HALF YEARLY EXAMINATION, 2017-18

MATHEMATICS

Time : 3 hrs. **Class - XII** **M.M. : 100**

Name of the student ____________________________**Section** _____ **Date – 19.09.2017 (Tuesday)**

General Instructions :

- All questions are **compulsory**.
- This question paper is divided into four sections, **Section A** contains **4 questions** each carrying **1 mark**, **Section B** contains **8 questions** each carrying **2 marks, Section C** contains **11 questions** each carrying **4 marks** and **Section D** contains **6 questions** each carrying **6 marks**.
- **Question No. 28** must be attempted in **graph paper**.
- Graph paper will be provided to you.
- Use of **calculator** or any other **electronic devices** is **not allowed**.

SECTION-A

Q.1 Let $f : \{1, 3, 4\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down *gof*.

Q.2 Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

Q.3 If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix X, such that 2A + 3X = 5B.

Q.4 Evaluate the determinant $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$.

SECTION - B

Q.5 Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as ,

$$a * b = \begin{cases} a + b & if\ a + b < 6 \\ a + b - 6 & if\ a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element *a* of the set is invertible with (6–*a*) being the inverse of *a*.

Q.6 Show that $\sin^{-1}\left(2x\sqrt{1 - x^2}\right) = 2\sin^{-1} x, \quad -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$.

Q.7 Simplify: $\cos x \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \sin x \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$

Q.8 Find $\frac{dy}{dx}$ of the given function: $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$.

Q.9 The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres ?

Q.10 Find the intervals in which the function f given by $f(x) = sin\,x + cos\,x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Q.11 Integrate the function: $\int (x^3 - 1)^{\frac{1}{3}} x^5\, dx$.

Q.12 Integrate the function: $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$.

SECTION-C

Q.13 Let $f : R \rightarrow R$ be the Signum Function defined as $f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$

and $g : R \rightarrow R$ be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then, does fog and gof coincide in (0, 1]?

Q.14 Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$

OR

Find the value of the following:

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right] \quad |\,x\,| < 1,\ y > 0\ and\ xy < 1.$

Q.15 Obtain the inverse of the following matrix using elementary rows operations,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Q.16 Show that: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1\right) = abc + bc + ca + ab$

OR

By using properties of determinants, show that: $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$

Q.17 Find all the points of discontinuity of f defined by $f(x) = |\,x\,| - |\,x + 1\,|$.

Q.18 Differentiate the function $y = (x\,cos\,x)^2 + (x \sin x)^{\frac{1}{x}}$ w.r.t. x.

OR

Differentiate the function $y = \left(x + \frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$ w.r.t. x.

Q.19 If $x = \sqrt{a^{\sin^{-1} t}}\ and\ \ y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

OR

If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, then show that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$

Q.20 If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2$

OR

If $y = 3\,cos\,(log\,x) + 4\,sin\,(log\,x)$, show that $x^2 y_2 + xy_1 + y = 0$

Q.21 Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

Q.22 Integrate the function: $\int \frac{x+3}{\sqrt{5-4x+x^2}}$ dx

Q.23 Integrate the function: $\int \frac{(3\sin\varphi - 2)\cos\varphi}{5-\cos^2\varphi - 4\sin\varphi}$ dφ

SECTION-D

Q.24 Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if A ⊂ B, (where ARB means A is related to B). Is R an equivalence relation on P(X)? Justify your answer.

OR

If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

Q.25 Show that the semi-vertical angle of the right circular cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Q.26 Show that the normal at any point θ to the curve

$x = a\,cos\theta + a\,\theta\,sin\,\theta,\ y = a\,sin\theta - a\theta\,cos\theta$ is at a constant distance from the origin.

OR

Prove that the curves $x = y^2$ and $xy = k$ cut at right angles, $if\ 8k^2 = 1$.

Q.27 Integrate the function: $\int \frac{x^2+x+1}{(x+1)^2(x+2)}$ dx

Q.28 A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

Q.29 If a + b + c = 0 and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

then, show that x = 0 or x = $\sqrt{(3/2)(a^2+b^2+c^2)}$