Common term A.P. :-

→ common terms' A.P's common difference $= d_1 \times d_2$ where d_1 & d_2 are common differences of two different A.P.s.

Example :-

(1.) Given two A.P. are 3, 7, 11, 15, 19, 23, 27, --- to 110 term & 2, 7, 12, 17, 22, 27, ---- to 100 terms

Solⁿ :- Here, Last term of first A.P. is,

$$t_{110} = 3 + (110-1)4 = 439$$

& Last term of second A.P. is,

$$t_{100} = 2 + (100-1)5 = 497.$$

So, common term A.P. is ... $(\underline{d = 4 \times 5 = 20})$

7, 27, 47, 67, -------

Also, $T_n = 7 + (n-1)20 \leq 439$

or, $(n-1)20 \leq 432$

or, $(n-1) \leq \frac{432}{20}$

i.e. $n \leq \frac{452}{20} = 22.6$

$\therefore \boxed{n = 22 \text{ terms}}$

$\therefore T_n = 7 + (22-1) \times 20$

$= 7 + 21 \times 20 = 420 + 7$

i.e. $\boxed{T_n = 427.}$

(2.) The number of terms common between the two series $2+5+8+\cdots$ to 50 terms & $3+5+7+\cdots$ to 60 terms is ---

(a) 24 (b) 26 (c) 25 (d) NOT.

Solnº:- Last term of first A.P., $= 2+(50-1)\times 2$

$= 147+2 = 149$

Last term of second A.P., $= 3+59\times 2$

$= 121$

$\therefore$ common difference of common term A.P.

$= d_1 \times d_2 = 3 \times 2 = 6$

So, common term A.P. is,

$5, 11, 17, 23, \cdots$

so, $T_n = 5+(n-1)6 = 5+6n-6 = 6n-1$

$\therefore \quad 6n-1 \le 121 \Rightarrow n \le \frac{122}{6}$

i.e. $n \le 20.33$

Hence, $\boxed{n = 20.}$

So, option 'D'.

ARITHMETIC MEAN :-

→ Arithmetic mean betⁿ two numbers 'a' & 'b' is

$$\boxed{A = \frac{a+b}{2}}.$$

Proof :- $a, A, b \Rightarrow A-a = b-A$

or, $2A = a+b$

i.e. $$\boxed{A = \frac{a+b}{2}}$$

Inserting n AMs between 'a' & 'b' :-

→ Let, $A_1, A_2, A_3, \ldots, A_n$ are n A.M.s bet^n a & b

$\therefore$ $a, A_1, A_2, A_3, \ldots A_n, b$ are in A.P.

So, $b = a + (n+2-1)d \Rightarrow b - a = (n+1)d$

$$\therefore \boxed{d = \frac{(b-a)}{(n+1)}}$$

So, $A_1 = a + d = a + \frac{b-a}{n+1} = \frac{an + \cancel{a} + b - \cancel{a}}{(n+1)}$

$$\text{i.e. } \boxed{A_1 = \frac{an+b}{n+1}}$$

$A_2 = a + 2d = a + \frac{2(b-a)}{n+1} = \frac{an+a+2b-2a}{(n+1)}$

$$\therefore \boxed{A_2 = \frac{(n-1)a + 2b}{(n+1)}}$$

$A_3 = a + 3d = a + \frac{2(b-a)}{n+1} = \frac{an+a+3b-3a}{(n+1)}$

$$\therefore \boxed{A_3 = \frac{(n-2)a + 2b}{(n+1)}}$$

$$\therefore \boxed{A_K = \frac{(n-K+1)a + Kb}{(n+1)}}$$

Note :- If A be the single A.M. bet^n two quantities & $A_1, A_2, A_2, \ldots A_n$ be 'n' AMs bet^n the same quantity then $\boxed{\sum_{i=1}^{n} A_i = nA}$

→ If there are 'n' AMs bet^n two quantities then sum of the AMs equidistant from the begining & the end is constant & is equal to $(a+b)$.

i.e. $A_1 + A_n = a + b = A_2 + A_{n-1} = \cdots$

Q. If $a_k = \dfrac{1}{k(k+1)}$, for $k = 1, 2, 3, \ldots, n$ then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

(A) $\dfrac{n}{n+1}$ (B) $\dfrac{n^2}{(n+1)^2}$ (C) $\dfrac{n^4}{(n+1)^4}$ (D) $\dfrac{n^6}{(n+1)^6}$.

sol^n:-

$$a_k = \frac{(k+1) - k}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

$$\therefore \sum_{k=1}^{n} a_k = \left(\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cdots + \cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

$$= \frac{n}{n+1}$$

i.e. $\left(\sum_{k=1}^{n} a_k\right)^2 = \dfrac{n^2}{(n+1)^2}$

Q. 20 AMs $A_1, A_2, A_3, \ldots, A_{20}$ are inserted bet^n 10 & 42, find A_{12}.

sol^n:- $10, A_1, A_2, A_3, \ldots, A_{20}, 42$

$$\therefore 42 = 10 + (22-1)d \Rightarrow d = 32/21$$

$$\text{so, } A_{12} = A + 12d$$

$$= 10 + \left(12 \times \frac{22}{21}\right)$$

$$\therefore A_{12} = \frac{198}{7}$$

Some important Questions on A.P. :-

Q. For, any three positive real nos. a, b & c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$ then

(a) a, b, c are in A.P. (b) b, c, a are in A.P. (c) c, a, b are in A.P. (d) NOT.

