

FIRST TERMINAL EXAMINATION, 2016

MATHEMATICS

Time : 3 hrs.

Class XI

M.M. : 100

Date – 14.09.2016

General Instructions :

- All questions are compulsory.
- This question paper is divided into three sections, **Section A** contains **4 questions** each carrying **1 mark**, **Section B** contains **8 questions** each carrying **2 marks**, **Section C** contains **11 questions** each carrying **4 marks** and **Section D** contains **6 questions** each carrying **6 marks**.
- Question Nos. **18 and 28** must be attempted in **graph paper**.
- Graph paper will be provided to you.
- Use of calculator or any other electronic devices is not allowed.

SECTION - A

- Q.1** In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?
- Q.2** Let $= \{(x, y) : x, y \in W, y = 2x - 4\}$. If $(a, -2)$ and $(4, b^2)$ belong to R, find the value of 'a' & 'b'.
- Q.3** Find the value of $\tan \frac{10\pi}{12}$.
- Q.4** Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$.

SECTION - B

- Q.5** Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.
- Q.6** Find the domain and range of following relation: $R = \left\{ \left(x, \frac{1}{x} \right) : x \text{ is an integer. } 0 < x < 6 \right\}$.
- Q.7** Convert 6 radians into degree measure.
- Q.8** Prove the following: $\tan 4x = \frac{4 \tan x - 4 \tan^3 x}{\tan^4 x - 6 \tan^2 x + 1}$
- Q.9** For any triangle ABC, prove that: $(b + c) \cos \frac{B+C}{2} = a \cos \frac{B-C}{2}$.
- Q.10** Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{\left(x - \frac{\pi}{2}\right)}$.
- Q.11** Find the derivative of $\frac{\sec x - 1}{\sec x + 1}$.
- Q.12** Solve: $\frac{1}{4} \left(\frac{2}{3}x + 1 \right) \geq \frac{1}{3}(x - 2), x \in R$.

SECTION - C

- Q.13** Let A and B be sets. If $A \cap X = B \cap X = \emptyset$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.
- Q.14** Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Are the following true? Justify your answer.
- $(a, a) \in R, \text{ for all } a \in N$
 - $(a, b) \in R, \text{ implies } (b, a) \in R$
 - $(a, b) \in R, (b, c) \in R \text{ implies } (a, c) \in R$

- Q.15** Let m be a given fixed positive integer.
let $R = \{(a,b) : a,b \in \mathbb{Z} \text{ and } (a-b) \text{ is divisible by } m\}$
show that R is an equivalence relation on \mathbb{Z} .

OR

Determine the range of the following function $f: \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbb{R} \right\}$.

- Q.16** Prove that: $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

OR

Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

- Q.17** State and show by Venn diagram 'DeMorgan's law'.

- Q.18** Draw the graph of $\tan x$, where $-\pi \leq x \leq 2\pi$.

- Q.19** In any triangle ABC, prove that $a \sin(B - C) + b \sin(C - A) + c \sin(A - B) = 0$

OR

Prove that : $\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$

- Q.20** A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

- Q.21** Evaluate : $\lim_{x \rightarrow 1} \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1}$.

- Q.22** Find the derivative of $\frac{x^2 \sin x}{1-x}$.

- Q.23** Find the general solution: $\sec x - \tan x = \sqrt{3}$

SECTION - D

- Q.24** In a class, 18 students offered physics, 23 offered chemistry and 24 offered mathematics. Of these, 13 are in both chemistry and mathematics; 12 in physics and chemistry; 11 in mathematics and physics and 6 in all the three subjects.

Find:

- (i) how many students are there in the class
- (ii) how many offered mathematics but not chemistry,
- (iii) how many are taking exactly one of the three subjects.

- Q.25** Find the domain and range of the real function $f(x) = \frac{1}{(1-x^2)}$. Also find $\left\{ \frac{f(5) - f(1)}{(5-1)} \right\}$.

- Q.26** Prove that : $\frac{\cos x}{1 - \sin x} = \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$.

OR

Prove that : $(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4 \cos^2 \left(\frac{x-y}{2} \right)$.

- Q.27** Find the general solution of the equation: $\sin 2x + \sin 4x + \sin 6x = 0$.

- Q.28** Solve the following system of inequalities graphically and shade the solution region.

$$3x + 2y \leq 150, x + 4y \leq 80, x \leq 15, y \geq 0, x \geq 0.$$

- Q.29** Find the derivative of $f(x) = \tan x$ by the first principle.

OR

Find the derivative of $f(x) = x \sin x$ by the first principle.

