FIRST TERMINAL EXAMINATION, 2016
MATHEMATICS

Time : 3 hrs. **Class - XII** **M.M. : 100**

Date : 19.09.2016

General Instructions :

- All questions are compulsory.
- The question paper consists of **29 questions** divided into **four** sections A, B, C and D.

  Section A contains **04 questions** of **1 mark** each, **Section B** contains **8 questions** of **2 marks** each, **Section C** contains **11 questions** of **4 marks** each and **Section D** contains **6 questions** of **6 marks** each.
- There is no overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. Attempt only one of the alternatives in all such questions.
- Use of calculator is not allowed.

SECTION : A

Q.1 For the set $\{1,2,3\}$ define a relation R as $\{(1,1)(2,2)(3,3)(1,3)\}$. Write the ordered pair to be added to R to make it the smallest equivalence relation.

Q.2 Find the principal value of $\sin^{-1}\left(\cos\left(\frac{13}{5}\pi\right)\right)$

Q.3 If $A = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then find AA^T.

Q.4 Find the area of the triangle with vertices (5,4), (- 2,4) and (2, - 6)

SECTION : B

Q.5 Evaluate: $\int \frac{dx}{\sin x \cos^3 x}$

Q.6 Integrate: $\frac{\sin x \cos^3 x}{\sqrt{7-6x-x^2}}$

Q.7 A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Q.8 Use differentials to find the approximate value of $(25)^{\frac{1}{3}}$

Q.9 Differentiate: $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Q.10 A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of Rs.1800

Q.11 Prove that: $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{(1-3x^2)}, |x| < \frac{1}{\sqrt{3}}$

Q.12 Given a non-empty set X, let $*$: P(X) × P(X) → P(X) be defined as A * B = (A – B) ∪ (B – A), ∀A, B ∈ P(X). Show that the empty set φ is the identity for the operation $*$ and all the elements A of P(X) are invertible with $A^{-1} = A$.

SECTION : C

Q.13 If the function $f: R \to R$ be given by $f(x) = x^2 + 2$ and $g: R \to R$ given by $g(x) = \frac{x}{x-1}$, find fog and gof

Q.14 Simplify: $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$, where $-\frac{3\pi}{4} \leq x \leq \frac{\pi}{4}$

OR

Prove that: $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2, -1 < x < 1$

Q.15 If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$ then show that $(A+B)^2 = A^2 + B^2$.

Q.16 Find A^{-1}, if $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2 - 3I}{2}$

OR

Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

Q.17 Find the value of 'k' for which the function: $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if\ -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & if\ 0 \leq x < 1 \end{cases}$ is continuous aat $x = 0$

Q.18 If $x = e^{x/y}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$

OR, If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

Q.19 If $x = 3\cos\theta - \cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$ find $\frac{dy}{dx}$ at θ for which $\sin\theta = \cos\theta, 0 < \theta < \frac{\pi}{2}$

Q.20 If $x = 3^{\cos 2t}$, $y = 3^{\sin 2t}$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Q.21 If $y = x - x^2$, prove that the sides of two squares such that the sides of the second square are x and y, find the rate of change of the area of square w.r. to the first when $x = 2$ units.

Q.22 Integrate: $\frac{1-x^2}{x(1-2x)}$

Q.23 Evaluate: $\int \frac{x(1-x^2)}{\sqrt{x}\sqrt{x+1}}\,dx$

OR

$\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$

SECTION : D

Q.24 Let $A = \{1,2,3,4,5\}$ (... ... 9} and R be the relation defined in AXA defined by $(a,b)R(c,d)$ if $a+d=b+c$ for $(a,b),(c,d)$ in AXA. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$.

OR

Let X be a non-empty set and let '*' be a binary operation on P(X), the power set of X defined by A*B=$A \cap B$ for A, B ϵ P(X).

i) Find the identity element w.r.to * in P(X)

ii) Show that X is the only invertible element of P(X).

ii) Show ment of P

Q.25 If $x+y+z=0$, then prove that $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Q.26 Find the points on the curve $\sqrt{x}+\sqrt{y}=4$ at which the tangents are equally inclined to the coordinate axes. Also find the equations of the tangents at those points.

OR

Find the equations of the normal lines to the curve $3x^2-y^2=8$ which are parallel to the line $x+3y=4$.

Q.27 L $+3y=4$. er of a circle and C be any point on the circle, then show that the area of ΔABC is maximum when it is isosceles.

OR

Among all the rectangles that can be inscribed in a given fixed circle, prove that its area is maximum when it is a square.

Q.28 Integrate: $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx$

Q.29 A manufacturer considers that men and women are equally efficient so he pays both of them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital are required to produce one unit of B. If A and B are priced at Rs. 100 and Rs. 200 per unit respectively, how should he use his resources to maximise the total revenue? Form a LPP and solve it graphically.

Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?