

Definite Integrals, Area under the Curve and Differential Equations:

1. Evaluate :  $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$
2. Evaluate :  $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
3. Show that  $\int_0^{\pi} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2\pi}$ .
4. Evaluate :  $\int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$
5. Evaluate :  $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$
6. Evaluate:  $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x} dx$
7. Evaluate:  $\int_0^5 (|x-2| + |x-3| + |x-5|) dx$
8. Evaluate :  $\int_0^{\frac{\pi}{2}} \log \sin x dx$
9. Evaluate :  $\int_1^3 (3x^2 + 2x) dx$  as limit of a sum
10. Evaluate:  $\int_0^{\frac{3}{2}} (|x \cos \pi x|) dx$
11. Using integration, find the area of the region bounded by the parabola  $y^2 = 4x$  and the circle  $4x^2 + 4y^2 = 9$ .
12. Find the area bounded by the lines  $x + 2y = 2$ ,  $y - x = 1$  and  $2x + y = 7$ .
13. Find the area of the region enclosed between the two circles:  
 $x^2 + y^2 = 4$ ,  $(x-2)^2 + y^2 = 4$
14. Using the method of integration find the area bounded by the curve  $x^2 = 4y$  and the line  $x = 4y - 2$ .
15. Find the area of the region enclosed between the two circles:  
 $x^2 + y^2 = 4$ ,  $(x-2)^2 + y^2 = 4$
16. Using integration, find the area of the region bounded by the parabola  $y = x^2$  and  $y = |x|$ .
17. Using integration, find the area of the region:  
 $\{(x, y): x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\}$ .
18. Using integration, find the area of the region:  $\{(x, y): 4x^2 + 4y^2 \leq 9 \text{ and } y^2 \leq 4x\}$ . using method of integration.
19. Find the area of the region included between the parabola  $4y = 3x^2$  and the line  $2y = 3x + 12$
20. Find the area of the region included between the parabola  $y^2 = 2x$  and the line  $y - x = 4$ .

21. Using integration, find the area of the region:

$$\{(x, y): 4x^2 + 4y^2 \leq 9 \text{ and } 4y^2 \leq 9\}.$$

22. Find the particular solution of the differential equation :

$$(\tan^{-1} y - x) dy = (1 + y^2) dx, \text{ given that } x = 0, y = 0.$$

23. Show that the differential equation  $\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x dy = 0$  is homogeneous.

Find the particular solution of this differential equation, given that  $x = 1$  and  $y = \frac{\pi}{4}$ .

24. Find the particular solution of the differential equation :

$$\frac{dy}{dx} + x \cot y = 2y + y^2 \cot y, y \neq 0, \text{ given that } x = 0 \text{ and } y = \frac{\pi}{2}.$$

25. Prove that the differential equation  $x^2 \frac{dy}{dx} - xy = 1 + \cos \frac{y}{x}, x \neq 0$  is homogeneous.

Find the particular solution of this differential equation, given that  $x = 1$  and  $y = \frac{\pi}{2}$ .

26. Show that the differential equation  $(x - y) \frac{dy}{dx} = x + 2y$  is homogeneous and solve it.

27. Solve the differential equation :  $\frac{dy}{dx} = 1 + x + y + xy$ , given that  $y = 0$  when  $x = 1$ .

28. Find the particular solution of the differential equation :  $x(1+y^2) dx - y(1+x^2) dy = 0$ , given that  $y = 1, x = 0$ .

29. Find the particular solution of the differential equation :

$$x \log\left(\frac{dy}{dx}\right) = 3x + 4y, \text{ given that } y = 0, x = 0$$

30. Show that the differential equation  $x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin \frac{y}{x} = 0$  is homogeneous.

Find the particular solution of this differential equation, given that  $x = 1$  and  $y = \frac{\pi}{2}$ .