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CLASS: XII MATHEMATICS

Application of Derivative and Indefinite Integrals

1. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2=1$.
2. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.
3. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere
4. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of base
5. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find maximum volume.
6. The Contentment obtained after eating x-units of a new dish at a trial function is given by the Function C(x) = $x^3 + 6x^2 + 5x = 3$.If the marginal contentment is defined as rate of change of C(x) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.
7. Sand is pouring from a pipe at the rate of $12cm^3/sec$ .the falling sand form a cone on the ground in such a way that the height is always one-sixth of the radius of the base . How fast is the height of the sand cone increasing, when the height is 4cm ?
8. Find the interval in which the function i) $f(x) =3x^4-4x^3-12x^2+5$

ii) $f(x)=\sin^4 x +\cos^4 x$ on Istqudrnt (a) strictly increasing (b) strictly decreasing.

9. A ladder 5m long is leaning against a wall, the bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 4m away from the wall.
10. Using differential, find the approximate value of $\sqrt{49.5}$, $\sqrt{0.037}$, $\sqrt{0.0036}$
11. Find the equation of tangent to the curve y= $\frac{x-7}{(x-2)(x-9)}$ at the point where it cuts the x-axis
12. a)For the curve $y = 4x^3-2x^5$, find all the points at which the tangent passes through the origin.

    b) Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).
13. Find the approximate change in the volume v of a cube of side x metres caused by increasing the side by 2%.

14. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3, If building of tank costs Rs. 70 per sq. metre for base and Rs. 45 per sq. metre for sides, what is the cost of least expensive tank?

15. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

16. Show that semi vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}(\frac{1}{3})$.

17. Evaluate : $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

18. Evaluate : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$

19. Evaluate : $\int \frac{\sin x}{(1-\cos x)(2-\cos x)}dx$

20. Evaluate : $\int \log(\sin x)dx$

21. Evaluate : $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx$

22. Evaluate : $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$

23. Evaluate : $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

24. Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

25. Evaluate : $\int sinx\ sin2x\ sin3x\ dx$

26. Evaluate : $\int (x-3)(\sqrt{x^2+3x-18})\ dx$

27. Evaluate : $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

28. Evaluate : $\int \frac{dx}{\sin x+\sin 2x}$

29. Evaluate : $\int e^{2x}\frac{1-sin2x}{1-cos2x}dx$

30. Evaluate : $\int \sqrt{\frac{a-x}{a+x}}dx$