SEQUENCE AND SERIES :-

Sequence:- An ordered selection with identified first term, second term & so on ---

ex :- 1, 3, 5, 7, ----

Note:- Minm. no. of terms to be a sequence = 3.

Series:- Represented sum of the sequence is series.

ex :- If 1, 2, 3, 4, 5, --- be the sequence then

1 + 2 + 3 + 4 + 5 + --- be the series.

Progression :- progression is sequence with defined rule.

(i) A.P. (ii) G.P. (iii) H.P. (iv) A.G.P. (v) Misc.

(i) A.P. (Arithmetic progression):-

Sequence whose terms increase or decrease by a fixed number is called A.P. fixed number is called 'Common difference'.

If $a \to$ first term & $d \to$ c.d.

So, the sequence be,

$a,\ a+d,\ a+2d,\ a+3d,\ ---,\ a+(n-1)d$

→ $T_n = a+(n-1)d$ is also known as n'th term of the A.P. or General term.

$\therefore T_2 = a+d,\quad T_6 = a+5d,\quad T_{50} = a+49d.$

→ General rule for any sequence to be an A.P.

$T_n - T_{n-1} = d \ \forall\ n \geq 2,\ n \in N$

→ $d>0$ (incr. AP), $d<0$ (decr. A.P.) & $d=0$ (const. AP)

Sum of 'n' terms of A.P. :-

Let $S_n = a + (a+d) + (a+2d) + --- + (a+(n-1)d)$ —— (i)

(where $a+(n-1)d \to l$)

$S_n = l + (l-d) + (l-2d) + --- + a$ —— (ii)

Adding both (I) & (II) $\Rightarrow$ $2S_n = \underbrace{(a+l) + (a+l) + \cdots + (a+l)}_{n \text{ terms}}$.

So, $2S_n = n(a+l)$ $\Rightarrow$ $\boxed{S_n = \frac{n}{2}(a+l)}$

Also, $S_n = \frac{n}{2}(a + a + (n-1)d)$ i.e. $\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$

→ Sum of first 'n' natural no. $= \boxed{n(n+1)/2}$

→ Sum of first 'n' odd nos $= \boxed{n^2}$

→ Sum of first 'n' even nos $= \boxed{n(n+1)}$

Q In an A.P. if $a_2 + a_5 - a_3 = 10$ & $a_2 + a_9 = 17$ then find the first term & common difference.

Soln:- Let $a \to$ first term & $d \to$ c.d.

$\therefore$ $a_2 + a_5 - a_3 = 10 \Rightarrow (a+d) + (a+4d) - (a+2d) = 10$

or, $a + 3d = 10$ --- (I)

& $a_2 + a_9 = 17 \Rightarrow 2a + 9d = 17$ --- (II)

Solving (I) & (II),

$a + 2d = 10) \times 2$
$2a + 9d = 17$
$-3d = 2$
$\boxed{d = -1}$

& $a = 10 - 3d = 10 + 2 = 12$

So, $\boxed{a = 12}$

$\therefore$ $\boxed{\begin{array}{l}\text{first term} = 12 \\ \text{\& c.d.} = -1.\end{array}}$

Q. If 16^{th} & 11^{th} term of an A.P. are resp. 17 & 32. Find T_{20}.

→ Students must try this.

<u>Props</u>:-

→ If each term of an A.P. is increased or decreased by a fixed no. then the resulting seq. is also an A.P. with same c.d.

→ If each term of an A.P. is multiplied by a same non-zero no. then the resulting sequence is A.P. with c.d. = k × d (where, 'k' be the fixed no. & 'd' be the c.d. of original A.P.).

→ 3 no.s can be taken in AP, $a-d, a, a+d$

→ 4 " " " " " " , $a-3d, a-d, a+d, a+3d$

→ 5 " " " " " " , $a-5d, a-2d, a+d, a+2d, a+5d$

→ For any series $T_n = S_n - S_{n-1}$, where T_n is a linear fn of 'n' & S_n is a quad. fn of 'n'.

$$S_n = T_1 + T_2 + T_3 + \cdots + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + \cdots + T_{n-1}$$

$\Rightarrow$ $\boxed{S_n - S_{n-1} = T_n}$,(valid of any types of progressions).

So, $T_1 = S_1$, $T_2 = S_2 - S_1$ etc.

→ From 'T_n' we can see that co-efficient of 'n' is 'd' i.e. c.d. & from 'S_n' we can see that coeff. of n^2 is '$d/2$'.

→ If a, b, c are in A.P. $\Rightarrow$ $b-a = c-b$

i.e. $2b = a+c$ $\Rightarrow$ $\boxed{b = \frac{a+c}{2}}$

Q If an A.P. has $T_m = 1/n$ & $T_n = 1/m$ then, find S_{mn}.