Solⁿ :- Multiplying by '2' on both sides of the given eqⁿ

$$2\left(9a^2(25)+9b^2+25c^2-3\times25ac\right)=(15\times3ab+15bc)\times2$$

or, $(9b^2-30bc+25c^2)+(9b^2-90ab+225a^2)+(25c^2-150ac+225a^2)=0$

or, $(3b-5c)^2+(3b-15a)^2+(5c-15a)^2=0$

$\therefore\ 3b=5c,\ 3b=15a,\ 5c=15a$

$\Rightarrow 3b=5c=15a$

or, $b=5a$
$c=3a$

so, $5a, 3a, a$ are in A.P.
$\downarrow b \quad \downarrow c \quad \rightarrow a$

so, $\boxed{b, c, a \text{ are in A.P.}}$ i.e. option 'b'.

Q. Sum of first 'm' terms of an A.P. = sum of next 'n' terms of an A.P. & sum of first 'm' terms of an A.P. = sum of next 'p' terms of an A.P. Then pt: $(m+n)\left(\frac{1}{p}-\frac{1}{m}\right)=(m+p)\left(\frac{1}{n}-\frac{1}{m}\right)$

Solⁿ :- A/c to question, $S_{m+n}=2S_m$

$$\Rightarrow \frac{m+n}{2}\{2a+(m+n-1)d\}=2\times\frac{m}{2}\{2a+(m-1)d\}$$

$$\Rightarrow (m+n)\{2a+(m+n-1)d\}=2m\{2a+(m-1)d\} \quad \text{--- (1)}$$

let $2a+(m-1)d=x$

So, eq^n (I) becomes.

$$(m+n)(x+nd) = 2mx \Rightarrow (m+n)nd = (m-n)x \quad \text{—— (II)}$$

Similarly, $S_{m+p} = 2S_m$

$$\therefore (m+p)pd = (m-p)x \quad \text{—— (III)}$$

(ii) $\div$ (iii), we get

$$\frac{(m+n)n}{(m+p)p} = \frac{(m-n)}{(m-p)} \Rightarrow (m+n)(m-p)n = p(m+p)(m-n)$$

$$\therefore \boxed{(m+n)\left(\frac{1}{p} - \frac{1}{m}\right) = (m+p)\left(\frac{1}{n} - \frac{1}{m}\right)}$$

Hence proved.

JEE

Q Let p & q be the roots of the eq^n $x^2 - 2x + A = 0$ & Let 'r', 's' be the roots of the eq^n $x^2 - 18x + B = 0$. If $p < q < r < s$ are in A.P., then $A = \ldots$, $B = \text{——}$.

Sol^n:- Let, four terms in A.P. be, $\underset{p}{a-3d}, \underset{q}{a-d}, \underset{r}{a+d}, \underset{s}{a+3d}$

$$\therefore p+q = 2 \text{ \& } r+s = 18 \Rightarrow p+q+r+s = 20$$

$$\text{i.e. } 4a = 20 \Rightarrow \boxed{a = 5}$$

$$\text{Also, } p+q = 2 \Rightarrow 2a - 4d = 2 \Rightarrow a - 2d = 1$$

$$\therefore \boxed{d = 2}$$

$$\text{so, } p = -1, \; q = 3, \; r = 7, \; s = 11$$

$$\text{so, } A = p \times q = (-1) \times 3 = -3$$

$$\boxed{A = -3}$$

$$\text{\& } B = r \times s = 7 \times 11 = 77$$

$$\text{so, } \boxed{B = 77}$$

JEE

Q. The real nos x_1, x_2, x_3 satisfying the eqn. $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which β & γ lie?

Soln:. Let, $x_1 = a-d$, $x_2 = a$, $x_3 = a+d$.

$$\therefore \quad 3a = 1 \Rightarrow \boxed{a = 1/3}$$

Also, $a(a-d)(a+d) = -\gamma \Rightarrow a(a^2-d^2) = -\gamma$

or $\frac{1}{3}\left(\frac{1}{9} - d^2\right) = -\gamma \Rightarrow \frac{1}{27} - \frac{1}{3}d^2 = -\gamma$

or $\frac{1}{27} + \gamma = \frac{1}{3}d^2 \geq 0 \Rightarrow \gamma \geq -\frac{1}{27}$

so $\boxed{\gamma \in \left[-\frac{1}{27}, \infty\right).}$

Also, $a(a-d) + (a-d)(a+d) + a(a+d) = \beta$

or $a^2 - \cancel{ad} + a^2 - d^2 + a^2 + \cancel{ad} = \beta \Rightarrow 3a^2 - d^2 = \beta$

or $\frac{1}{3} - d^2 = \beta \Rightarrow d^2 = \frac{1}{3} - \beta \geq 0$ so, $\beta \leq \frac{1}{3}$

i.e. $\boxed{\beta \in \left(-\infty, \frac{1}{3}\right].}$

JEE

Q. The fourth power of the common difference of an A.P. with integer entries is added to the product of any four consecutive terms of it. Prove that resulting sum is square of an integer.

Soln:. let, $a-3d, a-d, a+d, a+3d \Rightarrow 2d = \text{c.d} = \text{int.}$

$$\therefore P = (2d)^4 + (a-3d)(a-d)(a+3d)(a+d)$$

$$= (a^2-5d^2)^2$$

$$\therefore a^2 - 5d^2 = a^2 - 9d^2 + (2d)^2 = \underset{\text{int}}{\underset{\downarrow}{(a+3d)}}\underset{\text{int}}{\underset{\downarrow}{(a-3d)}} + \underset{\text{int}}{\underset{\downarrow}{(2d)^2}}$$

So, $a^2 - 5d^2 = \text{int.}$

Hence, $\boxed{P = (\text{int.})^2}$