4/3^n$ is a G.P. where $n \in N$.

Solⁿ:- Here, $a_n = 4/3^n$

so, $a_1 = 4/3$, $a_2 = 4/9$, $a_3 = 4/27$, ----

so, $a_1, a_2, a_3, ---$ are in G.P. with common ratio, $\boxed{'1/3'}$

Q which term of G.P. 2, 8, 32, --- upto n term is 131072?

Solⁿ:- Here, $t_n = 2 \times r^{n-1} \Rightarrow 131072 = 2 \times 4^{n-1}$

or, $4^{n-1} = 65536 = 4^8$

so, $n-1 = 8$

i.e. $\boxed{n = 9}$

Q If the first & nth term of a G.P. are a & b resp. & if 'p' is the product of first 'n' terms then prove that $p^2 = (ab)^n$.

Solⁿ:- $b = ar^{n-1} \Rightarrow r = \left(b/a\right)^{\frac{1}{n-1}}$

$P = a \cdot ar \cdot ar^2 \cdot ar^3 \cdot \dots\dots a \cdot r^{n-1}$

$= a^n r^{1+2+3+\dots+(n-1)}$

$= a^n r^{\frac{n(n-1)}{2}}$

$= a^n \left(\frac{b}{a}\right)^{n/2}$

$= (ab)^{n/2}$

so, $\boxed{P = (\sqrt{ab})^n}$

Q. The sum of three numbers in G.P. is 21 and the sum of their squares is 189, find the numbers.

Sol^n:- Let, three terms in A.P. are, a, ar, ar^2

$\therefore a(1+r+r^2) = 21, \quad a^2(1+r^2+r^4) = 189$

so, $\frac{1+r^2+r^4}{(1+r+r^2)^2} = 3/7$ or, $\frac{(1+r^2)^2 - r^2}{(1+r+r^2)^2} = 3/7$

or, $\frac{1+r^2-r}{1+r+r^2} = 3/7 \Rightarrow r = 2, 1/2.$

$\Rightarrow \boxed{3, 6, 12 \text{ or, } 12, 6, 3.}$

Note:-

→ If three numbers are in G.P. then take a/r, a, ar (if product is given).

→ If four numbers are in G.P. then take a/r^3, a/r, ar, ar^3 (if product is given).

Q. In a set of four numbers, the first three are in G.P. and the last three are in A.P. with c.d. '6'. If the first number is same as the fourth, find the four numbers.

Sol^n:- Let, $b, b+6, b+12$ are in A.P.

$$\therefore a = b+12 \quad \text{---} \ (I)$$

& $a, b, b+6$ are in G.P.

$$\therefore b^2 = a(b+6) \quad \text{---} \ (II)$$

Solving $b = -4$, $a = 8$

So, $\boxed{8, -4, 2 \ \& \ 8}$

Sum of first 'n' terms of a G.P. :-

Let $a, ar, ar^2, \ldots, ar^{n-1}$

Case (I):- when $r \neq 1$,

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \cdots + ar^n$$

$$(1-r)S_n = a - ar^n \Rightarrow \boxed{S_n = \frac{a(1-r^n)}{(1-r)}}, \ r \neq 1$$

Case (II):- when $r = 1$,

$$S_n = a + a + a + \cdots \text{ to } n \text{ terms.}$$

So, $\boxed{S_n = na}$

Generally, $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$, $|r| < 1$

& $\boxed{S_n = \frac{a(r^n - 1)}{r-1}}$, $|r| > 1$

Note :- $\boxed{S_n = \frac{lr - a}{r-1}}$, $\boxed{l = ar^{n-1}}$

Q. Find the sum to 'n' terms of the series $8+88+888+\cdots$

Soln:- Here,

$$S_n = 8+88+888+\cdots$$

$$= \frac{8}{9}[9+99+999+\cdots]$$

$$= \frac{8}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\cdots\right]$$

$$= \frac{8}{9}\left[(10+10^2+10^3+\cdots)-n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

So, $\boxed{S_n = \frac{8}{81}(10^{n+1}-10-9n)}$

Q. How many terms of the series $1+3+3^2+3^3+\cdots$ must be taken to make 3280?

Soln:- Here, $T_n = 1\cdot 3^{n-1} = 3280$

solving $n-1 = 7$

So, $\boxed{n=8}$

Q. If $f(x)$ is a fn. satisfying $f(x+y) = f(x)\cdot f(y)$ for all $x, y \in N$ such that $f(1) = 3$ & $\sum_{n=1}^{n} f(n) = 120$, find the value of n.

Soln:- Here, $f(x+y) = f(x)\cdot f(y)$

$\therefore f(2) = (f(1))^2 = 3^2$

$f(3) = f(2)\cdot f(1) = 3^3$

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So, $\sum_{n=1}^{n} f(n) = 3^1+3^2+3^3+\cdots+3^n$

~~$\therefore 3\cdot 3^{n-1} = 120$~~

~~or $3^n = 120$~~

$\therefore \frac{\cancel{3}(3^n-1)}{(3-1)} = \cancel{120}\ 40$

or $3^n = 81$

So, $\boxed{n=4}$