Soln:- Here, $T_m = 1/n \Rightarrow a + (m-1)d = 1/n$ —— (i)

& $T_n = 1/m \Rightarrow a + (n-1)d = 1/m$ —— (ii)

Solving (i) & (ii), $a = d = 1/mn$

So, $S_{mn} = \frac{mn}{2}\left[2a + (mn-1)d\right]$

$= \frac{mn}{2}\left[\frac{2}{mn} + 1 - \frac{1}{mn}\right]$

So, $\boxed{S_{mn} = \frac{1}{2}(mn+1)}$.

Q The sum of 1st 3 terms of an A.P. is 27 and the sum of their square is 293. Find S_n & T_n of A.P.

Solⁿ:- Let 3 terms in AP be $a-d, a, a+d$ & try to solve it.

Q. The sum of 'n' terms of two A.P.s are in the ratio $(7n+1):(4n+27)$ find the ratio of their 11th term.

Solⁿ:- $$\frac{(S_n)_1}{(S_n)_2} = \frac{7n+1}{4n+27} \Rightarrow \frac{\frac{n}{2}(2a_1+(n-1)d_1)}{\frac{n}{2}(2a_2+(n-1)d_2)} = \frac{7n+1}{4n+27}$$

or $$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

$$\therefore \frac{n-1}{2} = 10 \Rightarrow n = 21$$

i.e. $$\frac{a_1+10d_1}{a_2+10d_2} = \frac{7\times21+1}{4\times21+27} = \frac{148}{111}$$

so, $$\boxed{\frac{(T_{11})_1}{(T_{11})_2} = \frac{148}{111}}$$

Q How many terms of the seq. $20 + 19\frac{1}{3} + 18\frac{2}{3} + \cdots$ must be taken, so that their sum is 300.

→ Students must try this.

Q Find the sum of all the integers betⁿ 1 to 100 which are divisible by 2 or 3.

Solⁿ:- $(2+4+6+\cdots+98) + (3+6+9+\cdots+99) - (6+12+18+\cdots+96)$

$= 2450 + 1683 - 816$

$= \boxed{3317.}$

Q. Find the value of $S = 100^2 - 99^2 + 98^2 - 97^2 + \cdots + 2^2 - 1^2$.

Sol^n:- Here, $S = 100^2 - 99^2 + 98^2 - 97^2 + \cdots + 2^2 - 1^2$

$= (100+99)(100-99) + (98+97)(98-97) + \cdots + (2+1)(2-1)$

$= 100 + 99 + 98 + 97 + \cdots + 2 + 1$

$= \frac{100 \times 101}{2}$

$= \boxed{5050}$

Q. For any three positive real nos. a, b & c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a+c)$. Then

(a) a, b, c are in A.P. (b) b, c, a are in A.P. (c) c, a, b are in A.P.
(d) NOT.

Sol^n:- $2(9 \times 25a^2 + 9b^2 + 25c^2 - 3 \times 25ac) = (15 \times 3ab + 15bc) \times 2$

or $(9b^2 - 30bc + 25c^2) + (9b^2 - 90ab + 225a^2) + (25c^2 - 150ac + 225a^2) = 0$.

or $(3b - 5c)^2 + (3b - 15a)^2 + (5c - 15a)^2 = 0$

$\therefore 3b = 5c,\ 3b = 15a,\ 5c = 15a \Rightarrow 3b = 5c = 15a$

or $b = 5a,\ c = 3a$

$\therefore 5a, 3a, a \rightarrow$ A.P.

So, $\boxed{\text{option 'b'}}$

Q. Sum of first 'm' terms of an A.P. = sum of next 'n' terms of an AP & sum of first 'm' terms of an A.P. = sum of next 'p' terms of an A.P. then Pt: $(m+n)\left(\frac{1}{p} - \frac{1}{m}\right) = (m+p)\left(\frac{1}{n} - \frac{1}{m}\right)$.

Sol^n:- $S_{m+n} = 2S_m \Rightarrow (m+n)\{2a + (m+n-1)d\} = 2m\{2a + (m-1)d\}$

III^{ly}, $(m+p)\{2a + (m+p-1)d\} = 2m\{2a + (m-1)d\}$

let, $2a + (m-1)d = x$

So, $(m+n)nd = (m-n)x$ & $(m+p)pd = (m-p)x$

Dividing both,

$\boxed{(m+n)\left(\frac{1}{p} - \frac{1}{m}\right) = (m+p)\left(\frac{1}{n} - \frac{1}{m}\right)}$ PD//.